RECENT STUDIES OF FFAGS IN THE USA *

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Abstract

Muon accelerators have proved to be difficult to design because of potentially heavy losses from decay. The rapidity with which acceleration must occur precludes the use of conventional synchrotrons. The current baseline approach is a recirculating linac (RLA) in the shape of a racetrack with opposing linacs and separate, fixed-field arcs for each acceleration turn. However, the RLAs have proved costly and, in addition, they represent a bottleneck in the acceptance of a Neutrino Factory. An alternative, a fixed-field, alternating gradient (FFAG) accelerator is actively being revisited in recent work and this paper emphasizes the FFAG option.

1 INTRODUCTION

Because of potentially heavy losses from decay, acceleration must occur rapidly for any application requiring a high-energy, intense muon source; e.g. a Neutrino Factory or a Muon Collider. Linear acceleration is the most efficient, but above a GeV it becomes prohibitively expensive. Conventional synchrotons cannot be used because the rapid rate and cycle time required preclude ramping even normal conducting magnets[1]. The current baseline approach employs recirculating linacs with separate, fixed-field arcs for each acceleration turn. However, the RLAs with their separate return arcs have not only proved costly, they also represent a bottleneck in acceptance, and, as a result, set the present neutrino intensity limit for the U.S. Neutrino Factory designs[2]. (As an example, the combined longitudinal and transverse acceptance of the RLA is a factor of 4-5 below that of the Neutrino Factory storage ring[2].)

and expense of ultra-rapid cycling synchrotrons and recirculating linacs, the idea of using fixed-field, singlepath accelerators has been revisited in recent work. The arcs of such machines, composed of large-bore superconducting magnets, can be designed to accommodate the large energy range in acceleration. Lattices have been developed which can contain an energy change of a factor of four[3]. These recent studies of FFAG accelerators represent an effort to reduce cost and promote an acceptance which is better matched to the performance of the ionisation cooling system and the storage ring.

2 FFAGS FOR MUON ACCELERATION

An overriding consideration for a Neutrino Factory is to design an acceleration system which has an exceptionally

large acceptance, both transversely and longitudinally, thereby reducing as much as possible the degree of beam cooling required. Hence the naturally-large longitudinal acceptance of the FFAG makes it an attractive option to explore and potentially address the acceptance issues associated with the RLA designs. Further, revisiting FFAG lattices in light of present superconducting technology and magnet design has advanced their reach into the multi-GeV regime, making a chain of FFAG accelerators a potential candidate for a complete acceleration scenario—a scenario applicable to either the Neutrino Factory or Muon Collider.

2.1 Types of FFAG Lattices

A circular accelerator system can be designed with fields that remain constant for the duration of the acceleration cycle using an alternating gradient focussing lattice. The closed orbits are not fixed as in a ramped machine, but rather move across the magnet aperture during the cycle. There are three basic types of alternating gradient structures used in FFAG lattice design. These include:

- Traditional scaling FFAG
- Triplet-based scaling FFAG
- Non-scaling FFAG

The traditional scaling FFAG is comprised of combined-function short FODO cells with edge focussing and B fields which scale with momentum. The consequence of scaling the magnetic fields is that the orbit properties are maintained constant as a function of momentum and the optics are also independent. Such FFAG rings were first designed and studied at MURA[4]. The triplet-based FFAG is a recent innovation based on the scaled-field concept, but formed from a triplet quadrupole structure rather than a FODO one. (It was developed for the KEK Proof of Principle, or POP, Its primary advantage over the previous machine.) structure is incorporation of a significantly longer straight section in each cell facilitating injection, extraction, rf insertion, etc.

The non-scaling FFAG is a concept unique to muon acceleration where acceleration occurs so rapidly the beam experiences only a few turns in the accelerator. For such rapid acceleration, one does not have to avoid resonances or control lattice parameters as a function of momentum. Instead, one has the freedom to choose parameters optimal for muon acceleration such as minimizing circumference and requiring a large transverse dynamic aperture.

2.2 Example of a 16-64 GeV Scaling FFAG

The following, Figure 1 and Table 1, is an example of a scaling, radial-sector FFAG with a large field index designed to collapse a factor of 4 range in momentum (16-64 GeV) into a modest SC magnet aperture. Table 1 gives parameters of the ring and Figure 1 the optics of a single sector, including a long, dispersion-suppressed, 14 m straight section.

Table 1. Parameters of a 16-64 GeV scaling FFAG.

General	
FFAG type	Radial sector
Energy Range	16-64 GeV
Circumference	1356 m
Arc Cells	
Central Energy	40 GeV
Rigidity	133.4 T-m
Magnetic Field @ 40 GeV	4.86 T
Gradient	125 T/m
Field Index (n)	707
Radius of Curvature	27.5 m
"F' Length	1.79 m
"D" Length	1.13 m
Cell Tunes, v_x/v_y	0.416/0.084
Radial Displacements :	
16 GeV	-3.6 cm
64 GeV	1.8 cm



Figure 1. A sector of the 16-64 GeV scaling FFAG showing the 14 m dispersion-suppressed drift.

Even with careful design, the performance limitations of the scaling FFAG generally eliminate it as a candidate for high-energy muon acceleration when a large range in energy is involved. The primary obstacle comes from the large transverse muon beam size which must be transported along with the large change in energy. Keeping the orbit and optical properties consistent require the B field to scale with momentum, but achieving a largetransverse dynamic aperture requires that B' must be nearly constant. For a large energy range, this implies the horizontal spread of orbits becomes large. For example, at 160 T/m, the 16 and 64 GeV orbits would be separated by 0.75 m; apertures which are incompatible or far too expensive for SC magnets. Curtailing the magnet aperture means the field must rise sharply with radius through addition of higher-order field terms. (The most significant terms are generally sextupole and octupole.) Degradation of dynamic aperture follows.

In the above example, the stable dynamic aperture of the arc cell alone was found to be only 450π mm-mr (normalized) at 16 GeV, which is inadequate to transport even a muon beam cooled to the extent needed for a collider. (This represents only $\pm 3\sigma$ of the projected transverse emittance for a muon collider[1].)

2.3 Example of a 4-16 GeV Nonscaling FFAG

As mentioned earlier, muon acceleration occurs so rapidly that resonances are not a consideration. The rf systems are assumed to deliver on the order of 0.5-2 GeV per turn. In this case, the beam can be accelerated through an integer, or other resonance-driving "global" tunes if the tune is only valid for a fraction of a turn. With a fast acceleration cycle, the lattice's optical parameters are released from scaling with momentum as well. One is then allowed the freedom to choose machine parameters which are optimal for muon beam acceleration; i.e. minimizing the circumference to limit intensity loss from decay and maximizing the transverse dynamic aperture to accept a less-cool beam. This approach has been termed a non-scaling FFAG accelerator.

The lattice components, parameters, and functions of the non-scaling FFAG cell are given in Table 2 and plotted in Figure 2. The non-scaling approach yields the smallest design circumference of any lattice and can approach a factor of 2 less than that of a scaling lattice(1.3-1.7 in the present example).



Figure 2. Lattice functions at 54 GeV for a 16-64 GeV non-scaling FFAG.

Relaxing the requirement for consistency in closed orbits at different energies means that the orbits no longer remain parallel. The peak of the closed orbit excursion always occurs at the center of the F quadrupole. Orbit excursions at 16 GeV are plotted in Figure 3 (top). The corresponding orbit excursion at 64 GeV is almost an inversion of the 16 GeV curve as can be seen in Figure 3 (bottom). The need for a large transverse dynamic aperture is automatically satisfied in this design because only linear elements are used.

Table 2. Parameters of a 16-64 GeV Non-scaling FFAG.

General	
FFAG type	nonscaling
Energy Range	16-64 GeV
Central Energy	54 GeV
Circumference	900-1100 m
Half-cell straight length	1.3-1.8 m
Rigidity	180 T-m
Maximum Poletip field	8 T
Arc Cell	
Number	200
Length	5.5 m
Half-cell straight length	1.8 m
Bend/cell	0.0157 rad
Quadrupole Gradient	57.1 T/m
"F' length	0.76 m
"F" strength	0.33 m^{-1}
"D" length	1.14 m
"D" strength	0.32 m^{-1}
Cell Tunes:	
@16 Gev	0.45 (162°)
@64 GeV	0.08 (29°)
Bmax:	
@16 GeV	33 m
@54 GeV	13 m
@64 GeV	15 m
Maximum Displacements:	
16 GeV	-11 cm
64 GeV	14 cm



Figure 3. 16-GeV (top) and 64-GeV (bottom) orbit amplitudes for a 16-64 GeV non-scaling FFAG.

2.6 Pathlength Dependencies in FFAGs

A main drawback to FFAGs in both the scaling and the non-scaling versions is the large changes in pathlength as a function of energy. The pathlength depndence is clearly linear with momentum for radially-staggered, parallel orbits as in the scaling case, but it is parabolic in non-scaling FFAGs (Figure 4).

The circumference change is problematic because as the highly relativistic beam changes energy, its time of arrival is incorrect relative to the rf phase—it walks significantly away from the synchronous phase. For example, if the rf system is 200 MHz, then a pathlength change of 50 cm from the central orbit means the bunch arrives 1/6 of an rf wavelength out of phase and will not accelerate properly or even remain bunched.

16-64 GeV Non-scaling FFAG



Figure 4. Circumference dependence on Momentum.

Recently workshops have focussed on the phasing problems of the FFAGs and a number of solutions are being advanced. These are:

- Chicanes
- Broad-band rf
- Low frequency rf
- Multiple rf frequencies

In summary, scaling FFAGs for the most part have not been found to be applicable to muon acceleration in the multi-GeV regime due to their poor transverse dynamic aperture. It looks promising, however, to build a chain of muon accelerators from FFAGs and replace the costly and restrictive RLAs given any of the above solutions to the phase-slip problem are feasible.

3 REFERENCES

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