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COHERENT BEAM-BEAM EFFECTS

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Abstract

We have performed a strong-strong simulation to investigate the effect of a beam-beam interaction on the motion of circulating beams in a 50 GeV×50 GeV muon-collider ring. The phase-slip factor η_1 in the muon-collider ring is approximately -1×10^{-6} so that the synchrotron motion is nearly frozen during the muon-storage time. Thus, the energy variation of a particle in the beam due to the beam-beam interaction accumulates from turn to turn, resulting in increase in the energy spread of the beam. Through the strong-strong simulation, we have investigated the parametric dependencies of the energy spread and the beam size induced by the beam-beam interaction on various parameters. It is shown that the induced energy spread due to the beam-beam interaction affects the growth of the beam size. It is also shown that compensation of the energy spread by introducing one additional rf cavity in the ring can reduce the growth of the beam size.

1 INTRODUCTION

One of factors that may limit the performance of collider rings is the beam-beam interaction. A particle in a beam feels a nonlinear force due to electric fields produced by particles in an opposite beam. If the beam size varies significantly within the bunch length, the longitudinal effect which the beam energy changes has to be considered. Accordingly, the transverse kick depends on the longitudinal position as well as the transverse position [1]. This is the case for the proposed 50 GeV×50 GeV muon-collider ring [2].

We will show investigations of the beam-beam interaction in a 50 GeV×50 GeV muon-collider ring. We note that the muon-collider ring has several remarkable features: (i) the bunch has a large charge (number of muons in a bunch $n = 4 \times 10^{12}$), (ii) the beam-energy spread is very small ($\Delta E/E_0 = 3 \times 10^{-5}$), (iii) the beta function (β_{IP}) at the interaction point (IP) is comparable to the nominal bunch length, (iv) the muon has a short lifetime, $\tau_\mu \simeq 1.1$ ms at 50 GeV, corresponding to 1000 turns in a ring with a circumference (C) of 350 meters, and (v) the need to minimize the rf voltage leads to a small phase-slip factor $\eta_1 = -1\times 10^{-6}$ and the synchrotron-oscillation period is much longer than the muon-storage time. The small η_1 freezes the longitudinal positions of particles in the beam. Then, the energy variation of a particle in the beam due to the beam-beam interaction accumulates from turn to turn, and may increase the energy spread of the beam. Accordingly, it is necessary to maintain a small energy spread of the beams in a muon-collider ring because this makes it possible for a direct measurement of the resonance cross section of a low-mass Higgs boson. Maintaining an intense beam with a low energy spread provides a challenge to the ring design. Since the radiation damping and the Landau damping are also negligible in the muon-collider ring, it will be interesting to investigate the effects on growth of transverse beam size due to the induced energy spread.

In this paper, it is shown that the induced energy spread due to the beam-beam interaction may affect the growth of the beam size. It is also shown that compensation of the energy spread due to the beam-beam interaction can reduce the growth of the beam size. The strong-strong simulation was employed to investigate the beam-beam interaction in the 50 GeV × 50 GeV muon-collider ring. In the simulation, both transverse and longitudinal motions of each particle in the two strong beams are included. The dependence of the bunch length on the beam-beam interaction is considered because the beam size is a function of the longitudinal position in the interaction region. After we show that the energy spread and the beam size are increased by the beam-beam interaction, we present the parametric dependencies of the energy spread and the beam size on various parameters, such as the beam-beam parameter, bunch length and betatron function at IP. We then provide a means of controlling the energy spread by showing that the increase in the energy spread can be reduced by adding one rf cavity to the ring with a modest rf voltage.

2 EQUATIONS OF MOTION

We introduce scaled coordinates in transverse and longitudinal phase spaces. Each particle in the two strong beams is given by

$$Y = \frac{y}{\sigma_{y0}}, \quad Y' = \frac{\beta_{IP} y'}{\sigma_{y0}}, Z = \frac{z}{\sigma_{z0}}, \quad E(s) = \frac{\epsilon}{\sigma_{\epsilon 0}}, \quad (1)$$

where σ_{y0}, σ_{z0} and $\sigma_{\epsilon 0}$ are the nominal rms values at the IP of the transverse beam size, the bunch length and the relative energy spread, respectively. In the above equation, y and y' are the position and the slope of a particle in a beam in the transverse direction, respectively, and ϵ is the relative energy deviation from the nominal energy [$\epsilon = (E - E_0)/E_0$]. Here, we are assuming round beams, as is the case of the beams in the muon-collider ring.

At the IP, the change in scaled coordinates due to the beam-beam interaction is given by [1, 3]

$$Y \quad \rightarrow \quad Y - R_1(Z - Z_*)\delta P,$$

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$$\begin{aligned} Y' &\rightarrow Y' + \delta P, \\ E &\rightarrow E + R_2 \delta P (Y' + \frac{\delta P}{2}) - G. \end{aligned} \quad (2)$$

Here, the beam-beam interaction is modeled by the interaction between a macroparticle in one beam and one in a opposite beam with several slices. The center of each slice in two strong beams is represented by the longitudinal position z_*. In eq. (2), we have defined

$$\begin{aligned} \delta P &= 8\pi\xi \frac{n_*}{N_*} \frac{1}{Y + R_1(Z - Z_*)Y'} \left(e^{-A^2} - 1 \right), \\ G &= 8\pi\xi \frac{n_*}{N_*} \frac{R_1 R_2 Z - Z_*}{1 + R_1^2 (Z - Z_*)^2} e^{-A^2}, \\ A &= \frac{Y + R_1(Z - Z_*)Y'}{\sqrt{2(1 + R_1^2(Z - Z_*)^2)}}, \\ R_1 &= \frac{\sigma_{z0}}{2\beta_{IP}}, \quad (3) \\ R_2 &= \frac{\epsilon_0}{2\sigma_{\epsilon 0}\beta_{IP}}, \\ Z_* &= \frac{z_*}{\sigma_{z0}}. \quad (4) \end{aligned}$$

Here, ξ is the nominal beam-beam parameter defined by

$$\xi = \frac{N_* r_\mu}{4\pi\gamma\epsilon_0}, \quad (5)$$

where ϵ_0 is the transverse nominal emittance of each beam, r_μ is the muon radius and γ is the usual relativistic factor. More detailed discussions of eqs. (2) and (4) can be found in ref. 2.

Two strong beams are divided longitudinally into several slices, and a beam-beam kick given in eq. (2) for each slice is delivered at the barycenter Z_* [4]. Here, Z_* is the longitudinal barycenter of each slice in the opposite beam and each barycenter represents n_*/N_* of all particles in the opposite beam where N_* and n_* are the numbers of particles in the opposite beam and in each slice of the opposite beam, respectively.

The transformation in the transverse plane from IP to IP around the ring is simply a rotation in phase space:

$$\begin{pmatrix} Y \\ Y' \end{pmatrix} = \begin{pmatrix} \cos 2\pi\nu_y & \sin 2\pi\nu_y \\ -\sin 2\pi\nu_y & \cos 2\pi\nu_y \end{pmatrix} \begin{pmatrix} Y \\ Y' \end{pmatrix}, \quad (6)$$

where ν_y is the betatron tune of a ring.

Neglecting the synchrotron oscillation, as is the case for the muon-collider ring under consideration, the longitudinal phase-space transformation is given by

$$\begin{pmatrix} Z \\ E \end{pmatrix} = \begin{pmatrix} 1 & -\eta_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} Z \\ E \end{pmatrix}. \quad (7)$$

3 SIMULATION RESULTS

We simulate the collision of two strong μ^+ and μ^- beams. For initial parameters of eq. (1), 40000 macroparticles in each beam are distributed with a Gaussian distribution with an average of zero and a root-mean square (rms) value of one. Tracking up to 1000 turns is sufficient because it corresponds to the muon lifetime $\tau_\mu \approx 1.1$ ms.

3.1 Bunch-length effect

Two beams are divided into slices to take into account the effect of the initial bunch length on the motion of a beam. To examine this effect, the total number of slices n was varied from one to five. Figure 1(a) shows the resulting normalized rms energy spread as a function of the turn number with different numbers of slices (n=1, 2, 4, 5) in each beam. The figure clearly shows that the energy spread depends on n if it is too small. It is seen that the energy spreads for n=4 and n=5 are almost identical. The beam-beam parameter ξ in the two beams are given by 0.03.

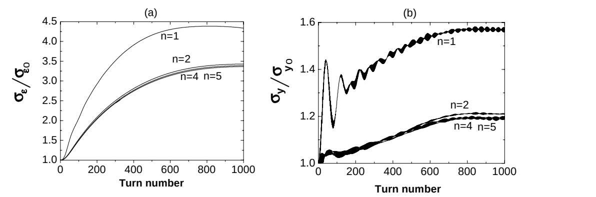

Figure 1: (a) Normalized rms energy spread ($\sigma_\epsilon/\sigma_{\epsilon 0}$) as a function of turn number and number of slices (n): $n = 1$, $n = 2$, $n = 4$ and $n = 5$ are shown from top to bottom. (b) Normalized rms transverse beam size (σ_y/σ_{y0}) as a function of turn number. In each figure, ν_y=6.239, ξ=0.03 and $\sigma_{yIP} = 294\mu m$.

3.2 Dependence on beam-beam parameter

Figure 2 shows the variation of beam size and energy spread as a function of ξ. This figure indicates that the transverse beam size increases by a factor of 1.5 between $\xi = 0.01$ and $\xi = 0.055$. On the other hand, it is seen that the energy spread is a strong function of ξ.

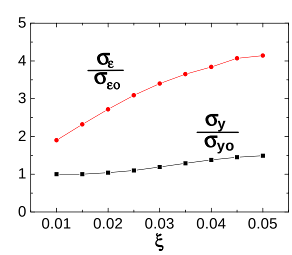

Figure 2: Beam-beam parameter (ξ) dependence on the scaled rms beam size and the scaled rms energy spread after 1000 turns. $\sigma_{yIP} = 294\mu m$, ν_y=6.239 and $n = 5$.

3.3 Collision with an initial offset

We looked the effect of colliding the beams with an initial y offest. Figure. 3 (a) and (b) show the beam sizes as a function of turn number when offsets in the two beams are given by 1σ and 0, respectively. The beam (a) with an initial offset shows a larger beam size than the beam (b) without initial offset. This effect becomes more pronounced as the maginitude of the offset is increased. Figure 3 (c) and (d) show the beam sizes as a function of turn number when two beams don't have initial offsets. The beam sizes in this case show both the same. The beam-beam parameters in the Figure 3 is given by 0.04 in each beam.

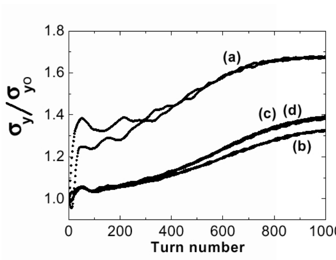

Figure 3: (a) and (b) show beam sizes for the cases that intial beam offsets in the two beams are given by $1\sigma_y$ and 0, respectively. (c) and (d) show beam sizes for the cases that intial beam offsets in the two beams are both zero. ν_y=6.239, $\sigma_{yIP} = 294\mu m$, $n = 5$ and $\xi = 0.04$.

3.4 *Compensation for the energy spread*

Figure 4(a) shows the increment of the energy spread of a beam due to the beam-beam interaction in the longitudinal direction after one turn. The beam-beam parameters of $\xi = 0.03$ are given by each beam. In order to compensate for the increase in the energy spread, a rf cavity can be utilized in such a way that the rf voltage almost cancels the induced energy spread by applying an opposite kick. Since the muon beam decays with time, the magnitude of the applied rf voltage also decreases with time according to

$$V_{rf} = V_0 \, e^{-\frac{NT_0}{\tau_\mu}} \sin(\omega_{rf} t + \phi_0), \tag{8}$$

where V_0 is the initial amplitude of the applied rf voltage, N is the number of turns, T_0 is the revolution period of a muon beam, τ_μ is the lifetime of a muon particle, and ϕ_0 is the initial rf phase. In eq. (8), the angular frequency ω_{rf} of the rf is determined by the bunch length, and if its value is appropriately chosen together with that of the voltage amplitude V_0, the induced energy spread due to the beam-beam interaction can be minimized.

Figure 4(b) shows dependence of the energy spread and beam size on the beam-beam parameters when $f_{rf} = 280$ MHz, V_0 is 16.4 kV and the phase offset ϕ=$-\pi$. Upon comparing this figure with Figure 3, it is clear that in the presence of the rf cavity the induced beam size and energy spread due to the beam-beam interaction are significantly reduced. Figure 4(b) clearly show that the energy spread and the beam size due to the beam-beam interaction in the 50 GeV$\times$50 GeV muon-collider ring can be reduced for by one rf cavity with a low rf voltage. Figures 4(c) and 4(d) show the beam phase space after 1000 turns in the presence of the rf voltage. We note that the beam phase spaces with a rf cavity do change noticeably after 1000 turns. The beam-beam parameters of $\xi = 0.04$ in Figure 4(c) and (d) are given by each beam.

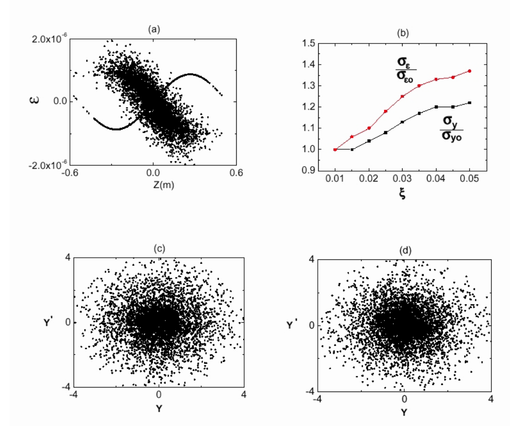

Figure 4: (a) Increments of the energy spread due to the beam-beam interaction after one turn. Energy spread with one rf voltage after 1 turn is also shown. $\xi = 0.03$ (b) The energy spread and the beam size due to the beam-beam interaction verse various beam-beam parameter. One rf voltage is applied to compensated for the energy spread. (c) and (d) The transverse phase space distribution for $\xi = 0.04$ with a rf cavity. In each figure, ν_y=6.239, $\sigma_{yIP} = 294\mu m$ and $n = 5$.

4 CONCLUSIONS

The effect of the beam-beam interaction in a 50 GeV$\times$50 GeV muon-collider ring has been investigated with a strong-strong simulation. The simulation demonstrated the necessity of including the bunch-length effect. Since the synchrotron motion in muon-collider ring is frozen during the storage time, the beam-beam interaction causes a large beam-energy spread. But, the growth of the energy spread can be controlled by employing an additional rf cavity with a modest rf voltage. It is shown that the energy spread due to the beam-beam interaction affects growth of the beam size. It is also shown that compensation of the induced energy spread can reduce the growth of the beam size.

5 REFERENCES

[1] K. Hirata, H. Moshammaer and F. Ruggiero: Part. Accel. **40** (1993) 205.

[2] C.M. Ankenbrandt, et al.: Phys. Rev. Special Topics: Accelerators and Beams **2** (1999) 081001.

[3] K. Hirata, H. Moshammaer and F. Ruggiero: CERN SL-AP/90-02, 1990.

[4] S. Krishnagopal and R. Siemann: Phys. Rev. D **41** (1990) 2312.