ANALYSIS OF THE FEEDBACK SYSTEM USED TO DAMP LONGITUDINAL QUADRUPOLE-MODE BUNCH OSCILLATIONS

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Abstract

Longitudinal quadrupole-mode oscillations of the electron bunches sometimes limit the beam quality in storage-ring FEL's or in other storage rings. Such oscillations can be damped by using a feedback system. In this paper, we analyze how the feedback system damps the longitudinal quadrupole oscillations. We also present a result of the feedback test experiment, which was carried out at the KEK Photon Factory.

INTRODUCTION

In high-intensity storage rings for free-electron lasers (FEL's) or for particle factories, higher-order bunch oscillations sometimes limit the ring performance. Problems due to longitudinal quadrupole-mode oscillations have been reported in the Super-ACO under FEL experiments [1] or in the DAΦNE Φ-factory [2]. Such quadrupole-mode oscillations can be damped by using a feedback system [1]. We analyze in this paper how the longitudinal feedback can damp the quadrupole oscillations. We limit our analysis to a single-bunch case, assuming that a single bunch is made up of two macroparticles. An approximate expression for describing damping oscillations is then derived. This analysis will be useful for understanding the feedback mechanism for the quadrupole oscillations, although its effectiveness has been reported. The same analysis can be used to understand the feedback damping of a π-mode coupled-bunch oscillation in a two-bunch system, where we assume that two macroparticles are stored in separate rf buckets.

FEEDBACK MECHANISM

Detection of Quadrupole Oscillation

A typical feedback system for damping the longitudinal quadrupole oscillations is shown in Fig. 1. We suppose that a single bunch is made up of two point-like marcroparticles, each of which executes synchrotron oscillation having an opposite phase to each other. This situation is illustrated in Fig. 2. Let τ_1 and τ_2 be the time advances of these macroparticles, which are relative to the synchronous particle, and δ_1, δ_2 be the relative energy deviations, respectively. Small-amplitude synchrotron oscillations for these particles are then approximately described by

$$\tau_1(t) = r(t)\cos\left[\omega_s t + \psi(t)\right], \tag{1}$$

and $\tau_2(t) = -\tau_1(t)$, where ω_s is the synchrotron-oscillation frequency. We assume that the amplitude $r(t)$ and the phase $\psi(t)$ vary slowly with time. When each particle has an equal charge of $q_b/2$, the beam current is given by

$$i_b(t) = (q_b/2)\sum_{n=-\infty}^{\infty}\left[\delta(t - nT_0 + \tau_1) + \delta(t - nT_0 + \tau_2)\right], \tag{2}$$

where $\delta(t)$ is the delta function and T_0 is the revolution time, respectively. Using Poisson's sum formula, it

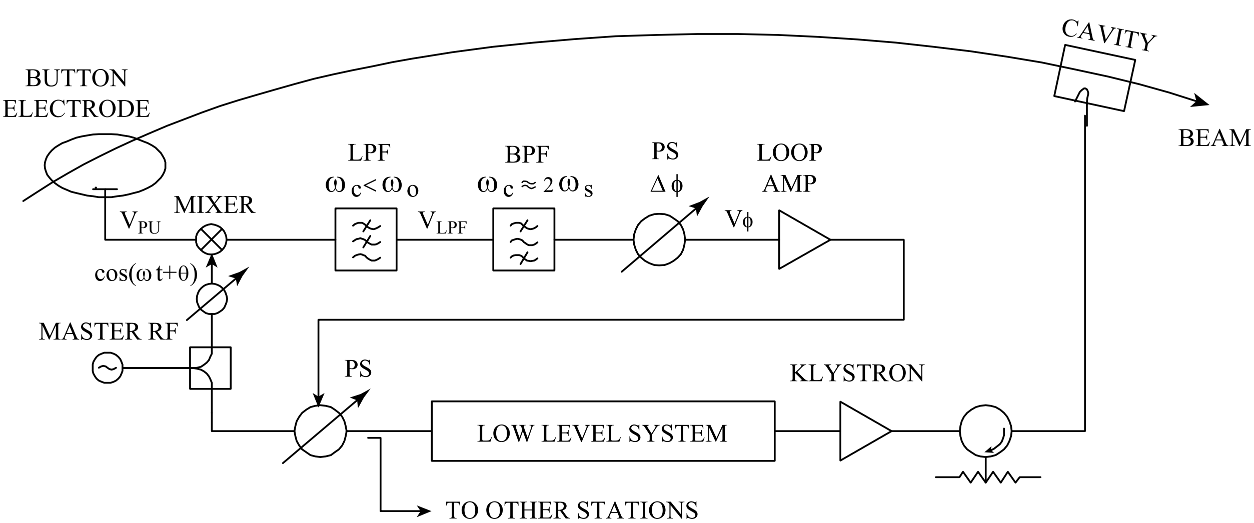


Figure 1: Typical feedback system for damping longitudinal quadrupole oscillations.

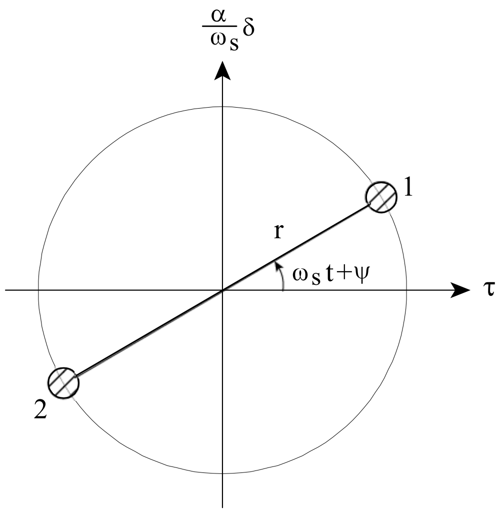


Figure 2: Two macroparticles in the longitudinal phase space.

follows

$$i_b(t) = I_0 \sum_{p=-\infty}^{\infty} \sum_{m:even} i^m J_m(p\omega_0 r) e^{i(p\omega_0 + m\omega_s)t + im\psi} , \quad (3)$$

where I_0 is the average beam current and $\omega_0 = 2\pi/T_0$ is the angular revolution frequency, respectively. When the frequency response (transfer function) for a pick-up electrode is given by $H(\omega)$, the output voltage from the electrode is given by

$$V_{PU}(t) = I_0 \sum_{p=-\infty}^{\infty} \sum_{m:even} i^m J_m(p\omega_0 r) H(p\omega_0 + m\omega_s) e^{i(p\omega_0 + m\omega_s)t + im\psi} .$$

In order to detect the phase oscillations, we multiply this signal by a distributed signal from a master rf oscillator:

$$V_{LO} = \cos(\omega_{rf} t + \theta) , \quad (4)$$

where the ω_{rf} is an rf frequency, and the θ is a certain phase angle which is adjusted later. Selecting low-frequency ($\omega < \omega_0$) signals using a low-pass filter (LPF), we have

$$V_{LPF}(t) = (I_0/2) \sum_{m:even} i^m J_m(h\omega_0 r) \times$$
$$[H(h\omega_0 + m\omega_s) e^{-i\theta} + H^*(h\omega_0 - m\omega_s) e^{i\theta}] e^{im(\omega_s t + \psi)} , \quad (5)$$

where $h = \omega_{rf}/\omega_0$ is the harmonic number, and the relation $H(-\omega) = H^*(\omega)$ was used. In most cases, we can think that the $H(\omega)$ is nearly constant ($\approx H_0 e^{i\theta_0}$) within a narrow frequency range of $|\omega - h\omega_0| << \omega_0$. We can then approximate eq. (5) by

$$V_{LPF} \approx I_0 H_0 \cos(\theta - \theta_0) \sum_{m:even} i^m J_m(h\omega_0 r) e^{im(\omega_s t + \psi)} . \quad (6)$$

In order to maximize the quadrupole-oscillation (m=2) signal, we adjust a phase angle θ of the local rf signal, so that $\cos(\theta - \theta_0) = -1$ holds. This can be experimentally done by adjusting a DC level in the V_{LPF} to its minimum value.

Next, we input the above signal to a bandpass filter (BPF) which can pass the frequency component around the frequency of $2\omega_s$, and then, apply a phase shift of $\Delta\phi$ using an electrical phase shifter. The output signal is given by

$$V_\phi = 2I_0 H_0 J_2(h\omega_0 r) \cos(2\omega_s t + 2\psi + \Delta\phi) . \quad (7)$$

After amplification by G_0-times, we apply this feedback signal to a phase shifter, which is used to modulate the phase of an rf accelerating voltage. When the amplitude r is small ($h\omega_0 r$ <<1), we can approximate $J_2(x)$ by $x^2/8$. Then, the phase modulation is approximately given by

$$\phi_m(t) \approx G r^2(t) \cos(2\omega_s t + 2\psi + \Delta\phi) , \quad (8)$$

with a feedback gain given by

$$G = G_0 I_0 H_0 (h\omega_0)^2 / 4 . \quad (9)$$

Damping quadrupole oscillation by feedback

When we apply a small phase modulation (ϕ_m) at a frequency of about $2\omega_s$, the synchrotron oscillation of a single macroparticle is approximately described by

$$\ddot{\tau} + 2\lambda\dot{\tau} + \omega_s^2 (1 + \phi_m \cot\phi_0)\tau = 0 , \quad (10)$$

where λ is the radiation damping rate, ϕ_0 is the synchronous phase ($\cos\phi_0 = U_0/eV_c$), ϕ_m is given by eq. (8), and the dot denotes the differentiation with respect to time. Here, we have ignored a less-important term of $\phi_m \omega_s^2/\omega_{rf}$ on the right hand side.

We can see that if the τ_1 for one of the macroparticles follows eq. (10), the other τ_2 automatically follows the same equation. We assume that the solution of eq. (10) is approximately given by eq. (1), which gives the first-order approximation by neglecting higher-frequency terms. Then, we substitute eq. (1) into eq. (10), and leave the terms having a fundamental frequency of ω_s. Because both the r and the ψ are assumed to be slowly varying functions of time, we omit higher-order terms of $\dot{\psi}$, $\dot{r}$ and λ. We then have

$$\dot{r} + \lambda r + k r^3 \sin\Delta\phi = 0 , \quad (11a)$$
$$r\dot{\psi} - k r^3 \cos\Delta\phi = 0 , \quad (11b)$$

with $k = (\omega_s G \cot\phi_0)/4$. Integrating eq. (11a), we obtain a solution for the r:

$$r(t) = \frac{r_0 e^{-\lambda t}}{\sqrt{1 + \frac{k r_0^2 \sin\Delta\phi}{\lambda} \cdot \left(1 - e^{-2\lambda t}\right)}} \ , \qquad (12)$$

where $r_0 = r(0)$ is the initial condition. When the feedback gain is zero ($k = 0$), the above equation (12) reduces to

$$r(t) = r_0 e^{-\lambda t} \ , \qquad (13)$$

which gives the radiation damping effect. When we apply a negative feedback ($\sin\Delta\phi > 0$), we can enhance the damping effect of the oscillation. A typical damping of the oscillation is shown in Fig. 3. The feedback is most effective when the phase shift is adjusted to be $\Delta\phi = \pi/2$. In this case, the above eq. (11b) gives

$$\psi(t) = \psi(0) \ , \qquad (14)$$

which means that there is no change in the oscillation phase. Note that somewhat unfamiliar expression in eq. (12) comes from the fact that the feedback signal is proportional to the square amplitude of the oscillation.

In the above analysis, we have assumed that there are two macroparticle in the same rf bucket, and that each macroparticle oscillates out-of-phase to each other. This analysis still holds under a different situation where two macroparticles are stored in different rf buckets. In this case, two macroparticles represent two bunches which oscillate out-of-phase to each other, indicating a π-mode coupled-bunch oscillation. Then, we can show that the π-mode coupled-bunch oscillation in this two-bunch system can be damped by using the same feedback system. This damping mechanism is essentially the same as the one used in the SLC damping ring [3]. Note that in case of the π-mode coupled-bunch oscillation, one can detect the oscillation more effectively by detecting $\pm\omega_s$ sidebands (m=1 mode) beside odd revolution harmonics, or by detecting each bunch oscillation independently, rather than by detecting $\pm 2\omega_s$ sidebands beside even revolution harmonics as shown in this paper.

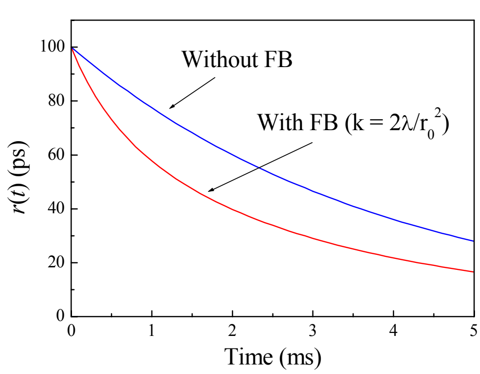
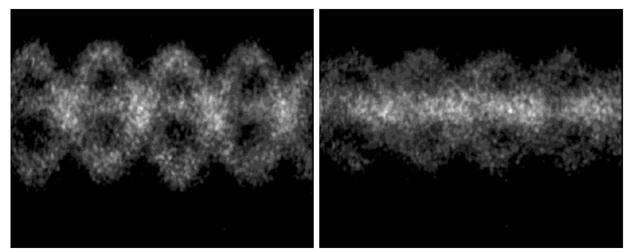


Figure 3: Calculated damping effect due to the feedback. Assumed r_0 = 100 ps, λ = 255 s^{-1}, and $\Delta\phi = \pi/2$.

EXPERIMENT

We carried out a feedback test experiment in the 2.5-GeV Photon Factory storage ring at KEK. While storing a single bunch of electrons, we artificially induced longitudinal quadrupole oscillations by exciting one of four rf cavities at a frequency of about ($\omega_{rf} - 2\omega_s$). Then, we tried to damp induced oscillations using a feedback system which is shown in Fig. 1. Most of the electronics were reuse of our old bunch-by-bunch feedback system [4]. Longitudinal bunch oscillations were observed using a dual-sweep streak camera. The results are shown in Fig. 4. The quadrupole oscillation was considerably reduced by using this feedback, as shown in Fig. 4(b).



(a) Without feedback. (b) With feedback.

Figure 4: Longitudinal quadrupole oscillations without and with the feedback, respectively, which were observed using a dual-sweep streak camera. Vertical axis: longitudinal bunch profile (1 ns full scale), horizontal axis: slow sweep (100 μs full scale). Beam current: 10 mA.

CONCLUSION

A feedback mechanism, which can damp longitudinal quadrupole-mode oscillations, was analyzed. It was shown that small-amplitude oscillation follows eq. (11), and its solution (12) gives the damping oscillation under both the feedback and the radiation damping. This analysis will be useful for designing such feedback systems, as well as for understanding its operations. Additionally, it was also shown that a π-mode coupled-bunch oscillation in two-bunch systems can be damped by using the same feedback system.

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