

# LATTICE OPTIONS FOR A 5 GEV LIGHT SOURCE AT CORNELL\*

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## Abstract

Cornell University has proposed an Energy Recovery Linac (ERL) based synchrotron-light facility [1] which can provide improved X-ray radiation due to the high beam quality which is available from a linac. To additionally utilize beam currents that are competitive with ring-based light sources, the linac has to operate with the novel technique of energy recovery. Cornell plans to address the outstanding issues of high-current injector, higher-order mode damping and extraction from superconducting RF environment, etc. in a downscaled prototype ERL prior to submitting a proposal for a full-scale machine. The flexibility of linacs allows for different modes of operating the ERL X-ray source, each of which requires specific manipulations of longitudinal phase-space that restrict the choice of the lattice. Here we discuss the different proposed modes of operating the ERL X-ray source and present options for corresponding lattices.

## INTRODUCTION

Linac-based accelerators have the potential to deliver beam of exceptional quality in terms of both transverse and longitudinal emittance. While the former is determined primarily by the properties of the electron source, the latter typically is an interplay of initial bunch length, RF waveform and beam optics with nonzero time of flight terms (sometimes referred to as momentum compaction).

To match the natural bandwidth of X-rays in the central cone from an  $N$ -period undulator, the fractional rms energy spread of the electron beam has to be  $\leq (5N)^{-1}$  (a factor of 2 here comes from the beam energy squared dependence of the emitted photon energy, and the additional factor is due to FWHM to rms conversion of the radiation bandwidth  $\sim 1/N$ ). This way spectral brightness in the fundamental is guaranteed to increase proportionally with the number of undulator periods. When linac-based accelerators reduce the beam energy spread to values below those achievable with storage rings, efficient use of very long undulators becomes possible, improving monochromaticity of X-rays.

The much shorter bunch length than that from storage rings is thought to enable new areas of ultra-fast X-ray science. Herein, we address longitudinal phase-space manipulations feasible in an ERL and consider various options for lattices and regimes of operation of such a light source.

The electron beam is created in a source [2], in which the space charge forces create an effective randomization of phase-space positions, so that we here assume a Gaussian

beam in the longitudinal phase-space between the injector system and the linac. In the Cornell project, beam energy is planned to be 10 MeV at that region. The bunch length and energy spread are planned to be  $\sigma_{z_0} = 0.64$  mm and  $\sigma_{\delta_0} = 10^{-3}$ . An energy recovering linac will accelerate this beam to 5 GeV. Magnetic lattice after the linac with adjustable time of flight terms, together with an off-crest acceleration of the beam, will then be used to reduce the bunch length to the desired amount. We herein compute limits on the achievable bunch length and provide analytical expressions that describe longitudinal dynamics in an ERL.

## ANALYTICAL EXPRESSIONS

Significant insight into the beam's longitudinal phase-space after the linac and the bunch compressor can be obtained by investigating higher-order transfer maps. These describe the change of the phase-space variables,  $\delta$  (fractional energy deviation from that of central ray) and  $z$  (difference of path lengths between a particle and that of central particle respectively) by a Taylor series. We will indicate phase-space coordinates between injector and linac by an index 0, after the linac by an index 1, and after the bunch compressor by an index 2. Retaining leading orders only, one writes:

$$\delta_1 \simeq \frac{E_0}{E_1} \delta_0 + \alpha_1 z_0 + \frac{\alpha_2}{2!} z_0^2 + \frac{\alpha_3}{3!} z_0^3 + \dots \quad (1)$$

The first term,  $\delta_0$ , is due to the uncorrelated energy spread after the injector, normalized by the full beam energy,  $\alpha_n$  is partial derivative at the location of the central ray:  $\alpha_n = \partial^n \delta_1 / \partial z_0^n |_{z_0=0}$ . Injected beam is accelerated from energy  $E_0$  to  $E_1$  according to  $E_1 - E_0 = E_{\max} \cos(\varphi + k_{RF} z_0)$ , i.e. coefficients  $\alpha_n$  are:

$$\begin{aligned} \alpha_{2n-1} &= (-1)^n \frac{E_{\max}}{E_1} k_{RF}^{2n-1} \sin \varphi, \\ \alpha_{2n} &= (-1)^n \frac{E_{\max}}{E_1} k_{RF}^{2n} \cos \varphi, \end{aligned}$$

here  $E_{\max}$  is the maximum energy gain in the linac,  $\varphi$  is the off-crest phase, and  $k_{RF} = 2\pi / \lambda_{RF} = 27 \text{ m}^{-1}$  is the wave number corresponding to the fundamental RF frequency. Note that we have transformed the particle position  $z$  into a time of flight term assuming that the velocity is  $c$  along the complete linac. Assuming a Gaussian beam between injector and linac with rms bunch length  $\sigma_{z_0} = \sqrt{\langle z_0^2 \rangle}$  of the injected bunch and no initial energy-length correlation, the rms energy spread  $\sigma_{\delta_1}^2 = \langle \Delta \delta_1^2 \rangle$ , with  $\Delta \delta_1 = \delta_1 - \langle \delta_1 \rangle$ , and emittance  $\epsilon_{z_1}^2 = \langle z_1^2 \rangle \langle \Delta \delta_1^2 \rangle - \langle z_1 \Delta \delta_1 \rangle^2$  after the linac become

$$\sigma_{\delta_1}^2 = \left( \frac{E_0}{E_1} \sigma_{\delta_0} \right)^2 + \alpha_1^2 \sigma_{z_0}^2 + \frac{\alpha_2^2 + 2\alpha_2 \alpha_3}{2} \sigma_{z_0}^4 + \dots,$$

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$$\epsilon_{z1}^2 = \left( \frac{E_0}{E_1} \sigma_{\delta_0} \right)^2 \sigma_{z0}^2 + \frac{\alpha_2^2}{2} \sigma_{z0}^6 + \dots$$

The energy spread from the injector,  $\frac{E_0}{E_1} \sigma_{\delta_0} \approx 2 \times 10^{-6}$ , is extremely small and will be neglected here,  $\sigma_{z0} = 0.64$  mm corresponds to  $\sigma = 0.017 = 1^\circ$  of the 1.3 GHz RF phase or 2.1 ps. The off-crest RF-phase  $\varphi \ll 1$ . Thus, for all practical cases of interest, energy spread and emittance are estimated as

$$\begin{aligned} \sigma_\delta^2 &\approx (\sigma\varphi)^2 + (\sigma^2/\sqrt{2})^2, \\ \epsilon &\approx \sigma^3/\sqrt{2}, \end{aligned} \quad (2)$$

here the bunch length and emittance without a subscript  $z$  are given in units of the RF phase, i.e  $\sigma = k_{RF}\sigma_z$  and  $\epsilon = k_{RF}\epsilon_z$ .

### Bunch compression

After an achromat, i.e. when only purely chromatic terms matter but transverse particle coordinates do not influence the time of flight, longitudinal phase-space transform using TRANSPORT notations is written as:

$$\begin{aligned} z_2 &= z_1 + R_{56}\delta_1 + T_{566}\delta_1^2 + U_{5666}\delta_1^3 + \dots, \\ \delta_2 &= \delta_1. \end{aligned} \quad (3)$$

Inserting  $\delta_1$  from equation (1) leads to the function  $z_2(z_0, \delta_0)$ . Ignoring the energy spread  $\frac{E_0}{E_1} \sigma_{\delta_0}$  due to the injector leads to

$$\begin{aligned} z_2 &= (1 + R_{56}\alpha_1)z_0 + (T_{566}\alpha_1^2 + R_{56}\frac{\alpha_2}{2})z_0^2 \\ &+ (U_{5666}\alpha_1^3 + T_{566}\alpha_1\alpha_2 + R_{56}\frac{\alpha_3}{6})z_0^3 + \dots \end{aligned}$$

Since the initial coordinates  $z_0$  and  $\delta_0$  are distributed by a Gaussian, the rms bunch length after the bunch compressor  $\sigma_{z2}^2 = \langle z_2^2 \rangle - \langle z_2 \rangle^2$  evaluates to

$$\begin{aligned} \sigma_{z2}^2 &\approx (1 + R_{56}\alpha_1)^2 \sigma_{z0}^2 + 2(T_{566}\alpha_1^2 + R_{56}\frac{\alpha_2}{2})\sigma_{z0}^4 \\ &+ 6(1 + R_{56}\alpha_1)(U_{5666}\alpha_1^3 + T_{566}\alpha_1\alpha_2 + R_{56}\frac{\alpha_3}{6})\sigma_{z0}^6. \end{aligned} \quad (4)$$

When the first and second order optics is chosen to minimize the bunch length,  $R_{56} = -1/\alpha_1$  and  $T_{566} = \alpha_2/(2\alpha_1^2)$ , the final bunching is given by

$$\sigma_{z2}^2 = 15(U_{5666}\alpha_1^3 + \frac{3\alpha_2^2 - \alpha_1\alpha_3}{6\alpha_1^2})^2 \sigma_{z0}^6. \quad (5)$$

After the third order bunch compression with  $U_{5666} = (\alpha_1\alpha_3 - 3\alpha_2^2)/(6\alpha_1^5)$  the leading order term becomes

$$\sigma_{z2}^2 = 96(V_{56666}\alpha_1^4 - \frac{15\alpha_2^3 - 10\alpha_1\alpha_2\alpha_3 + \alpha_1^2\alpha_4}{24\alpha_1^3})^2 \sigma_{z0}^8. \quad (6)$$

It is instructive to estimate various terms in expressions (4-6) in case of bunch compression to different order assuming typical ERL parameters, e.g.  $\sigma = 1^\circ$ ,  $\varphi = 10^\circ$ .

The leading order in the initial energy spread  $\sigma_{\delta_0}$  of the rms bunch length after the bunch compressor is neglected in equation (4) since it is very small. In case of linear bunch compression,  $R_{56} = -1/\alpha_1$ , it only amounts to  $R_{56}\frac{E_0}{E_1}\sigma_{\delta_0} \approx 2 \times 10^{-6}/(k_{RF}\varphi) = c \cdot 1.4$  fs. In case of an ideally linear compressor, i.e.  $T_{566} = U_{5666} = 0$ , equation (4) is estimated to  $\sigma_{z2} \approx \sigma^2/(\sqrt{2}k_{RF}\varphi) = c \cdot 150$  fs. When the second order time of flight term is chosen for second order bunch compression, equation (5) leads to  $\sigma_{z2} \approx \sqrt{15}\sigma^3/(2k_{RF}\varphi^2) = c \cdot 40$  fs. Expression (5) provides maximum compression for the parameters  $R_{56} \approx k_{RF}^{-1}/\varphi = 21$  cm,  $T_{566} \approx k_{RF}^{-1}/(2\varphi^3) = 3.4$  m. The third order bunch compression would be obtained for  $U_{5666} \approx k_{RF}^{-1}/(2\varphi^5) = 113$  m. Assuming  $V_{56666} = 0$ , equation (6) then leads to  $\sigma_{z2} \approx 15\sigma^4/(\sqrt{6}k_{RF}\varphi^3) = c \cdot 13$  fs.

Note that  $R_{56}$ ,  $T_{566}$ , and  $U_{5666}$  should be of the same sign. Although achieving this is very hard in a conventional chicane bunch compressor [3], it can be done with relatively small effort in the arc of a flexible storage ring [4].

It is also straightforward to estimate tolerances for the RF phase  $\varphi$  from (4). If the optics were adjusted to third order bunch compression for a phase  $\varphi$  and the phase varied by  $\Delta\varphi$ , the bunch length would change by  $(\frac{\Delta\sigma_{z2}}{\sigma_{z0}})^2 = (\frac{\Delta\varphi}{\varphi})^2 [1 + 8(\frac{\sigma}{\varphi})^2 + 2\sigma^2] \approx (\frac{\Delta\varphi}{\varphi})^2$ . Again, for the example at hand, assuming that tolerable bunch lengthening from RF phase errors is  $\Delta\sigma_{z2} = c \cdot 50$  fs, the allowed phase jitter is estimated to be  $0.2^\circ$ . Assuming state-of-the-art RF phase stability of  $0.1^\circ$ , the bunch length would be stable up to 20 fs. Since a better stability seems unrealistic, this suggests that a bunch compression of higher order than 2 will not lead to further improvements.

### Energy Recovery

Energy recovery requires that the accelerated bunch passes through the linac a second time with a phase shift of  $\pi$  with respect to the first pass. This puts constraints on the time of flight terms of the lattice. Ideally, the lattice should be isochronous (including higher orders), so that the RF waveform that the linac imposes in the longitudinal phase-space is cancelled when the bunch passes through the linac the second time.

For slight deviations from isochronicity, it is sufficient to analyze the energy distribution at the location of the beam dump, indicated by an index 3:

$$\begin{aligned} E_3 &= E_0 + 2E_{\max} \sin[\varphi + k_{RF}(z_0 + \frac{\Delta z}{2})] \sin k_{RF}\frac{\Delta z}{2}, \\ z_3 &= z_0 + \Delta z. \end{aligned} \quad (7)$$

Here  $\Delta z = z_2(z_0, \delta_0) - z_0$  is a position shift of a particle in the bunch that was initially at  $z_0$ , which it experiences due to nonzero time of flight terms of the recirculating arc.

If a lattice deviates significantly from the isochronous condition, the two RF waveforms will not cancel out and some parts of the bunch can have an energy deviation from the mean energy that is relatively large compared to the

ejection energy. This is especially severe for particles with less energy than the mean, since they can be over-focused and lost in the last section of linac. Such scenario can be avoided by proper control of the time of flight terms of the return arc.

An additional energy loss due to synchrotron radiation in the recirculating arc has to be included in equation (7), effectively lowering the injection energy  $E_0$ . In case when  $k_{RF}z_0 \ll \varphi \ll 1$ , the condition for successful energy recovery simplifies to  $-E_{\max}\varphi k_{RF}\Delta z \ll E_0$ , e.g. for  $E_{\max} = 5$  GeV,  $E_0 = 10$  MeV, and  $\varphi = 10^\circ$ , the shift  $\Delta z$  of any electron's position in the bunch after the return arc should be less than 0.4 mm. Note that the sign is important here, since only particles with too little energy are lost in the linac.

## LATTICE OPTIONS

### *Optics of the Return Arc*

The requirements for the optics of the return arc include: adjustable time of flight terms, dispersion-free regions for insertion devices, and low aberrations for emittance preservation. Different approaches were considered such as a lattice composed of identical achromats with adjustable  $R_{56}$ , etc. We favor a lattice design with non-periodic Twiss parameters, where negative dispersion of several meters is generated at certain locations, as opposed to a more conventional lattice made of compact achromatic cells with intrinsically small dispersion. Our choice allows effective control of higher-order time of flight terms with only a few moderately strong sextupoles for the whole return arc. Such an approach has been used in the design of an ERL in the CESR tunnel [4]. Furthermore, it was discovered that the only important terms for clean transport of the transverse phase-space for a beam with ERL parameters are the purely chromatic ones, thus, a lattice with corrected second-order dispersion  $T_{166}$  and its derivative  $T_{266}$  has sufficiently low aberrations to enable transport of low emittance ERL beam for all practical cases with virtually no emittance growth due to nonlinearities in transverse phase-space. Therefore, a second-order achromat where both geometric and chromatic aberrations cancel to zero is thought to be an unnecessary sophistication, especially since its qualities are achieved at the expense of relatively inflexible higher-order time of flight terms [3].

### *Linac optics*

Various linac configurations are under consideration for ERLs. These include single-pass straight-section RF structures, split linac arrangement, and multi-pass scenarios. Computer simulations of beam break-up instability [5], [6] suggest that a single-pass configuration is essential for a high-current ( $\sim 100$  mA) machine assuming state-of-the-art damping of higher-order modes in superconducting RF cavities, although for a lower current ERL a multi-pass scheme seems very attractive.

A single-pass straight-section linac is thought to have an advantage of containing a minimal number of bend sections and reducing the danger of coherent synchrotron radiation effects, i.e. the linac could be used to produce very short, high brightness electron bunches, which might be of use for certain applications (e.g. for SASE produced light). On the other hand, the split linac option allows better control of the transverse beam envelope due to smaller mismatch of the accelerated and decelerated beam energies in each of the linac sections. Furthermore, a split linac can allow advanced longitudinal phase-space manipulations, such as energy spread compression. It is clear from equation (2) that the minimum energy spread is achieved for on-crest running and scales as the bunch length squared. On the other hand, longer bunches are preferable since they produce less higher-order mode power. Also, longer bunches require less compression in the injector at low energies, leading to better transverse emittances.

As an illustration, consider a linac split in two parts separated by a transport line with adjustable time of flight terms. The first section is operated with nonzero phase  $\varphi$ , which generates a correlated energy spread. One then uses this spread to induce a positive quadratic term in  $\delta(z)$ . The second section runs with a phase of  $-\varphi$  removing both the linear and quadratic correlation and leading to a small energy spread after the second linac section. The optimum condition for the transport line separating the two linac parts in this case is given by  $R_{56} = 0$ ,  $T_{566} = \alpha_2/\alpha_1^3 \approx k_{RF}^{-1}/\varphi^3$  and  $U_{5666} = 0$ . The remaining leading order in the energy spread for this case, and with  $V_{56666} = 0$ , becomes  $\sigma_{\delta_1} \approx 5\sqrt{\frac{3}{2}}\frac{\sigma^4}{\varphi^2}$ . E.g. for a bunch length of  $\sigma = 1.5^\circ$  and  $\varphi = 10^\circ$ , one obtains  $\sigma_{\delta_1} = 9 \times 10^{-5}$ . For longer bunches the remaining energy spread would quickly become worse. Also, the RF tolerances set a limit to the smallest energy spread achievable with this scheme. If there is a mismatch  $\Delta\varphi$  between the phases of the two sections, the energy spread is limited to  $\sigma|\Delta\varphi|/2$ . Thus, to preserve the small energy spread in the given example, the phase mismatch  $\Delta\varphi$  has to be  $\Delta\varphi < 0.4^\circ$ .

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