SELF-CONSISTENT 3-D PIC CODE FOR MODELLING OF HIGH-BRIGHTNESS BEAMS

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Abstract

The 3D numerical code RETAR has been developed to model the beam dynamics of high-brightness electron beams in photoinjectors and in magnetic compressors, where the interaction of the beam with its self-field is of crucial relevance to optimize the performances of these devices, either in the case of space charge dominated dynamics or under strong coherent synchrotron radiation effects, respectively. The code is fully relativistic and calculates the self-fields directly from convenient integral forms that can be obtained from the usual retarded expressions. Numerical results are presented so far for the dynamics of an electron bunch in a radio-frequency photo-injector taking into account solenoidal magnetic focusing fields, RF focusing, thermal emittance, image charges inside the cathode surface, acceleration and space-charge effects.

INTRODUCTION

RETAR is a 3D code for the description of the motion of an electron beam under any kind of (given) external electromagnetic (em) fields plus the fields created by the beam itself (space-charge and radiation-fields). At the present time, the code is limited to consider only propagation in vacuum. Metallic or other kind of boundaries will be added in the future.

The trajectory of each single electron of the beam satisfies the following equations:

$$\frac{d\mathbf{x}_j}{dt} = \mathbf{v}_j \quad ; \quad \frac{d\mathbf{p}_j}{dt} = -e(\mathbf{E}_{tot} + \frac{\mathbf{v}_j}{c} x \mathbf{B}_{tot})$$

where $\mathbf{E}_{tot}$ and $\mathbf{B}_{tot}$ are the total fields acting on each electron, taking into account both external and self fields. The equations of motion are integrated by means of a second-order Runge-Kutta scheme: this does not cause any particular difficulty provided all forces acting on the electrons are known at each time step. The electron itself is actually a *macro-electron*, with total charge and mass that are several orders of magnitude larger than the charge and mass of a true electron. These macro-electrons are considered as point like particles but their number everywhere within the bunch is supposed to be sufficiently large as to allow suppression of divergent or very large contributions to the em self-fields, when they happen to be calculated at points very near to the electrons or exactly on the electrons. As regards the external forces, they are obviously considered to be known as functions of time and everywhere within the region of space crossed by the beam in its motion outside the cathode surface. The em self-fields are calculated directly (*i.e.* without calculating first the em potentials) in terms of the values of the charge density $\rho(\mathbf{x},t)$ at time t and at all preceding times, through the following equations that can be readily obtained by an easy manipulation of the usual retarded forms

$$\mathbf{E}(\mathbf{x},t) = \int d\mathbf{x}'\rho(\mathbf{x}',\tau)\mathbf{Q}_{\mathbf{E}}(\mathbf{x}-\mathbf{x}',\tau) \qquad (1)$$

$$\mathbf{B}(\mathbf{x},t) = \int d\mathbf{x}'\rho(\mathbf{x}',\tau)\mathbf{Q}_{\mathbf{B}}(\mathbf{x}-\mathbf{x}',\tau) \qquad (2)$$

where $\tau = t - \frac{1}{c}|\mathbf{x}-\mathbf{x}'|$ and

$$\mathbf{Q}_{\mathbf{E}} = \frac{\mathbf{n}\times((\mathbf{n}-\beta)\times\dot{\beta})}{c|\mathbf{x}-\mathbf{x}'|(1-\mathbf{n}\cdot\beta)^2} + \frac{(\mathbf{n}-\beta)(1-\mathbf{n}\cdot\beta)^{-2}}{\gamma^2|\mathbf{x}-\mathbf{x}'|^2} \qquad (3)$$

$$\mathbf{Q}_{\mathbf{B}} = -\frac{\mathbf{n}\times(\dot{\beta}(1-\mathbf{n}\cdot\beta)+\beta(\mathbf{n}\cdot\dot{\beta}))}{c|\mathbf{x}-\mathbf{x}'|(1-\mathbf{n}\cdot\beta)^2} - \frac{(\mathbf{n}\times\beta)(1-\mathbf{n}\cdot\beta)^{-2}}{\gamma^2|\mathbf{x}-\mathbf{x}'|^2} \qquad (4)$$

In addition, $\mathbf{n} = (\mathbf{x}-\mathbf{x}')/|x-x'|$, $\boldsymbol{\beta} = \mathbf{v}(t)/c$, and all time dependent quantities in (3) and (4) are calculated at the retarded time τ.

The 3D integrals in (1) and (2) are calculated in the following way:

i) The whole space around the point $\mathbf{x}$ at which the fields are to be computed, is divided into a set of spherical shells of a conveniently small thickness .

ii) In each shell the code sums the values of the vector $\mathbf{Q}_{\mathbf{E}}$ (and those of $\mathbf{Q}_{\mathbf{B}}$) at all points that happen to be occupied at the corresponding retarded time by one of the electron of the beam. The retarded time itself depends only on the radius of the shell and is the same everywhere inside each shell.

iii) The integrals giving the electric field $\mathbf{E}$ and the magnetic induction $\mathbf{B}$ at the position $\mathbf{x}$

and time t, are then obtained by summing the contributions from all spherical shells.

The basic advantages of this method seem to consist in i) the absence of convergence difficulties that may arise when one chooses to calculate the em self-fields from direct numerical integration of the relevant propagation equations and ii) in the fact that the retardation is very simply taken into account instead of having to find by means of a Newton iteration, for instance, the root of a complicated equation, when one chooses to calculate the em self-fields by summing the fields produced by each single electron as given by the Liénard-Wiechert equations. The CPU time is expected to scale as the product of the number of macro-electrons times the number of spherical shells, implying a linear scaling with the number of macro-electrons: we are in the process of checking in details this scaling law which represents a clear advantage over particle-to-particle integral codes based on Liénard-Wiechert retarded potentials.

The values of the self-fields obtained inside and outside the electron beam have been accurately tested by comparing them with well-known analytical and numerical estimates in various situations where space charge field is dominant (cylindrical and elliptic cross-section bunches with relativistic velocities) and the results have always shown fairly good agreement.

This algorithm is fully self-consistent in a 3D fashion, so it is suitable for the study of a class of beams strongly interacting with space charge and radiation fields: since it is based on an integral description for the field components, there is no mesh discretization, hence no limitations arising from space discretization of the field propagation like in case of differential PIC codes. Time (frequency) and space resolutions are only set by the number of spherical shells used in the simulation to reconstruct the retarded times.

NUMERICAL RESULTS

We are applying the code to the specific purpose of simulating numerically the relativistic motion of the electrons that are emitted by short laser pulses from the photo cathode surface and immediately subjected to a strong acceleration and focusing by suitable given electromagnetic fields.

The analysis includes the self-fields of the electrons that leave the cathode, as well as the fields produced by the image charges inside the cathode. The electron injection is simulated by fixing the electron flux at the cathode surface (for instance the plane $z=0$), *i.e.* by giving the number of electrons streaming out of the cathode per time interval and surface element. Thermal fluctuations in both spatial locations and velocity distribution of the electrons emitted are taken into account.

The photoinjector lay-out considered in these simulations is that of SPARC, described in a separate paper [1]: the external magnetic field has a solenoidal shape with the maximum value $B_z = 0.27\ T$ on the axis at $z=20$ cm, while the axial component of the electric field is applied with a peak field on the cathode $E_0 = 120\ MV/m$. The beam is accelerated in the RF gun up to about 6 MeV and propagated along 1.2 m of downstream drift space. 2000 macro-electrons and 100 spherical shells have been used in this simulation with an integration time step of 0.5 ps : the CPU time was about 3 hours on a 1.8 Ghz processor.

In Figure 1 the time evolution of the beam is presented at different times. Significant values of the beam parameters are: the total beam charge $Q=1\ nC$, the laser pulse length 10 ps (uniform flat top time distribution for the laser pulse intensity), initial beam radius $R=1$ mm, initial thermal emittance $\varepsilon_n = 0.3$ μm, launching phase 30 °RF.

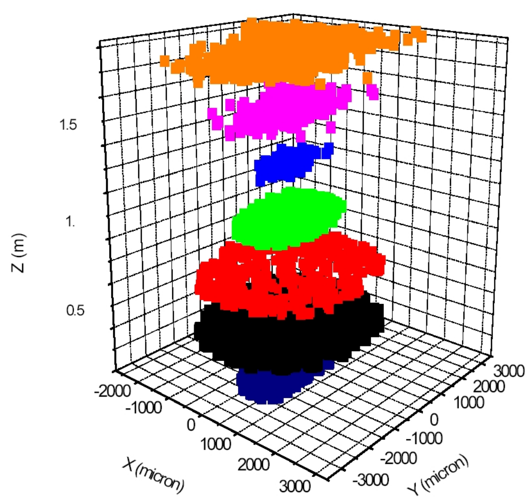

Figure 1: temporal evolution of the beam through the SPARC photoinjector

In figure 2 the rms radius σ_r and the rms normalised emittance:

$$\varepsilon_x = \sqrt{<x^2><p_x^2> - <xp_x>^2}$$

are plotted as functions of z for the values of the parameters given previously showing that the minimum value of ε_x takes place at $z=105$ cm in correspondence of the minimum of the rms radius.

No systematic investigations has been done yet on the optimum parameter set in order to minimize the emittance according to the Ferrario criterium for the working point of integrated photo-injectors [4]. Therefore, the comparisons to other simulations are quite preliminary, with the aim just to test the absence of errors in the code.

In Figure 3 the $r-p_r$ phase space is shown at the same time instants as in Figure 1.

After the focusing, a degradation of the beam quality can be seen, apparently due to non linear space charge effects in combination to an incorrect matching into the invariant envelope [3,4] conditions.

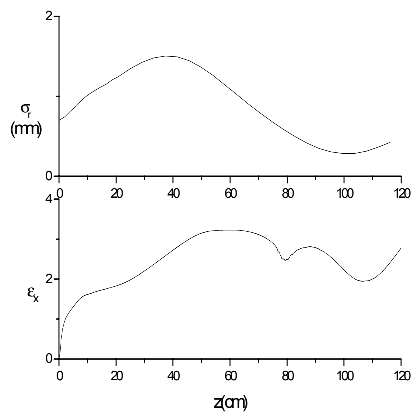

Figure 2: rms radius σ_r in mm and normalised emittance ε_x in µm vs z in the case of SPARC parameters

The RETAR code has been compared with other beam evolution codes as for instance Homdyn [2] based on the envelope approximation [3].

In Figure 4 the comparison between the average kinetic energy as given by RETAR (black curve) and Homdyn (red curve) for the same parameters is shown for the first 12 cm.

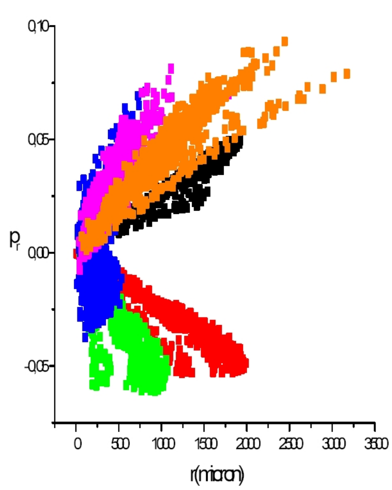

Figure 3:beam evolution in the $r-p_r$ phase space

As for the beam envelope and the rms transverse emittance, we found a slight discrepancy with Homdyn, which predicts an emittance minimum located at 1.5 m with a minimum value of 0.6 µm. We believe most of this discrepancy is due to an incorrect setting of the most critical parameters (solenoid peak field, launching phase, cathode spot size) and we are in the process of investigating it thoroughly.

In conclusion, the code RETAR is under test in the context of a class of general beam phenomena where beam-field interaction is of crucial relevance: we reported here the preliminary results of testing the code in photoinjector beam dynamics. Next step will be the study of CSR effects in magnetic compressors.

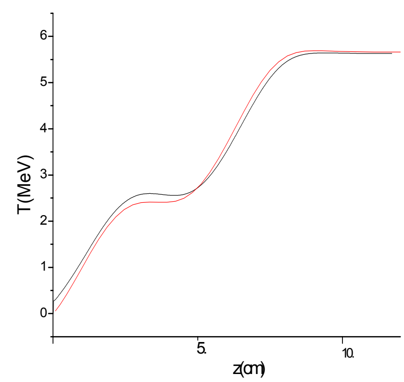

Figure 4: comparison between RETAR and Homdyn

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