MEASUREMENT AND APPLICATION OF BETATRON MODES WITH MIA*

Chun-xi Wang†, Argonne National Laboratory, Argonne, IL 60439, USA

Abstract

We present Model-Independent Analysis (MIA)-based methods for measuring lattice properties of a storage ring such as phase advance, beta function, chromaticity, and tune shift with amplitude. Using beam position histories of excited betatron oscillations that are simultaneously measured at a large number of beam position monitors (BPMs), the spatial-temporal modes of betatron oscillation can be extracted with MIA-mode analysis. The resulting spatial vectors are used to determine linear phase advance and beta function, and the temporal vectors are used to determine nonlinear chromaticity and tune shift with amplitude. Measurements done at the Advanced Photon Source are reported.

INTRODUCTION

Assuming weak coupling and nonlinearity, the betatron oscillation of a single particle can be described by

$$x_\beta(s) = \sqrt{2J\,\beta(s)}\cos[\phi + \psi(s)], \tag{1}$$

where $\{J, \phi\}$ are the action-angle variables specifying a specific trajectory, $\beta(s)$ is the beta function, and $\psi(s)$ is the phase advance. In a perfectly linear machine and on a time scale much shorter than the radiation damping time, the action J is conserved and the angle evolves as $\phi = \phi_0 + 2\pi\nu_0 p$, where ν_0 is the lattice tune and p is the number of turns. However, weak nonlinearities generate energy-dependent and amplitude-dependent tune shifts $\Delta\nu = \xi\delta + aJ$, where ξ is the chromaticity and a is the coefficient of amplitude-dependent tune shift. For a bunched beam, particles' energies and amplitudes have certain distribution, thus the bunch centroid observed at the BPMs is the phase-space average of Eq. (1). The resulting centroid oscillation of an excited beam may not follow Eq. (1), for example when decoherence occurs. However, for each turn the centroid is expected to follow single particle behavior closely, i.e., the centroid oscillation can still be written as

$$b_p^m = \sqrt{2J_p\beta_m}\cos(\phi_p + \psi_m), \tag{2}$$

where b_p^m is the beam centroid position at the m-th monitor for the p-th turn. Note that both action and angle are carrying the turn index so that Eq. (2) can accommodate much more complicated centroid motion.

In recent years, Model-Independent Analysis (MIA) [1, 2] has emerged as a new approach to study beam dynamics by analyzing beam histories simultaneously recorded at a large number of BPMs, i.e., the data matrix $B_{P\times M} = (b_p^m)/\sqrt{P}$, where P is the number of turns and M is the number of BPMs. B is normalized such that B^TB is the variance-covariance matrix of BPM measurements. A basic MIA technique is the spatial-temporal mode analysis via singular value decomposition (SVD) of B, which yields

$$B = USV^T = \sum_{\text{modes}} \sigma_i u_i v_i^T, \tag{3}$$

where $U_{P\times P} = [u_1, \cdots, u_P]$ and $V_{M\times M} = [v_1, \cdots, v_M]$ are orthonormal matrices comprising the temporal and spatial eigenvectors, and $S_{P\times M}$ is a rectangular matrix with nonnegative singular values σ_i along the upper diagonal. Similar to the Fourier analysis, this mode analysis decomposes the spatial-temporal variation of the beam centroid into superposition of various orthogonal modes by effectively accomplishing a major statistical data analysis, namely the Principal Component Analysis. A pair of spatial and temporal vectors $\{v_i, u_i\}$ characterizes a spatial-temporal eigenmode, and the corresponding singular value σ_i gives the overall amplitude of the mode. It can be shown that when beam motion is dominated by betatron oscillations, there are two orthogonal eigenmodes (referred to as betatron modes) that correspond to the normal coordinates of betatron motion. In the following, we give the explicit expression of the betatron modes and their measurements, then show how to use them to determine phase advances, beta function, chromaticity, and tune shift with amplitude.

BETATRON MODE MEASUREMENT

When B is dominated by the excited betatron motion given by Eq. (2) with action and angle independently distributed, it can be decomposed into [2]

$$B \simeq \sigma_+ u_+ v_+^T + \sigma_- u_- v_-^T, \tag{4}$$

where the spatial and temporal vectors are given by

$$\begin{cases} v_+ = \frac{1}{\sigma_+}\left\{\sqrt{\langle J\rangle\beta_m}\cos(\phi_0+\psi_m),\ m=1,\cdots,M\right\} \\ v_- = \frac{1}{\sigma_-}\left\{\sqrt{\langle J\rangle\beta_m}\sin(\phi_0+\psi_m),\ m=1,\cdots,M\right\} \end{cases} \tag{5}$$

and

$$\begin{cases} u_+ = \left\{\ \ \sqrt{\frac{2J_p}{P\langle J\rangle}}\cos(\phi_p-\phi_0)\,,\ p=1,\cdots,P\right\} \\ u_- = \left\{-\sqrt{\frac{2J_p}{P\langle J\rangle}}\sin(\phi_p-\phi_0)\,,\ p=1,\cdots,P\right\}. \end{cases} \tag{6}$$

Here $\langle\ \rangle$ means sample average. Note that the spatial vectors are orthogonal single-particle trajectories of Eq. (1) even though the centroid follows the more complicated

* Work supported by U.S. Department of Energy, Office of Basic Energy Sciences, under Contract No. W-31-109-ENG-38.

† wangcx@aps.anl.gov

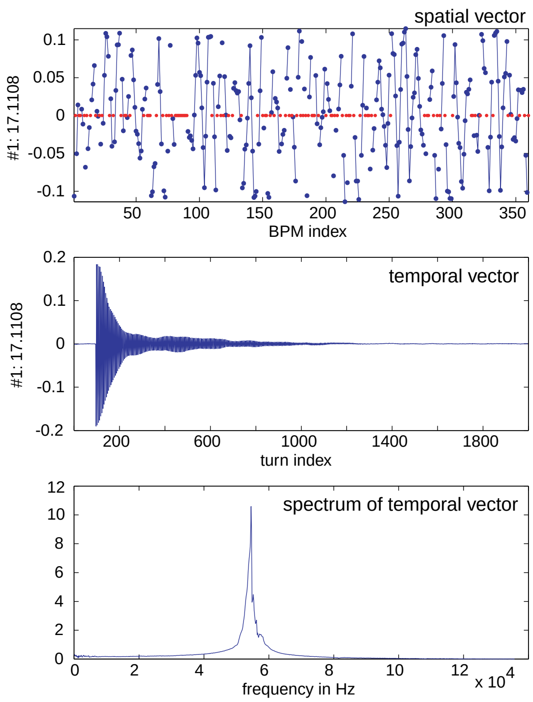

Figure 1: The first betatron mode of a kick excitation. Measured values are solid dots and joined by lines for consecutive BPMs. Bad BPMs are dots at zero. The mode number and its singular value are shown on the left-side label.

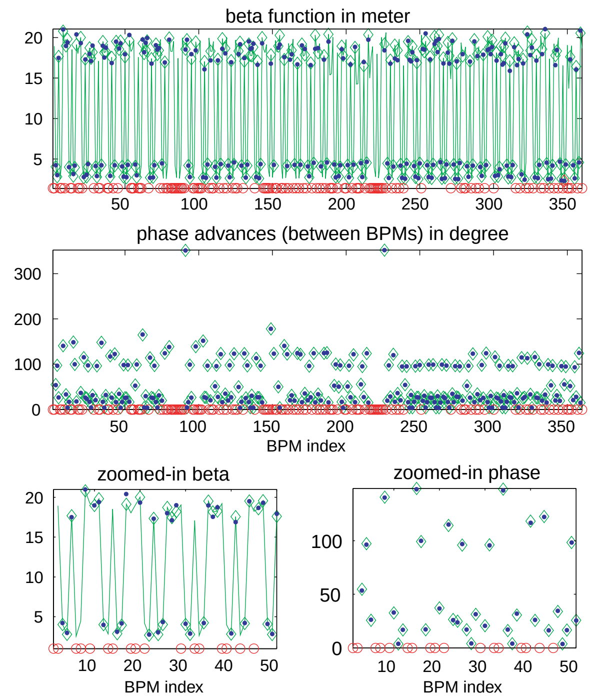

Figure 2: Horizontal beta function and phase advance. The solid dots are MIA measurements. The solid lines are calibrated machine model. Diamonds are model values at used BPMs. Circles are bad BPMs. Figures in the third row are blowups of the above figures for the first 50 BPMs.

Eq. (2). Furthermore, the temporal vectors clearly relate to the normal coordinates that depict time-evolution in phase space.

As an example, Fig. 1 shows the first betatron mode of a horizontally kicked beam in the APS ring. The second is very similar and not shown for lack of space. The spatial vector is a betatron orbit, although due to the unusable BPMs the orbit is broken into pieces and looks irregular. The temporal vector clearly shows a beam that is kicked at about the 100th turn then decohered and damped. The Fourier spectrum of the temporal vector shows the betatron frequency with a broadened peak due to decoherence. Note that the synchrotron and vertical tunes as well as other nonlinear resonance frequencies are invisible, even though they do exist and show up in other modes [3]. This indicates the quality of the betatron modes.

PHASE ADVANCE AND BETA FUNCTION

From the spatial betatron vectors, Eq. (5), the phase advances can be determined as

$$\psi = \tan^{-1}\left(\frac{\sigma_- v_-}{\sigma_+ v_+}\right), \tag{7}$$

where the phase ϕ_0 is absorbed by shifting the reference point. Since we are interested in only the phase advance between BPMs, the reference point does not matter. The beta function can be written as

$$\beta = \langle J\rangle^{-1}\left[(\sigma_+ v_+)^2 + (\sigma_- v_-)^2\right]. \tag{8}$$

Note that, except for an overall scaling factor $\langle J\rangle$ in β, the phase advance and beta function can be computed from the spatial betatron vectors. The phase measurement can tolerate BPM gain errors, but the beta function measurement cannot.

Using the measured spatial betatron vectors as shown in Fig. 1, the beta function and phase advances are computed and shown in Fig. 2. For comparison, the values from a model fitted with response-matrix measurements is also shown, which agrees well with the MIA measurements. See [4] for more details on phase advance and beta function measurements.

CHROMATICITY AND TUNE SHIFT WITH AMPLITUDE

Chromaticity and tune shift with amplitude are basic parameters describing the nonlinear energy- and amplitude-dependent tune shifts. Usually they are determined by directly measuring the tune slope versus different beam energies and kick amplitudes. One major limitation of such measurements is that machine tunes often wobble around from measurement to measurement such that the chromatic and nonlinear tune shifts may be obscured. An alternative approach is to measure chromatic decoherence [5] and nonlinear decoherence [6] of the beam centroid due to the tune shift $\Delta\nu = \xi\delta + aJ$. Assuming the initial state is a well-damped Gaussian distribution in (x, x') and δ, the centroid position is given by Eq. (2) with [7]

$$\sqrt{J_p} = \sqrt{J_{\max}}\frac{1}{1+\theta^2}\, e^{-\frac{Z^2}{2}\frac{\theta^2}{1+\theta^2}}\, e^{-2\left(\frac{\xi\sigma_\delta}{\nu_s}\right)^2 \sin^2(\pi\nu_s p)} \quad (9)$$

and

$$\phi_p = \phi_0 + 2\pi\nu_0 p + \frac{Z^2}{2}\frac{\theta}{1+\theta^2} + 2\tan^{-1}\theta, \quad (10)$$

where $\theta = 2\pi a\epsilon p$, ϵ is the emittance, $Z = \sqrt{2J_{\max}/\epsilon}$ is the kick strength, σ_δ is the energy spread, and ν_s is the synchrotron tune. Fitting these with measured centroid evolution gives the products $a\epsilon$ and $\xi\sigma_\delta$.

The temporal betatron vectors such as the one shown in Fig. 1 can be used in both approaches and may result in significant improvement by reducing the random noise by a factor of about $1/\sqrt{M}$. (Differences in BPM resolutions and beta values at BPM locations must be taken into account for better estimate.) For the direct tune measurement approach, one simply does a spectrum analysis of the temporal vectors. Since the signal often decoheres very fast, one may need to apply the NAFF technique [8] on a small number of turns to determine the tune for each kick. For decoherence measurement, J_p and ϕ_p can be simply obtained from the temporal vectors of Eq. (6) as

$$J_p = P\langle J\rangle\frac{u_+^2 + u_-^2}{2}\ , \quad \phi_p = -\tan^{-1}\left(\frac{u_-}{u_+}\right). \quad (11)$$

Note that both J_p and ϕ_p are smooth functions without fast betatron oscillation. They provide independent constraints for fitting the decoherence parameters.

As examples, we show preliminary results of two sets (at 0.2 and 1.5 mA) of horizontal measurements using horizontally kicked single bunches (at five different amplitudes in each set) in the APS ring. We choose the low current in order to minimize the wakefield effect since the above decoherence model does not take such an effect into account. The temporal vectors at the lowest kick are shown in Fig. 3,

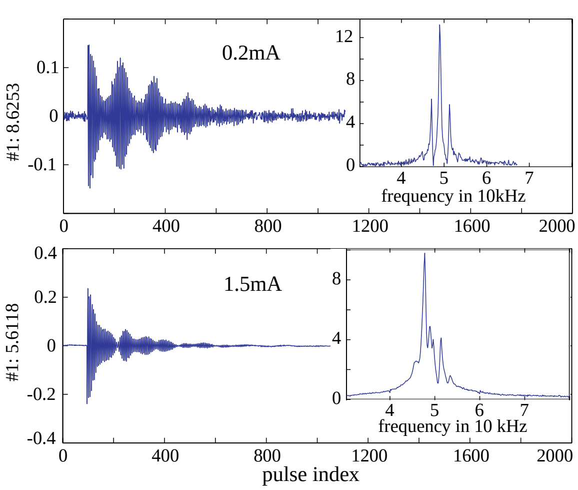

Figure 3: The temporal betatron vectors of 0.3 kV kick excitation ($Z = 3.2$) at 0.2 mA (top) and 1.5 mA (bottom).

where the wake effect is obvious. Since the APS BPM system is not intended for measuring such a low current, the resolution at 0.2 mA is very poor (five times worse than at 1.5 mA). Thanks to MIA noise reduction, we are still able to obtain decent decoherence measurements.

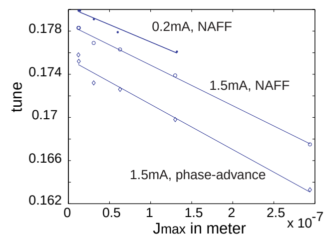

Figure 4: Tune slopes versus kick amplitudes.

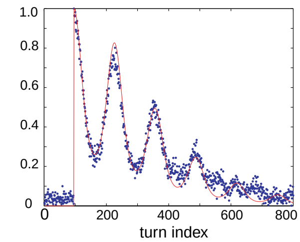

Figure 5: Measured and fitted decoherence.

The results of tune slope measurements are shown in Fig. 4, where the NAFF results are based on the first 20 turns of temporal vectors, which yield $a \simeq -3.2 \times 10^{-4}m^{-1}$ and $-3.8 \times 10^{-4}m^{-1}$ for 0.2 mA and 1.5 mA, respectively. The phase-advance results are based on Eq. (7), which yields $a \simeq -4.2 \times 10^{-4}m^{-1}$ for 1.5 mA and failed at 0.2 mA due to noise. The apparent offset is unclear at this point. The result of fitting decoherence at 0.2 mA is shown in Fig. 5. Fitting for 1.5 mA failed due to the wakefield effect. Since the beam completely decohered well before θ reached 1, the phase cannot constrain θ due to uncertainty in ν_0. Thus, only the fit for J_p is shown, which yields $\xi\sigma_\delta \simeq 6.2 \times 10^{-3}$ and $a\epsilon \simeq 1. \times 10^{-4}$. With $\sigma_\delta = 0.9 \times 10^{-3}$ and $\epsilon = 2.4nm$, we have $\xi \simeq 6.9$ and $|a| \simeq 4 \times 10^4 m^{-1}$. The measured chromaticity and tune shift with amplitude are consistent with the model.

Thanks to V. Sajaev for providing information about the APS storage ring model.

REFERENCES

[1] J. Irwin, C.X. Wang, Y. Yan et al., Phys. Rev. Lett. **82**(8), 1684 (1999).

[2] Chun-xi Wang, Model Independent Analysis of beam centroid dynamics in accelerators, Ph.D. dissertation, Stanford University (1999). Also available as SLAC-R-547.

[3] Chun-xi Wang, "Spatial-Temporal Modes Observed in the APS Storage Ring using MIA," in this proceedings.

[4] Chun-xi Wang, Vadim Sajaev, and Chih-Yuan Yao, "Phase advance and beta function measurements using Model-Independent Analysis," to be published.

[5] See, for example, I. Hsu, Particle Accelerators **34**, 43 (1990).

[6] See, for example, J.M. Byrd and D. Sagan, Proc. of the 1991 Part. Accel. Conf., p. 1080 (1991).

[7] A. W. Chao and M. Tigner, Eds., *Handbook of Accelerator Physics and Engineering*, World Scientific, River Edge, NJ (1998).

[8] J. Laskar et al., Physica D **56**, 253 (1992).