EVALUATION OF THE HORIZONTAL TO VERTICAL TRANSVERSE IMPEDANCE RATIO FOR LHC BEAM SCREEN USING A 2D ELECTROSTATIC CODE

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Abstract

The classical 2 wires method is not suitable for high precision transverse impedance measurements on a homogeneous copper beam-pipe with non circular cross-section due to measurement noise limitations in case of narrow wire spacing. Thus we tackled the problem by simulating the 2D electrostatic field and image charge distribution of that setup and subsequently calculating the corresponding surface current for a TEM wave excitation. In this computer simulation the 2 wires can be assumed lossless, which is not possible in a practical bench setup. The theoretical justification for the method and certain limitations are discussed. The results compare very well to several independent numerical and analytical results.

1 INTRODUCTION

We discuss a method to calculate the ratio between the horizontal and vertical transverse impedances (Z_h and Z_v) for a given beam pipe cross-section. In particular, we consider the case of a cylindrical homogeneous metallic beam pipe, such as the LHC beam screen (*BS*) geometry (the effect of the pumping slots present in the beam pipe wall will be neglected). Our results may become a useful tool to calculate the transverse impedance for non-analytical solution cases.

The basic principle is that an ultra-relativistic off center beam produces a surface current distribution in the inner surface of the beam pipe similar to a (statically) charged off center inner conductor. It is possible to reproduce the beam pipe cross section geometry and the inner conductor for use in an electrostatic field code which can then be used to obtain the surface current distribution.

Using the electrostatic field code *Superfish* [1], one can obtain the normal electric field (E) at each point on a perfectly conducting surface for a given 2-D geometry. Two inner conductors in an odd mode are placed inside the beam screen geometry of the LHC (Fig. 2) to simulate the dipole component of the beam field. Since the transverse impedance (Z) is proportional to $\int E^2 dl$ (losses in the surface wall), the ratio between the Z_h and Z_v is given by the ratio of $\int E^2 dl$ for the horizontal and vertical excitations.

The *BS* is expected to be the main contributor to the resistive wall effect in the LHC [2]. In order to properly calibrate this method, some cases with a known analytical solution have been simulated. We describe here the method used to calculate $\int E^2 dl$ from the *SuperFish* output and compare numerical and analytical results for the *BS*.

2 MATHEMATICAL METHOD AND THE USE OF *SUPERFISH*

For an arbitrary pipe cross section the ratio between Z_h and Z_v can be computed by [3]:

$$\frac{Z_v}{Z_h} = \frac{\int |J_v^2| \, dl}{\int |J_h^2| \, dl}, \qquad (1)$$

where $\boldsymbol{J}$ is the surface current in the inner surface of the outer conductor, and $\boldsymbol{J_v}$ and $\boldsymbol{J_h}$ stand for an off center beam displaced in the vertical axis and horizontal axis, respectively. In order to compute $\boldsymbol{J}$, the first step is to solve the Poisson equation using *SuperFish* and obtain the static electric field (E_x, E_y) for any position of the inner wires. Then, the dynamic solution of the TEM wave can be obtained by the electrostatic transverse field distribution:

$$E = (E_x, E_y) \cdot e^{j(\omega t - \beta z)}. \qquad (2)$$

In a TEM line, at any point of the outer conductor inner surface, the electric field ($\boldsymbol{E}$) is always orthogonal to the magnetic field ($\boldsymbol{H}$), and both are related by:

$$\frac{|E|}{|H|} = Z_0, \qquad (3)$$

where Z_0=377Ω is the vacuum impedance. For very high frequencies, and considering perfect conductors, $\boldsymbol{J}$ in the inner surface of the outer conductor is equal to the tangential $\boldsymbol{H}$ in the same surface:

$$J = H_t \hat{\varphi}. \qquad (4)$$

Joining Eqs. 3 and 4 to Eq. 1 leads us to:

$$\frac{Z_v}{Z_h} = \frac{\int |E_v^2| \, dl}{\int |E_h^2| \, dl}, \qquad (5)$$

where $\boldsymbol{E}$ has to be considered normal to the outer surface, since we are assuming a perfectly conducting beam pipe.

In such a perfect conductor the skin depth is null and therefore the transverse impedance (Z_T) will not depend on the frequency. This dependence can be introduced as a perturbation in a second order calculation, but the first order field distribution is not affected by the conductivity of the material. Therefore, the $\boldsymbol{J}$ distribution is related only to beam parameters, such as the beam displacement, i.e. the wire displacement in our model. One way to measure Z_T (and also the longitudinal impedance, Z_L) is to measure the variation of $\boldsymbol{J}$ versus the lateral displacement of the beam (wire) [3]. For that reason, we use the classical method of the two wires (polarized in the odd mode) to do bench measurements and calculations. In a first order approximation, the field distribution produced by an offset wire can be assumed to be generated by a centered wire plus the effect of a dipole field, which is illustrated in Fig. 1. The ratio of $\int E^2 dl$ for the total electric field (centered wire plus dipole) is the same as the ratio of the $\int E^2 dl$ for a dipole: a single offset wire (beam) can be represented for the surface charge or current distribution by a centered wire plus the contribution of a dipole moment. Since we are only interested in Z_T, we only consider the dipole moment (wire pair).

SuperFish provides the normal ***E*** at each point of the surface. The number of points into which the surface is divided is given by the mesh size. In order to get good resolution, the optimum value for the mesh size has been set to 0.1 mm for a total length ranging from 36 and 44mm (*BS* dimensions). Different values for the inner conductor diameter (Φ_{int}) and the separation between the inner wires (s) have been used (Fig. 1) in order to find the best combination for the dipole size.

When calculating $\int E^2 dl$, it is worth mentioning that the differential length fixed by the mesh size, *dl*, is not constant in the circular as in the flat zones of the *BS* geometry (Fig. 2). In order to ensure all points have the same specific weight when computing the $\int E^2 dl$, normalization for the 'bin' length has been implemented.

3 RESULTS FOR THE DIFFERENT GEOMETRIES

We are interested in calculating the ratio Z_v/Z_h for the *BS* following Eq. 5. In order to calibrate the method, some cases with a known analytical solution have been evaluated. Each case involves a cylindrical tube with a circular cross section (see Fig. 1) and a different radius (R=18, 20, and 22 mm). In each case, neglecting the skin depth, it turns out that Z_T depends inversely on the third power or R (see Ref. [4]). For a set of two different external radii, R_1 and R_2, this ratio is:

$$\frac{Z_T(R_1)}{Z_T(R_2)} = \frac{R_2^3}{R_1^3} \,. \qquad (6)$$

Since the *SuperFish* output will be $|E_i^2|$, the calibration of this method will be done joining Equations 5 and 6:

$$\frac{\int |E_{R_1}^2|\,dl}{\int |E_{R_2}^2|\,dl} = \frac{R_2^3}{R_1^3} \,. \qquad (7)$$

3.1 Two Inner Conductors in a Round Geometry

Figure 1 describes the general calibration geometry. The two inner conductors are numerically polarized to +1V and -1V respectively. The dimensions and the corresponding results for these cases are given in Table 1. Basically, the goal is to check how well the method agrees with the theoretical results and find an optimum value for the dipole size, i.e., an optimum combination of Φ_{int} and s.

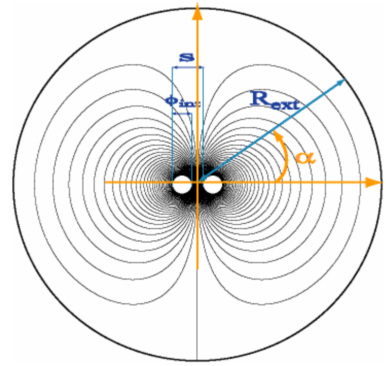

Figure 1. General round geometry used to calibrate the method. R_{ext}, stands for the external radius (in the text stated as R); Φ_{int} is the diameter of the internal wire; s is the distance between the inner conductors; and α marks the angle. The two lossless inner conductors are numerically polarized to +1V and -1V, and the internal lighter lines represent the equipotential lines.

Table 1: Ratio $\int E_{R_2}^2 dl / \int E_{R_1}^2 dl$ for different combinations of R_1 and R_2 and the different dipole sizes. Last row shows the theoretical value according to Eq. 7.

	R_1=18mm R_2=20mm	R_1=20mm R_2=22mm	R_1=18mm R_2=22mm
Φ_{int}=2mm s=3mm	*1.3780*	*1.3352*	*1.8399*
Φ_{int}=2mm s=6mm	*1.3888*	*1.3430*	*1.8653*
Φ_{int}=4mm s=6mm	*1.4035*	*1.3531*	*1.8992*
Theoretical value: R_2^3/R_1^3	***1.3717***	***1.3310***	***1.8258***

Table 1 shows the results of the calibration done using Eq. 7. Results for the smallest dipole case (Φ_{int}=2mm and s=3mm) show very good agreement: the maximum error for this geometry is within 1%. For a given combination of R_1 and R_2, the error increases with the dipole size. We can see that the discrepancy in the case of the largest dipole, i.e., Φ_{int}=4mm and s=6mm, can be as big as 8%. Note that for a fixed combination of Φ_{int} and s, the maximum error corresponds to the maximum difference between R_1 and R_2. Table 1 gives also an indication of the mistake one makes with the classical 2 wires method for Z_T measurements as a function of s and Φ_{int}.

According to results shown on Table 1, the best configuration when computing the ratio Z_v/Z_h for the *BS* geometry, is the combination of Φ_{int}=2mm and s=3mm.

3.2 Results for the Beam-Screen Geometry

Figure 2 shows the *BS* cross section with the two inner conductors placed in the horizontal plane. For the calculations in the vertical case, the dipole is rotated 90º.

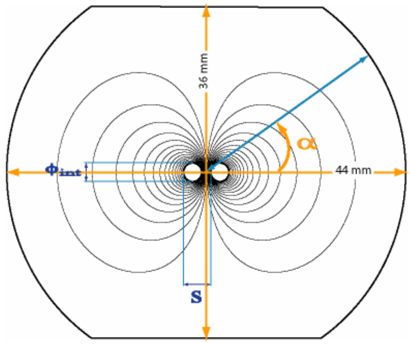

Figure 2. Cross section of the *BS* geometry with two wires inside simulating the dipole field. The inner wires are placed in the horizontal plane and held to +1V and -1V. The closed lighter lines represent the equipotential lines. For the vertical case, the dipole is rotated 90º.

In this case, we will use Eq. 6 to evaluate the Z_v/Z_h. Figure 3 shows the $|E_i^2|$ distribution for both vertical and horizontal configurations. The final results can be found in Table 2.

Table 2. Final results used to compute Z_v/Z_h and the value of the ratio of the horizontal and vertical Z_T's.

$\int E_{VER}^2 dl$	11.2830 (V/cm)^2
$\int E_{HOR}^2 dl$	7.8678 (V/cm)^2
Z_v/Z_h	***1.43***

The prediction for Z_v/Z_h is in a good agreement with other methods, such as the Boundary Element Method (BEM) from H. Tsutsui in [5], who recently obtained 1.42 ±1%. Our error bar is estimated to be 1%, as stated in Sec. 3.1.

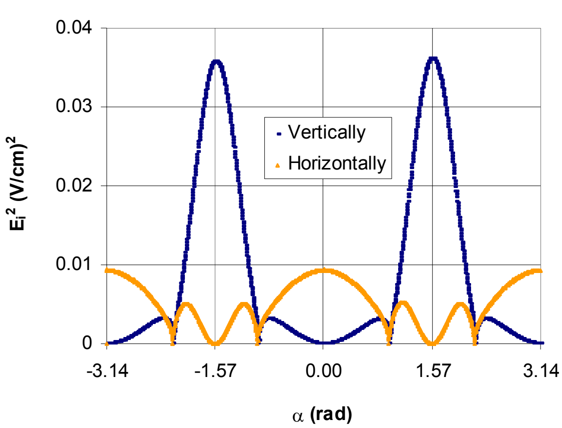

Figure 3. Distribution of $|E_i^2|$ at the surface of the external conductor of the *BS* surface as a function of the angle α when the inner conductors are placed vertically (blue trace) and horizontally (orange).

4 CONCLUSIONS

By means of an electrostatic computer code, one can compute a reasonable approximation for Z_v/Z_h for any beam pipe cross section. Assuming 2 lossless wires, the classical two wire method has been implemented by simulating the 2D electrostatic field and image charge distribution of the *BS* geometry, and subsequently calculating the corresponding surface current for a TEM wave excitation. The electrostatic computer code used was *SuperFish*, which as mentioned in Sec. 2, requires length normalization, especially for non-circular surfaces. The appropriate dipole size is critical to get reasonable results, and it should be as small as possible. Theoretical justification to compute the ratio Z_v/Z_h from the ratio of $\int E^2 dl$ is given. For the LHC *BS* geometry we predict Z_v/Z_h=1.43 ± 1%, which is in a good agreement with other theoretical results [5]. The calculation of coupling impedance of beam pipes of general cross section has also been solved by Yokoya [6] by means of a field formula perturbation solution, which is not always applicable. However, the use of this method is only restricted by the use of the electrostatic computer code. In the case of *SuperFish*, all arbitrary surfaces can be simulated and therefore, this can become a useful tool to calculate the transverse impedance.

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