# RF UNDULATOR FIELD AND ION BEAM ACCELERATION IN LINAC

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*Abstract*

The possibility to use radio frequency undulator fields for ion beam focusing and acceleration in linac is discussed. In periodical structure we consider has no space harmonics in synchronism with the beam. The accelerating force is produced by the combination of two or more space harmonics of RF undulator field. The particle motion equations in Hamilton's form are devised by means of smooth approximation. The analysis of 3D effective potential permits to find the conditions under which focusing and acceleration of the particles occur simultaneously. Examples illustrating the efficiency of the proposed method of acceleration are given for longitudinal and transverse undulators. The results are compared with a conventional linac and the other possibility of ion beam acceleration in UNDULAC-E(M) where electrostatic and magnetic fields are used.

## 1 INTRODUCTION

In a conventional RF linac the beam is accelerated by a synchronous wave. Another method to accelerate ions –in the fields without a synchronous wave- was suggested in Ref. [1]. In this case accelerating force is to be driven by a combination of two non-synchronous waves (two undulators). In an undulator linac in question one of undulators must be RF type (it drives nonsynchronous RF wave field), the second one being, optionally, of magnetic, electrostatic or RF type.

Some versions of the undulator accelerator using magnetic and electrostatic undulator was suggested in papers [2]–[4]. The combined acceleration field can be created without using of the magnetic and electrostatic undulator. In this paper ion beam dynamics is considered in the periodical RF structure without synchronous wave. Methods of realisation of RF undulator accelerator for ion are investigated.

## 2 PARTICLE MOTION EQUATION

The transverse $\boldsymbol{E}_\perp$ and longitudinal $E_z$ electric RF field components in periodical cavity can be expanded into Fourier series:

$$\boldsymbol{E}_\perp = \sum_n \boldsymbol{E}_{n,\perp}(x,y)\cdot\sin(h_n z+\alpha)\cdot\cos(\omega t),$$
$$E_z = \sum_n E_{n,z}(x,y)\cdot\cos(h_n z+\alpha)\cdot\cos(\omega t), \quad (1)$$

where $h_n = \mu/D + 2\pi n/D$, $\mu$ are the propagation factors and phase advance per period of RF structure, $n$=0,1,2,... . The interaction of the beam with each harmonic can be examined as ion interaction with radio frequency undulator (UNDULAC-RF) when the phase velocities $v_{ph,n} = \omega/h_n$ of harmonics differ significantly from the average velocity of the particles $v_b$. For longitudinal undulator $\boldsymbol{E}_{n,\perp}(0,0)=0$ and $\alpha$=0, for transverse one $E_{n,z}(0,0)=0$ and $\alpha=\pi/2$. The combined field of two harmonics $n$ and $p$ will accelerate the beam if beam velocity $v_b = v_c = \omega/k_z$, where the propagation factor $k_z$ of combined wave field: $k_z=(h_n \pm h_p)/2$ ($k_z \neq h_n \neq h_p$ , $n$=0, 1, 2 ; $p$=0, 1, 2 )

In general the trajectories of the individual particles of the beam have complicated shape but can always be represented as the sum of a slow variation $\bar{\mathbf{r}}$ and rapid oscillation $\tilde{\mathbf{r}}$. Accordingly, the beam momentum $\mathbf{p}$ can be represented as the sum of a slowly varying and a rapidly oscillating component, $\mathbf{p}=\bar{\mathbf{p}}+\tilde{\mathbf{p}}$. On averaging over fast oscillations, as it was done in [1], one obtains a time-averaged equation of motion for a nonrelativistic ion:

$$\frac{d^2\boldsymbol{R}}{d\tau^2} = -\frac{d}{d\boldsymbol{R}}\tilde{U}_{eff}, \quad (2)$$

where dimensionless slow co-ordinate $\boldsymbol{R}=2\pi\bar{\boldsymbol{r}}/\lambda$, $\tau=\omega\cdot t$. If all space harmonics of RF field are taken into account terms composing effective potential function $U_{eff}$ in Eq (2) are determined by dimensionless harmonics amplitudes of field $\boldsymbol{e}_\nu = e\boldsymbol{E}_\nu\lambda/(2\pi mc^2)$:

$$\tilde{U}_{eff} = \tilde{U}_1+\tilde{U}_2, \quad (3)$$

$$\tilde{\boldsymbol{U}}_{\boldsymbol{1}} = \frac{1}{16}\sum_{n=0}^{\infty}\boldsymbol{e}_n^2\left[\left(\Delta^-_{n,c}\right)^{-2}+\left(\Delta^+_{n,c}\right)^{-2}\right], \quad (3a)$$

$$\tilde{\boldsymbol{U}}_{\boldsymbol{2}} = \frac{1}{16}\sum_{h_p+h_n=2k_z}\left(\boldsymbol{e}^z_n\cdot\boldsymbol{e}^z_p-\boldsymbol{e}^\perp_n\cdot\boldsymbol{e}^\perp_p\right)\cdot\left(\Delta^-_{n,c}\right)^{-2}\cdot\cos(2\psi+2\alpha)+ \quad (3b)$$
$$+\frac{1}{8}\sum_{|h_p-h_n|=2k_z}\left(\boldsymbol{e}_n\cdot\boldsymbol{e}_p\right)\cdot\left(\Delta^-_{n,c}\right)^{-2}\cdot\cos(2\psi),$$

where is slow variation particle phase $\psi=\int\frac{dZ}{\beta_s}-\tau$, $\Delta^\pm_{n,c}=(h_n\pm k_z)/k_z$

The 3D dynamics of the ion beam in an undulator linear accelerator (UNDULAC) is defined by the particular type

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of undulator and transverse structure of its field [5]. As an example we shall consider the task about acceleration and focusing of the ribbon beam in the plane UNDULAC-RF. Three possible variants of realisation of such RF system by means of a periodic sequence of transverse electrodes are shown in Fig 1, where RF potential $V_v = V\cos\omega \cdot t$ is applied on electrodes. The depending on a phase difference of $V_v$ between adjacent pairs of electrodes it is possible to realise longitudinal (Fig.1a) or transverse (Fig.1b) undulator. In these simplest examples the RF field can have a phase advance per period $\mu_v = 0$ and $\mu_v = \pi$.

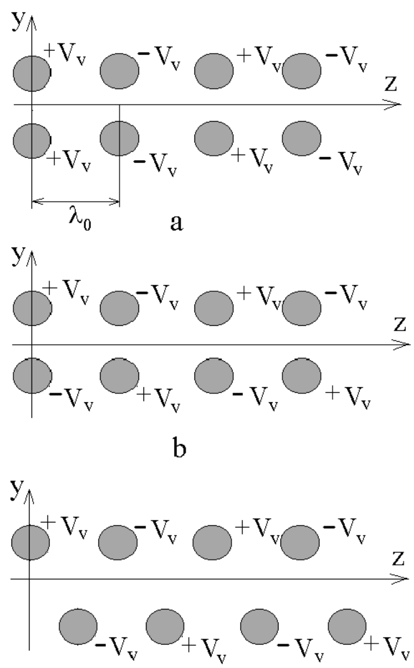


Figure 1: The periodic sequence of transverse electrodes for $\mu_v = \pi$

For plane structure the electric field depends only on longitudinal co-ordinate $Z$ and transverse $Y$. The amplitudes of field harmonics can be written as:

$$E^z_{v,n} = E_{v,n}\cosh(Y/\beta_{ph,n}),\quad E^y{}_{v,n} = E_{v,n}\sinh(Y/\beta_{ph,n}) \quad (4)$$

for longitudinal undulator and

$$E^z_{v,n} = E_{v,n}\sinh(Y/\beta_{ph,n}),\quad E^y_{v,n} = E_{v,n}\cosh(Y/\beta_{ph,n}) \quad (5)$$

for transverse undulator.

## 3 BEAM FOCUSING AND ACCELERATION

Let's find the conditions of focusing and acceleration for ribbon beam, using expressions for $U_{\text{eff}}$. In the items (3) it is enough to keep first two harmonics with $n$=0, 1. If the field has a phase advance per period $\mu_v = 0$, and beam velocity $\beta_b \approx \beta_c = 2\lambda_0/\lambda$ the dimensionless effective potential can be written as:

$$U_{eff} = \frac{1}{8}\Big[e_0^2 + \frac{5}{9}e_1^2 \cdot \cosh(4Y/\beta_c) + \\ + 2e_0 e_1 \cdot \cosh(2Y/\beta_c)\cdot\cos\ 2\psi\Big] \quad (6)$$

From (6) we obtain the equation for longitudinal particle velocity:

$$\frac{d\ \beta_z}{d\tau} = \frac{e_0 e_1}{2\beta_c}\cdot\cosh(2Y/\beta_c)\cdot\sin\ 2\psi\,. \quad (7)$$

The acceleration and phase stability of the beam are possible when the phase of a synchronous particle $\psi_c$ in the combined field is disposed in intervals $[\pi/4, \pi/2]$ and $[5\pi/4,\ 3\pi/2]$. In this case the frequency of ion beam bunching doubles ($\omega_b = 2\omega$). The transverse focusing of the beam is possible if there is a minimum of potential function (6) with respect to the transverse co-ordinates $Y$. Expanding the function $U_{\text{eff}}$ near axis one can find that amplitude of the first harmonic $e_1 > 0{,}9e_0$.

This result is interesting to compare with another possibility of acceleration in UNDULAC-RF in which the field has a phase advance per period $\mu_v = \pi$. If the beam velocity $\beta_b \approx \beta_c = \lambda_0/\lambda$, effective potential can be written as:

$$U_{eff} = \frac{1}{4}\Big[e_0^2\cosh(Y/\beta_c) + e_1^2\cosh(3Y/\beta_c) + \\ + 2e_0 e_1\cosh(Y/\beta_c)\cos 2\psi\Big] \quad (8)$$

The equation of longitudinal particle motion has the form:

$$\frac{d\beta_z}{d\tau} = \frac{e_0 e_1}{\beta_c}\cdot cosh(Y/\beta_c)\, sin\ \ 2\psi\,. \quad (9)$$

In this case the acceleration is possible for the same values of synchronous particle phase but the rate of energy gain will be twice as high. The analysis of the potential well has shown that it has a minimum in the transverse plane (focusing condition holds) for any relation between amplitudes of zero and first harmonics of RF field. This is the other important advantage against undulator for which field has a phase advance $\mu_v = 0$. It is interesting to mark that the all results obtained above concern both longitudinal and transverse UNDULAC-RF. In the plane undulator there is a possibility to select such arrangement of transversal electrodes (see fig.1c), when the condition $E^y{}_{v,n} = E^z{}_{v,n}$ will be carried out in any cross-section of the channel In this case terms $\tilde{U}_2 = 0$ and the effective potential $U_{\text{eff}}$ is independent of particle phases $\psi$. It signifies, that ions will be no acceleration, but here is possible to consider RF undulator as a device ensuring transverse focusing of ribbon beam.

## 4 UNDULAC AND LINAC

The rate of energy gain is given (7) and (9): $dW_c/dz = (1/2)\cdot eT_{rf}E_{v,0}\cdot\sin 2\psi$, where acceleration efficiency factor $T_{rf} = \dfrac{eE_{v,1}\lambda}{2\pi mc^2\beta_c}$ for μ=0 and $T_{rf} = \dfrac{eE_{v,1}\lambda}{\pi mc^2\beta_c}$ for μ=π. The rate of acceleration is proportional to amplitudes of RF field harmonics $E_{v,0}$ and $E_{v,1}$. In the other versions of UNDULAC-E (M) [2-4] the rate of acceleration is proportional to the amplitude of RF field $E_v$ and undulator field $E_o$ ($B_o$). For these cases the acceleration efficiency factor

$$T_m = \frac{eB_0\lambda_0}{2\pi mc}, \quad T_e = \frac{eE_0\lambda^2}{2\pi mc^2\lambda_0}. \tag{10}$$

The choice of the undulator field amplitudes is no arbitrary because simultaneously with acceleration it is necessary to keep up the transverse focusing of the beam. The choice of field amplitude $E_o$ ($B_o$) depend on $E_v$, since it is necessary to provide for the focusing and large transmission coefficient $K$. As show in [5] for this case the acceleration efficiency factors $T_m \approx T_e = \dfrac{eE_v\lambda}{2\pi mc^2\beta_c}$. This value is equal to $T_{rf}$ for μ=0.

It is interesting to compare these factors with linac in which axisymmetric RF focusing is realized. In this case a synchronous harmonic $E_s$ accelerates and defocusing particles and nonsynchronous one $E_n$ only focusing beam. As was shown in [5], [6] the trajectory of the particle with arbitrary ψ is stable in the transverse direction when

$E_s < \dfrac{eE_n^2\lambda}{2\pi mc^2\beta_c}\overline{\alpha}_{ns}g$. To express the acceleration rate with nonsynchronous harmonics amplitude $E_n$, we use:

$$\frac{dW_c}{dZ} = T_L E_n \cos\psi_c, \tag{11}$$

where the factor $T_L = \dfrac{eE_n\lambda}{2\pi mc^2\beta_c}\overline{\alpha}_{ns}g$. For the case, when s=0, n=1, coefficient α≈1 and $T_L$ is close to $T_m$ and $T_e$ but is twice lower than $T_{rf}$ for UNDULAC-RF, where the field has a phase advance per period $\mu_v = \pi$. Besides, the condition of focusing in an UNDULAC-RF can be fulfilled for any value of $E_0/E_1$, and the frequency of ion beam bunching is doublet $\omega_b=2\omega$. Peak values of $E_0$ and $E_1$ being found from RF performance data of the resonator, transverse acceptance and beam current.

## 5 COMPUTER SIMULATION

In order to test all basic obtained results the computer simulation was done. The calculations of average energy, energy spread, transverse and longitudinal acceptances, transmission coefficient were carried out both in polyharmonic fields and in averaged field. In the range of proton energy from 0.1 to 2 MeV the results of calculation for energy and transverse dynamics coincide up to 5-10%. It is good confirmation of smooth approach for transverse and longitudinal RF undulators.

## 6 CONCLUSION

Theoretical studies of using undulators in acceleration system showed a possibility to create a new type of ion linear accelerator (UNDULAC ). The requirement of particle focusing for all phases ψ imposes a limitation on the field amplitudes in all types undulator besides UNDULAC-RF with $\mu_v = \pi$, where focusing condition holds for any relation between harmonic amplitudes. The rate of energy gain in UNDULAC is comparable with analogous value for RFQ and conventional linac, where RF focusing is realised. But new accelerator has series of the advantage. It's possible to use not only longitudinal but also transverse radio frequency field in these accelerators. In these accelerators one can increase ion beam intensity [7]. There are two ways: *i*) to enlarge beam cross-section; *ii*) to compensate for charge by accelerating ions with opposite charge signs within the same bunch.

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