Institute for Research in Electronics & Applied Physics

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Optimizing the Discovery of Underlying Nonlinear Beam Dynamics

Bright Beams Collective

Liam A. Pocher, Levon Dovlatyan, Irving Haber, Thomas M. Antonsen Jr., Patrick G. O'Shea

We are addressing one of the grand challenges in Beam Physics.

Grand Challenge #4: Beam Prediction – "How do we develop predictive 'virtual particle accelerators'?"

- **Aim**: speed up commissioning and design studies of accelerators by uncovering underlying physics in virtual and real accelerators
- **Approach**: apply an existing method from the data-driven, nonlinear dynamics community called **SINDy**

What is SINDy and how can it be used for Beam Physics?

- SINDy = **S**parse **I**dentification of **N**onlinear **Dy**namics
- Uncover physics in problems that can't be solved analytically.
- *Predictive* and *Productive*

$$\mathbf{x} \in \mathbb{R}^n \qquad \frac{d}{dt}\mathbf{x} = \mathbf{f}(\mathbf{x})$$

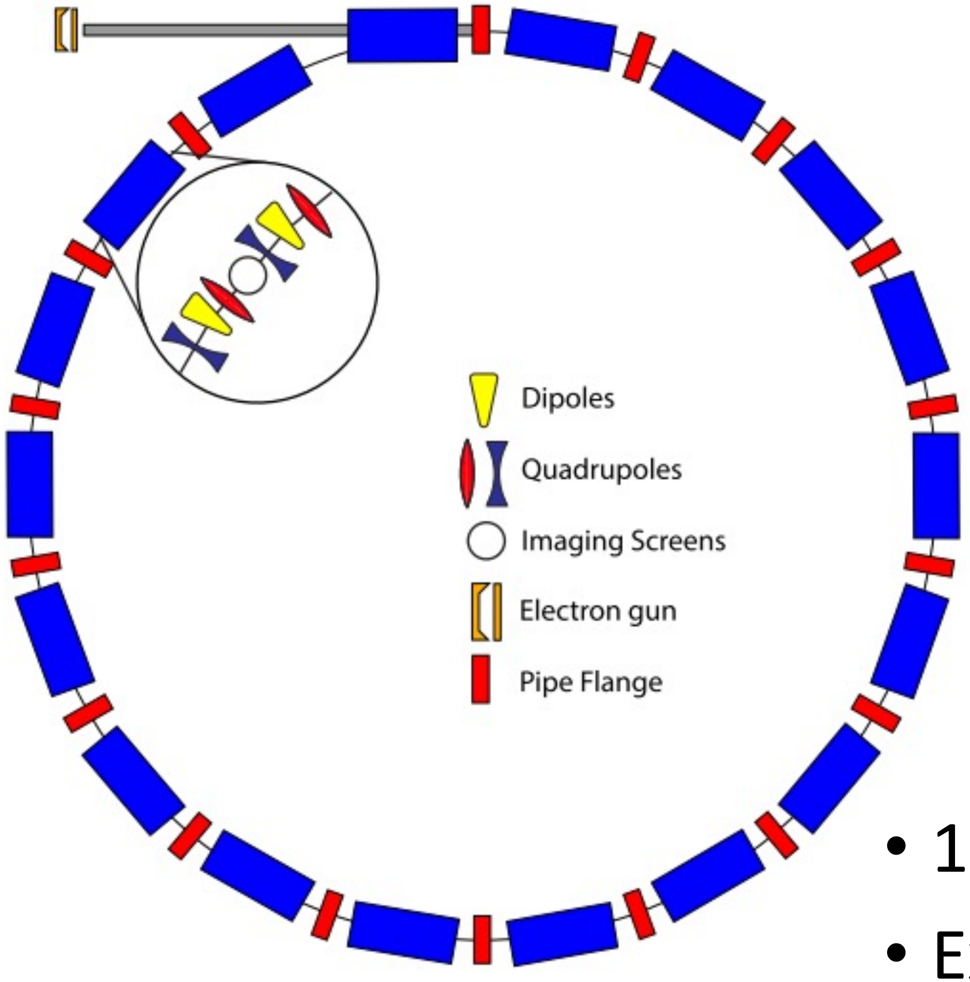
$$\mathbf{X} = \begin{bmatrix} \mathbf{x}^T(t_1) \\ \mathbf{x}^T(t_2) \\ \vdots \\ \mathbf{x}^T(t_m) \end{bmatrix} = \overset{\text{state}\ \longrightarrow}{\begin{bmatrix} x_1(t_1) & x_2(t_1) & \cdots & x_n(t_1) \\ x_1(t_2) & x_2(t_2) & \cdots & x_n(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ x_1(t_m) & x_2(t_m) & \cdots & x_n(t_m) \end{bmatrix}} \downarrow \text{time}$$

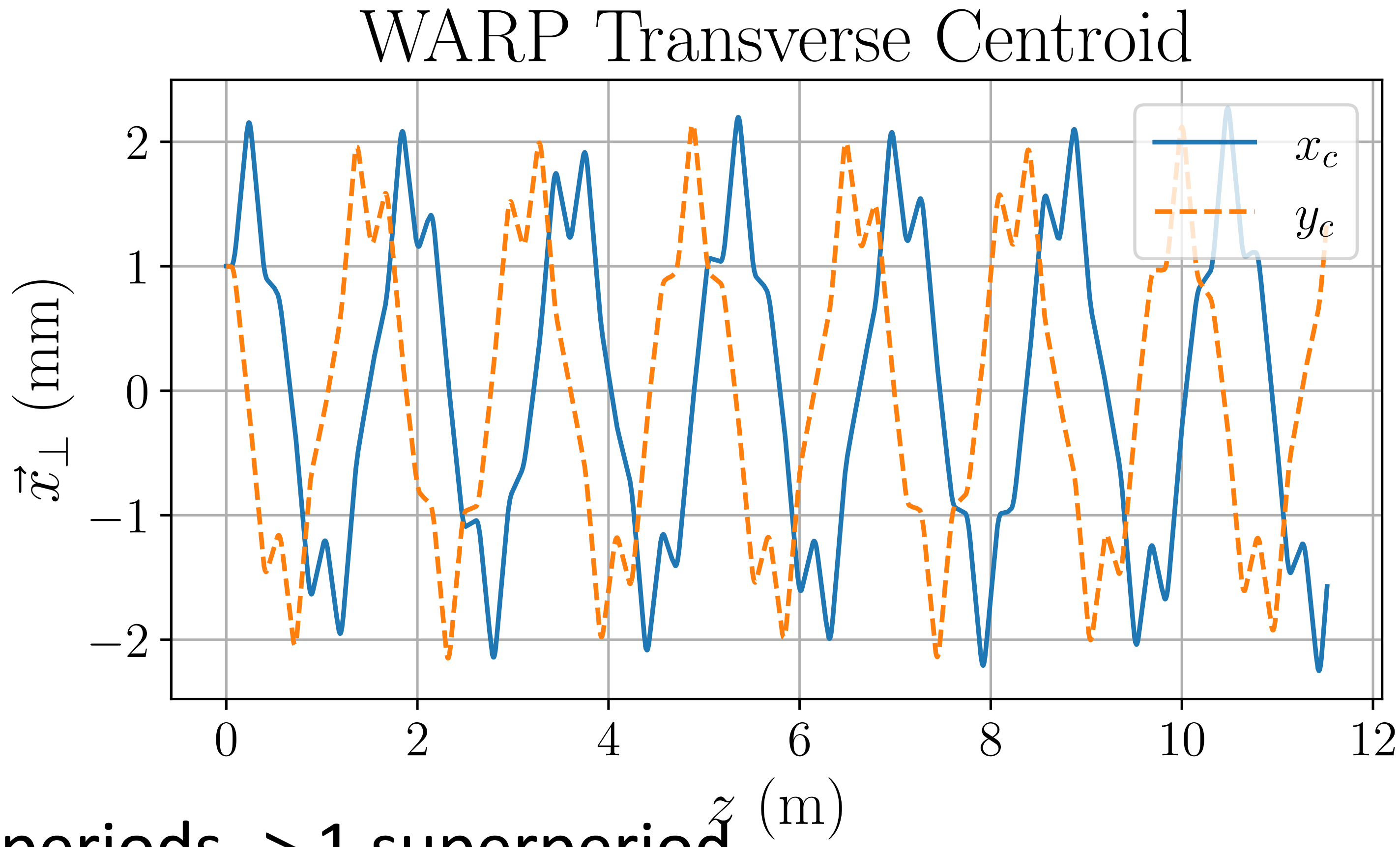
$$\dot{\mathbf{X}} = \begin{bmatrix} \dot{\mathbf{x}}^T(t_1) \\ \dot{\mathbf{x}}^T(t_2) \\ \vdots \\ \dot{\mathbf{x}}^T(t_m) \end{bmatrix} = \begin{bmatrix} \dot{x}_1(t_1) & \dot{x}_2(t_1) & \cdots & \dot{x}_n(t_1) \\ \dot{x}_1(t_2) & \dot{x}_2(t_2) & \cdots & \dot{x}_n(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ \dot{x}_1(t_m) & \dot{x}_2(t_m) & \cdots & \dot{x}_n(t_m) \end{bmatrix}.$$

$$\Theta(\mathbf{X}) = \begin{bmatrix} | & | & | & | & & | & | & \\ 1 & \mathbf{X} & \mathbf{X}^{P_2} & \mathbf{X}^{P_3} & \cdots & \sin(\mathbf{X}) & \cos(\mathbf{X}) & \cdots \\ | & | & | & | & & | & | & \end{bmatrix}$$

$$\Xi = [\boldsymbol{\xi}_1\ \boldsymbol{\xi}_2\ \cdots\ \boldsymbol{\xi}_n] \qquad \dot{\mathbf{X}} = \Theta(\mathbf{X})\Xi$$

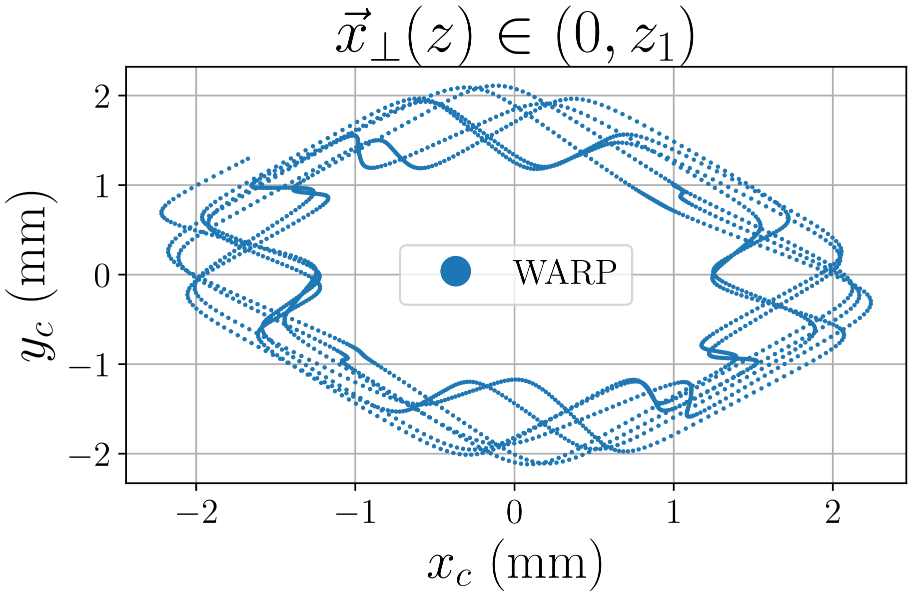
Example Problem: University of Maryland Electron Ring



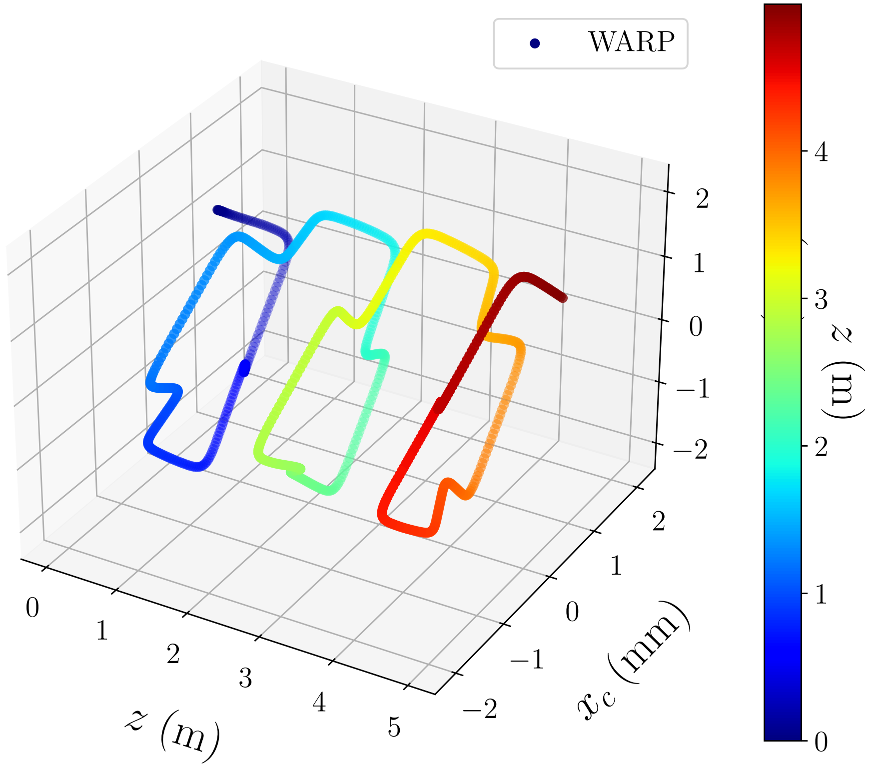


- 18 periods -> 1 superperiod
- Examine transverse displacement $\vec{x}_\perp(z)$ for 1 turn
- Using WARP data-source for virtual data

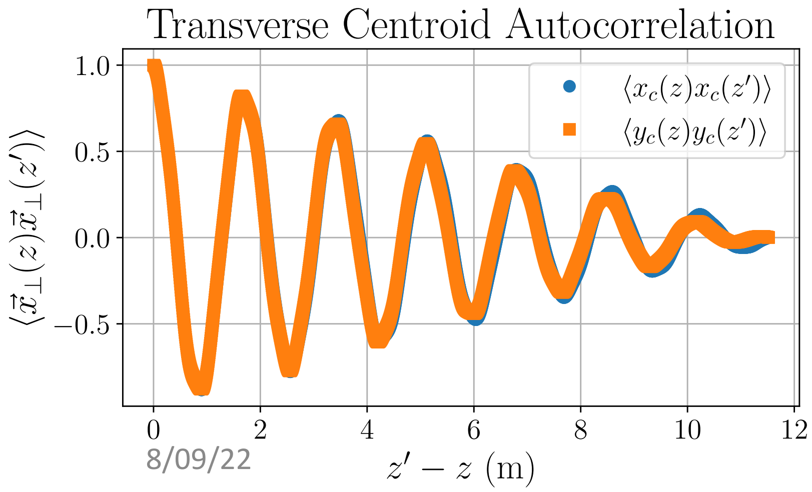
Examining the Data

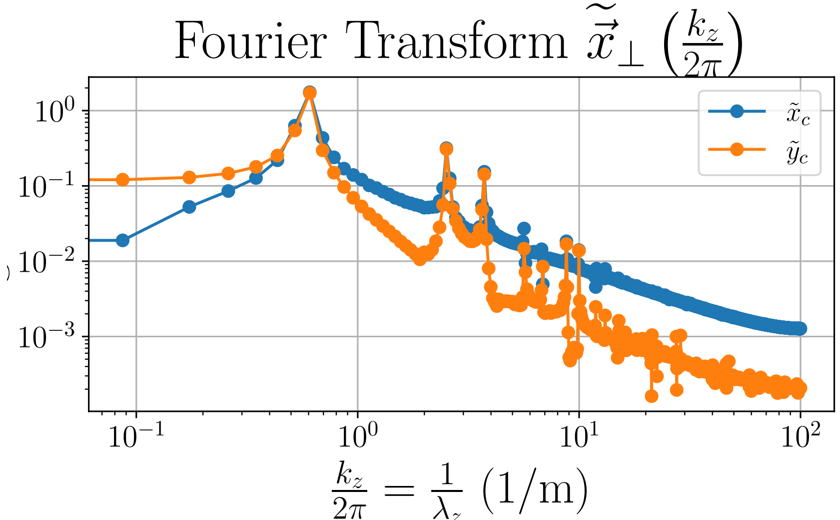


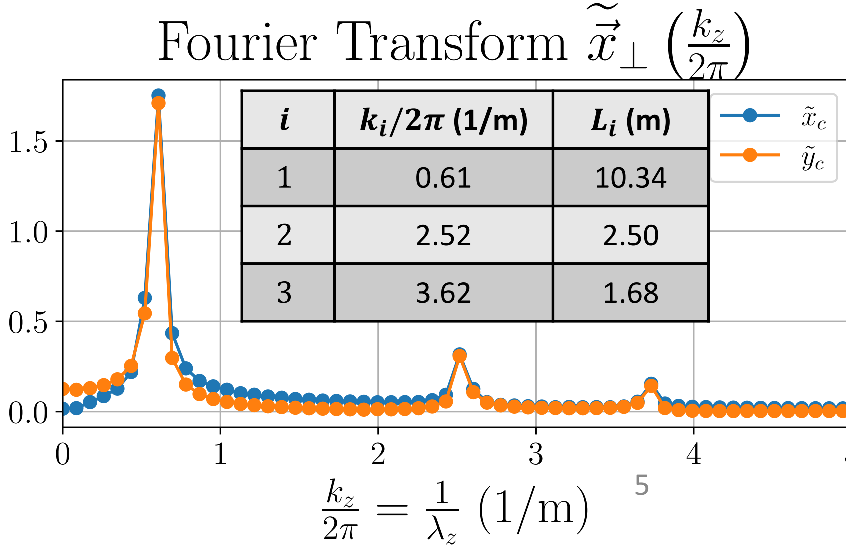
- Self intersecting ≠ ODE
- Use independent variable z to ensure ODE behavior



Figures of Merit:

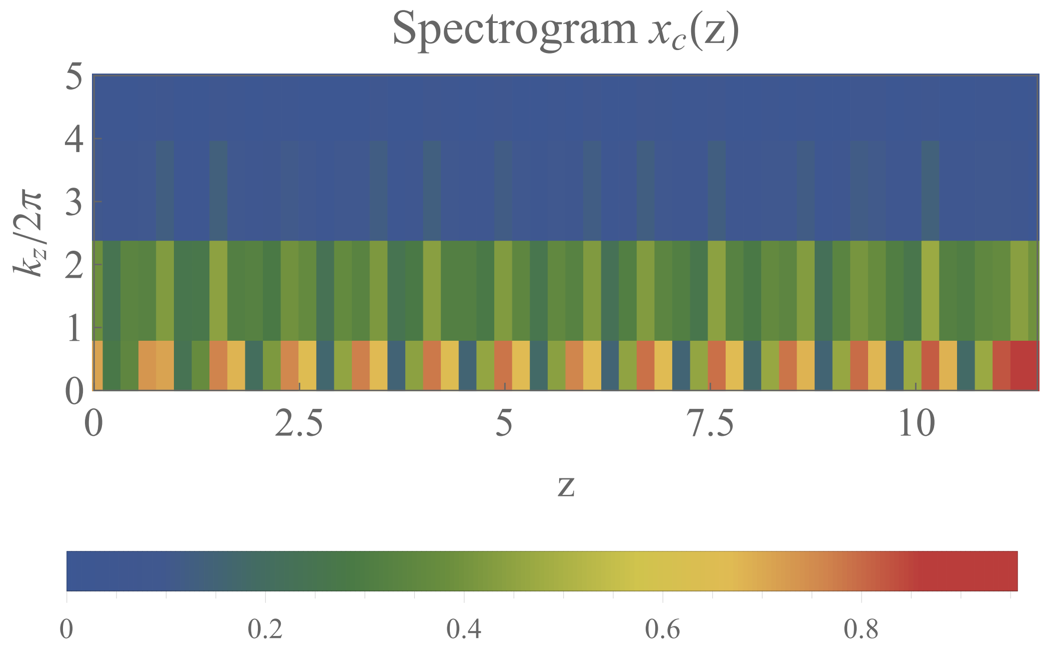






i	$k_i/2\pi$ (1/m)	L_i (m)
1	0.61	10.34
2	2.52	2.50
3	3.62	1.68

Choosing Basis Functions



- Choice of basis function guided by physics and dynamics inherent within the WARP simulation and embedded within the data.

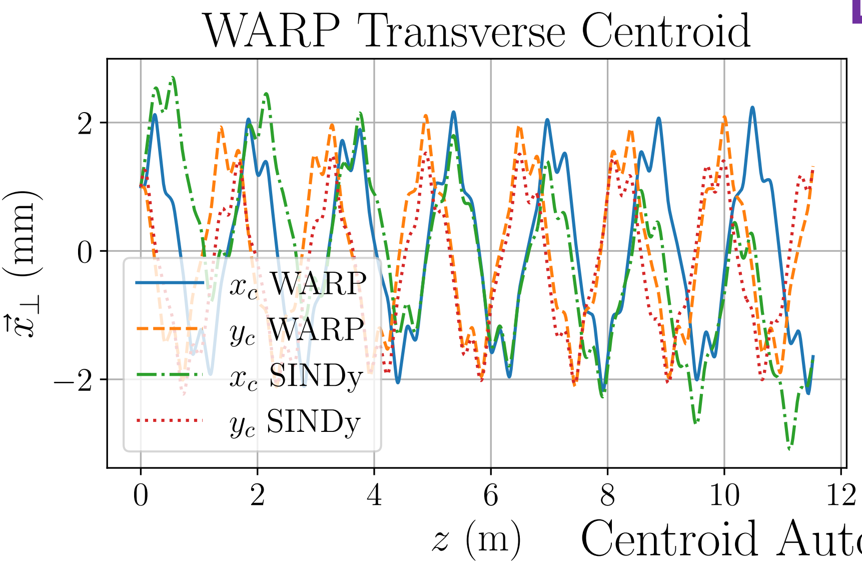
$$\mathbf{x} \in \{z, x_c, y_c\} \qquad \frac{d}{dz}\mathbf{x} = \mathbf{f}(\mathbf{x}) \qquad \boldsymbol{\Xi} = [\boldsymbol{\xi}_0 \quad \boldsymbol{\xi}_1 \quad \boldsymbol{\xi}_c \quad \boldsymbol{\xi}_s \quad \boldsymbol{\xi}_{nc} \quad \boldsymbol{\xi}_{ns}]$$

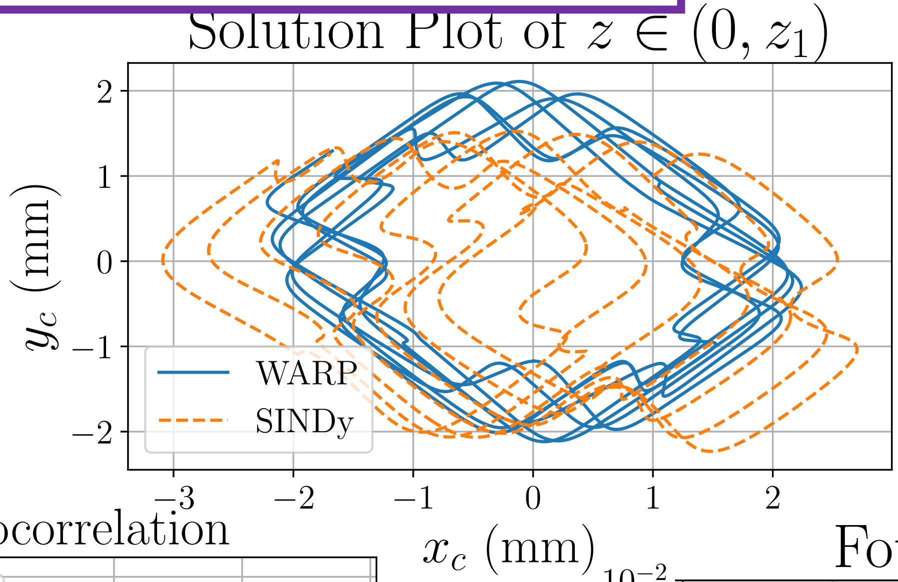
Simple Harmonic Motion — Lattice Elements: Fourier — Nonlinear Interaction

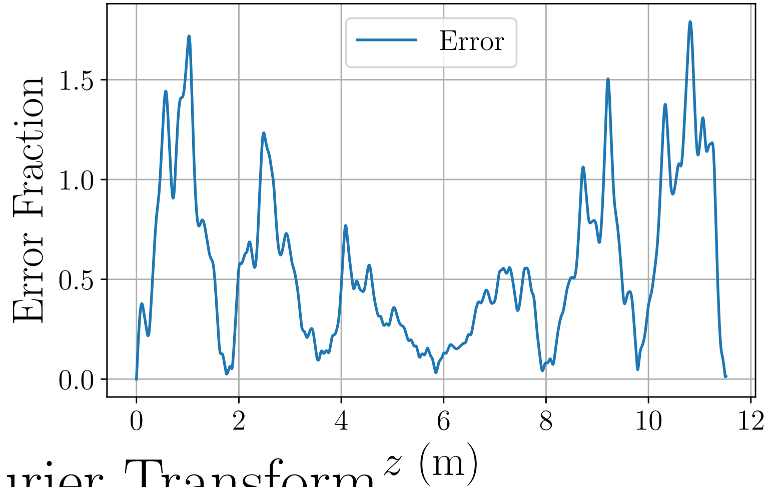
$$\mathbf{f}(\mathbf{x}) \approx \boldsymbol{\xi}_0\mathbf{x_0} + \boldsymbol{\xi}_1\mathbf{x} + \sum_{i=1}^{3}[\boldsymbol{\xi}_c\cos(k_i z) + \boldsymbol{\xi}_s\sin(k_i z) + \boldsymbol{\xi}_{nc}\,\mathbf{x}\cos(k_i z) + \boldsymbol{\xi}_{ns}\,\mathbf{x}\sin(k_i z)]$$

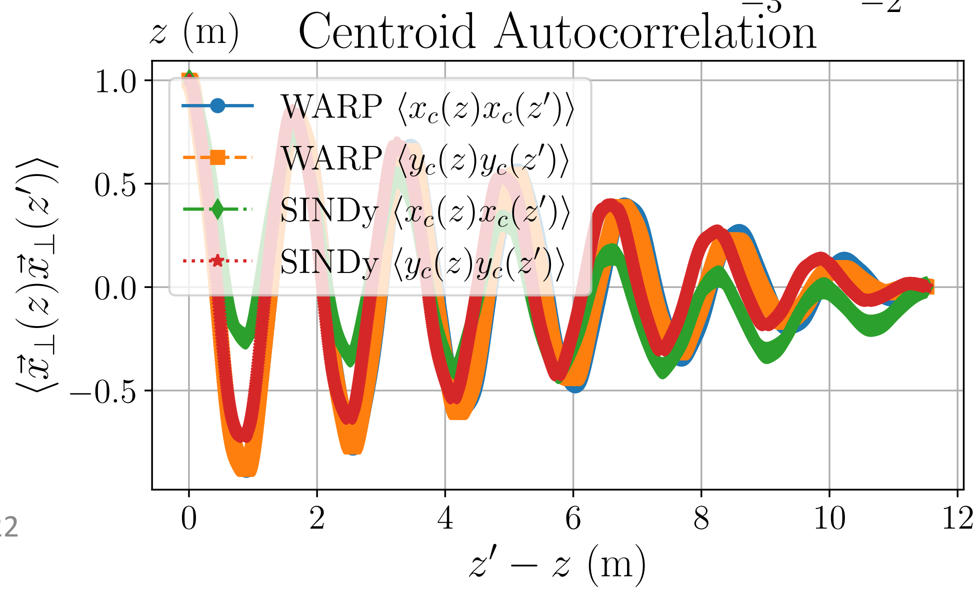
1st Try: Fourier

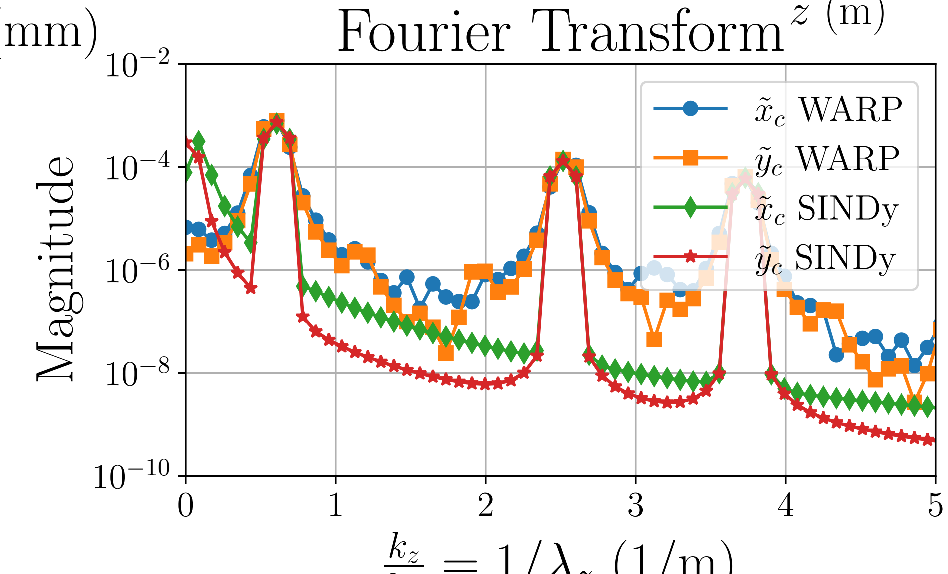
$$\mathbf{f}(\mathbf{x}) \approx \boldsymbol{\xi}_0\mathbf{x_0} + \boldsymbol{\xi}_1\mathbf{x} + \sum_{i=1}^{3}\left[\boldsymbol{\xi}_c\cos(k_i z) + \boldsymbol{\xi}_s\sin(k_i z) + \boldsymbol{\xi}_{nc}\,\mathbf{x}\cos(k_i z) + \boldsymbol{\xi}_{ns}\,\mathbf{x}\sin(k_i z)\right]$$





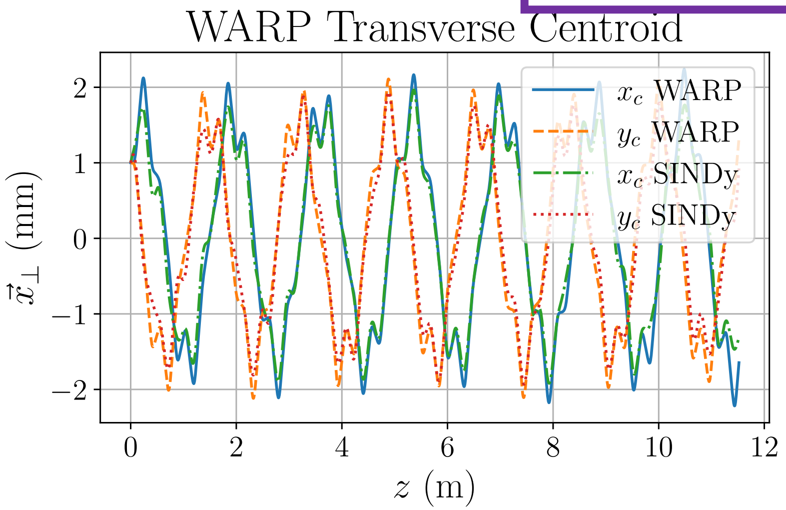


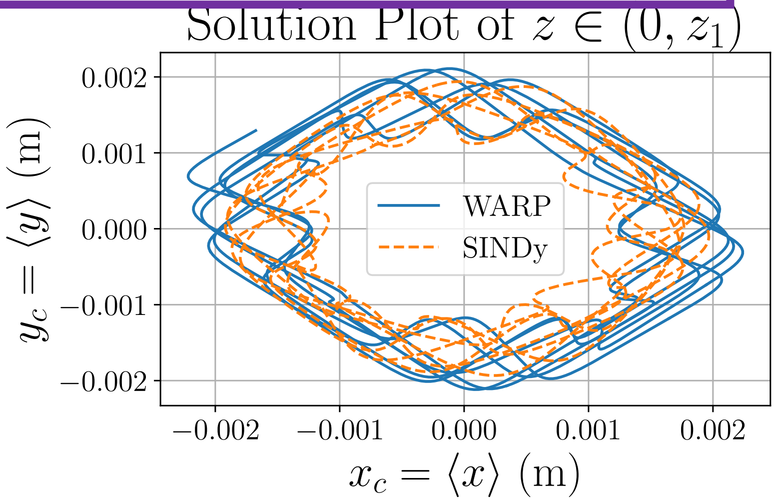


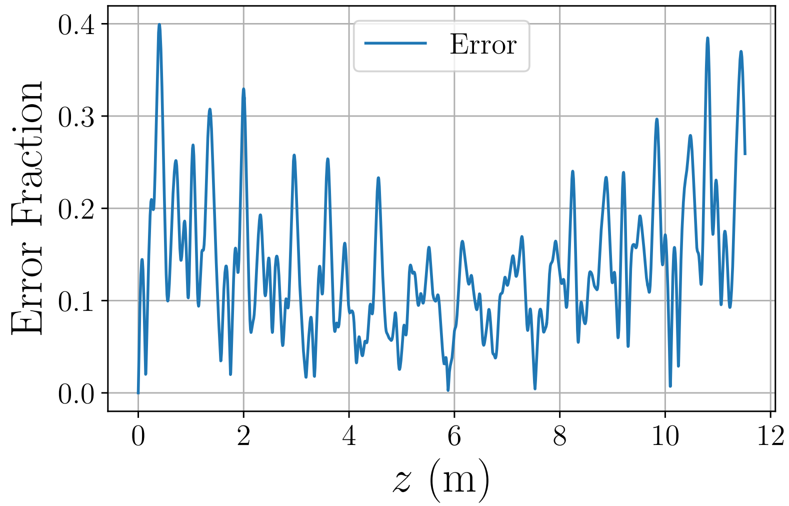


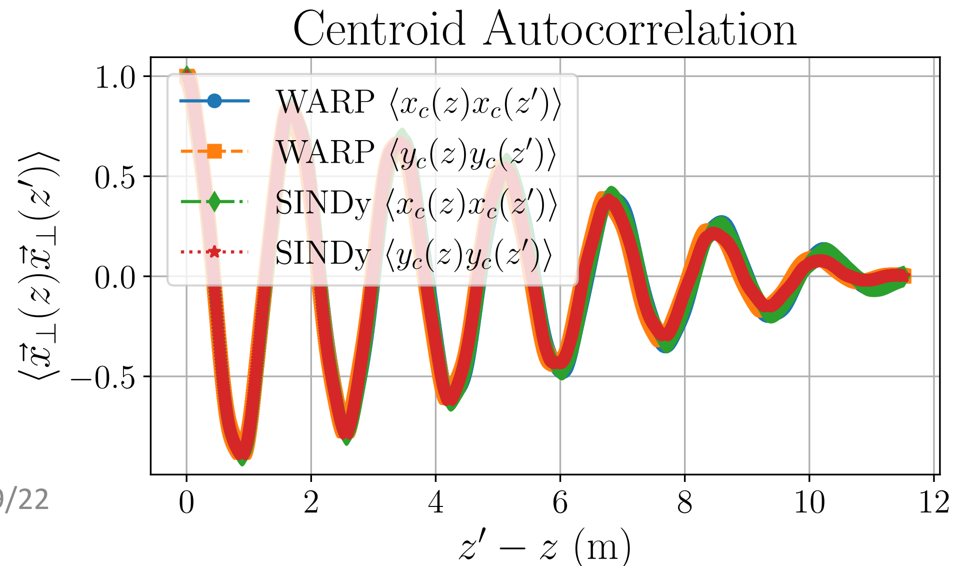
2nd Try: Fourier + Simple Harmonic Motion (SHM)

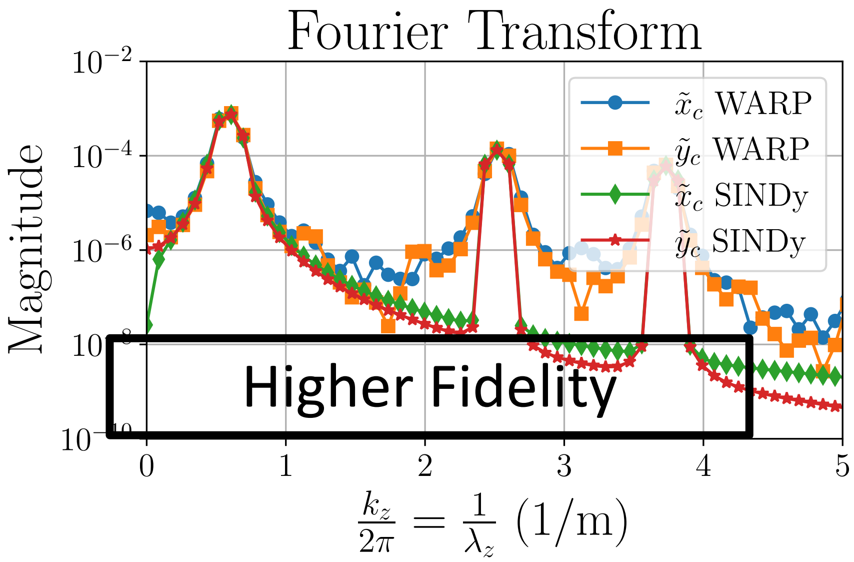
$$\mathbf{f}(\mathbf{x}) \approx \boldsymbol{\xi}_0 \mathbf{x_0} + \boldsymbol{\xi}_1 \mathbf{x} + \sum_{i=1}^{3} [\boldsymbol{\xi}_c \cos(k_i z) + \boldsymbol{\xi}_s \sin(k_i z) + \boldsymbol{\xi}_{nc}\, \mathbf{x} \cos(k_i z) + \boldsymbol{\xi}_{ns}\, \mathbf{x} \sin(k_i z)]$$





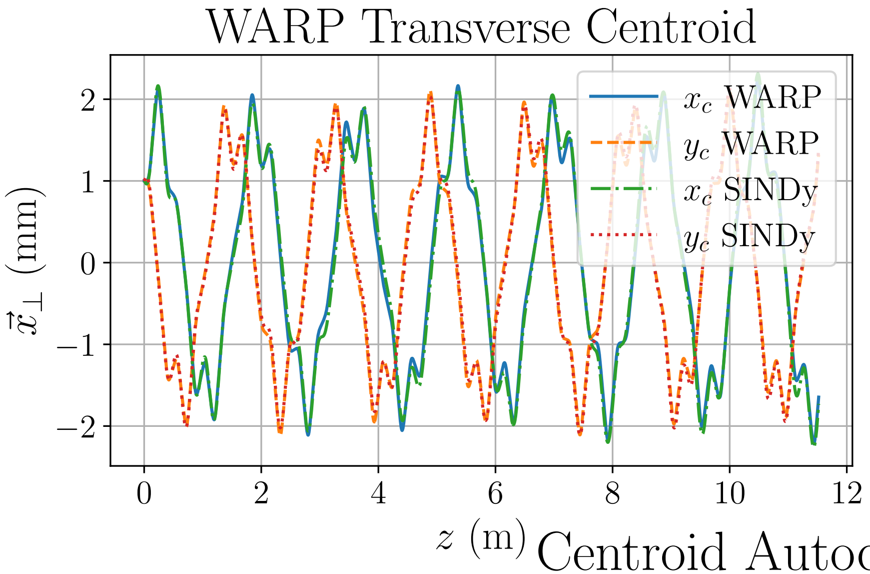


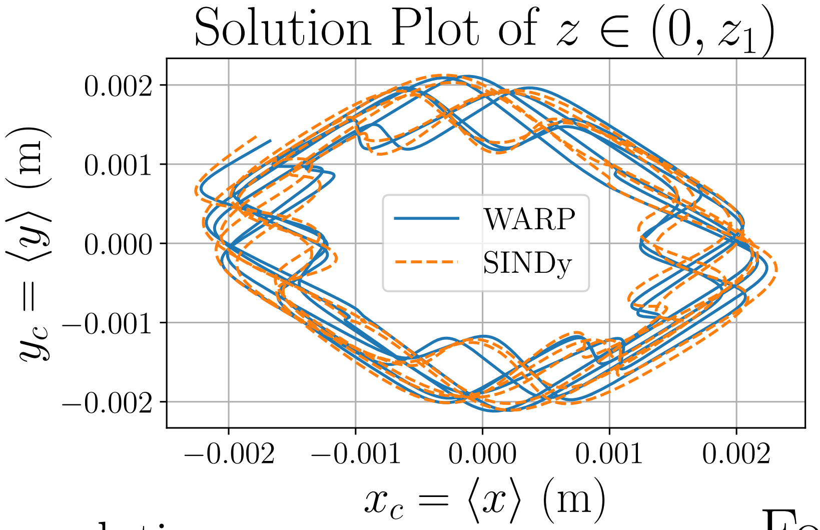


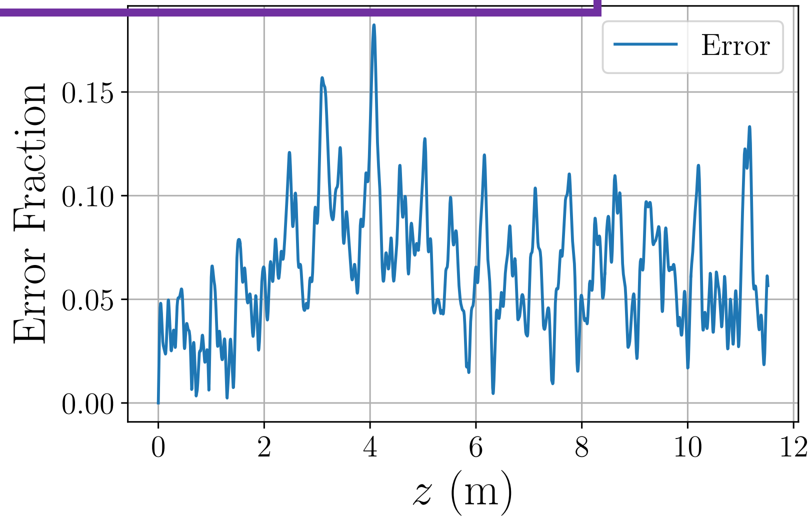


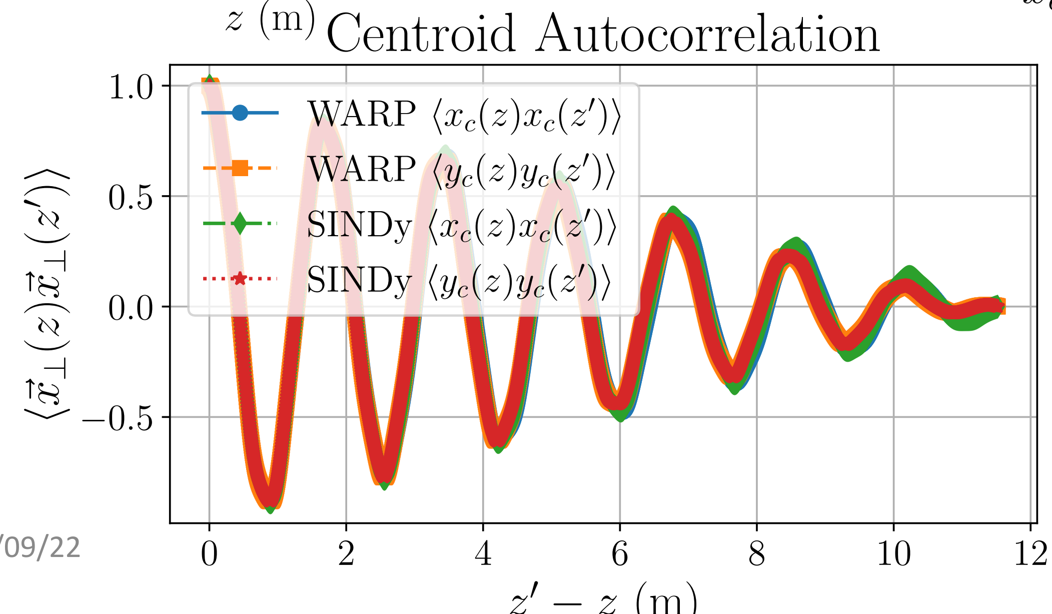
3rd Try: Fourier + SHM + Nonlinear (NL) Interaction

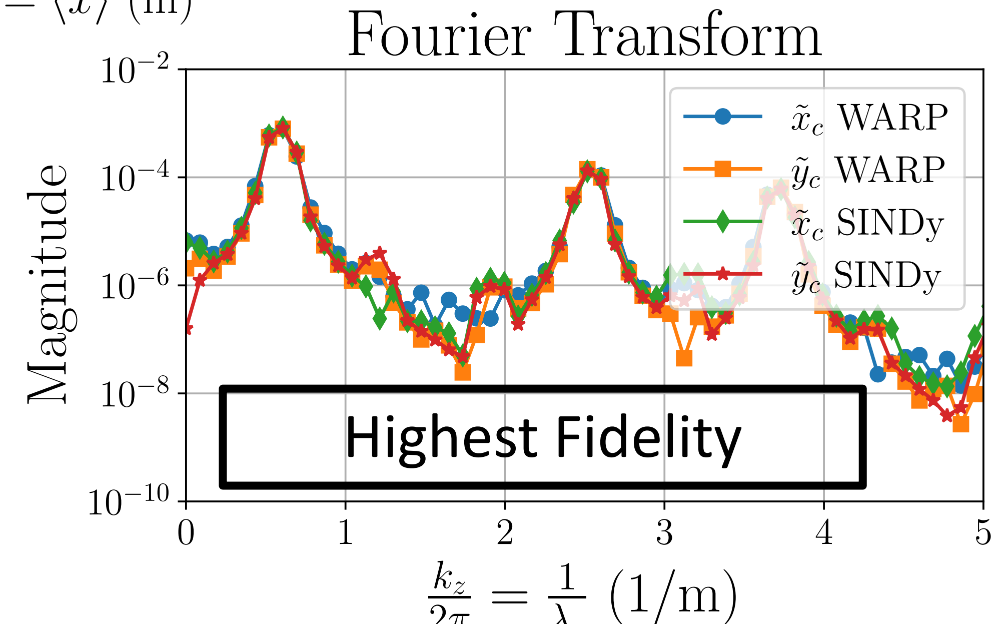
$$\mathbf{f}(\mathbf{x}) \approx \boldsymbol{\xi}_0\mathbf{x_0} + \boldsymbol{\xi}_1\mathbf{x} + \sum_{i=1}^{3}\left[\boldsymbol{\xi}_c\cos(k_i z) + \boldsymbol{\xi}_s\sin(k_i z) + \boldsymbol{\xi}_{nc}\,\mathbf{x}\cos(k_i z) + \boldsymbol{\xi}_{ns}\,\mathbf{x}\sin(k_i z)\right]$$

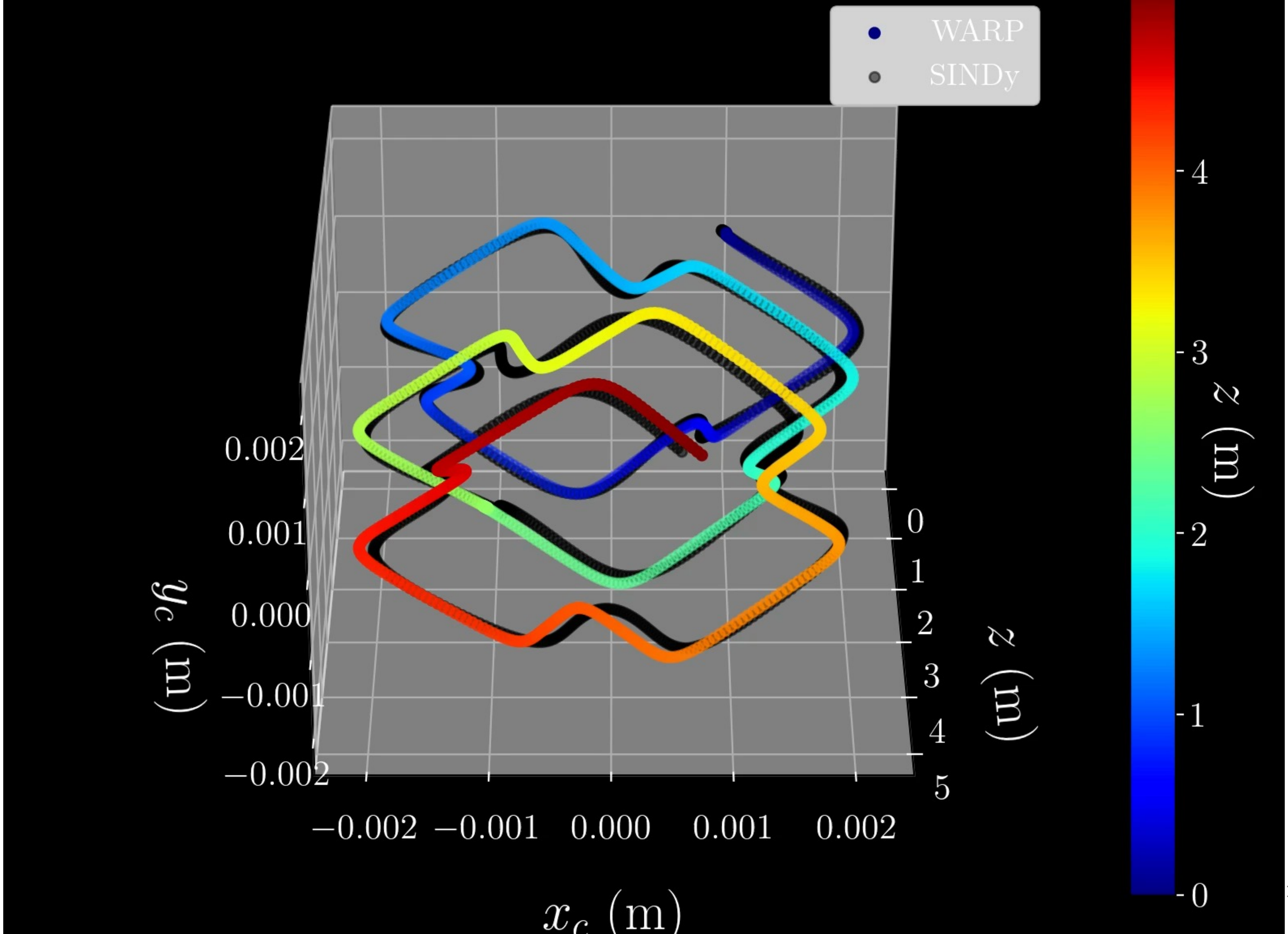








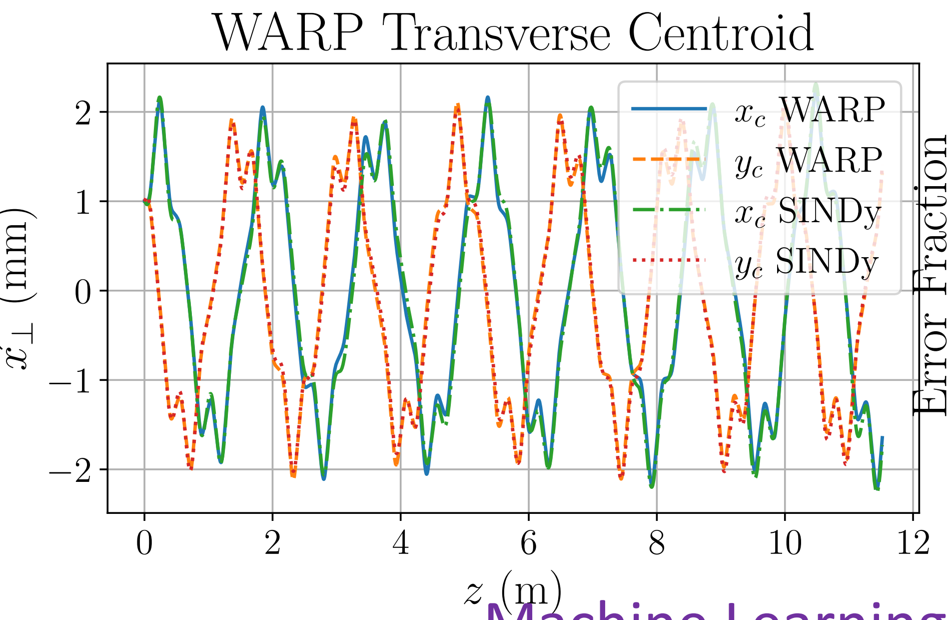


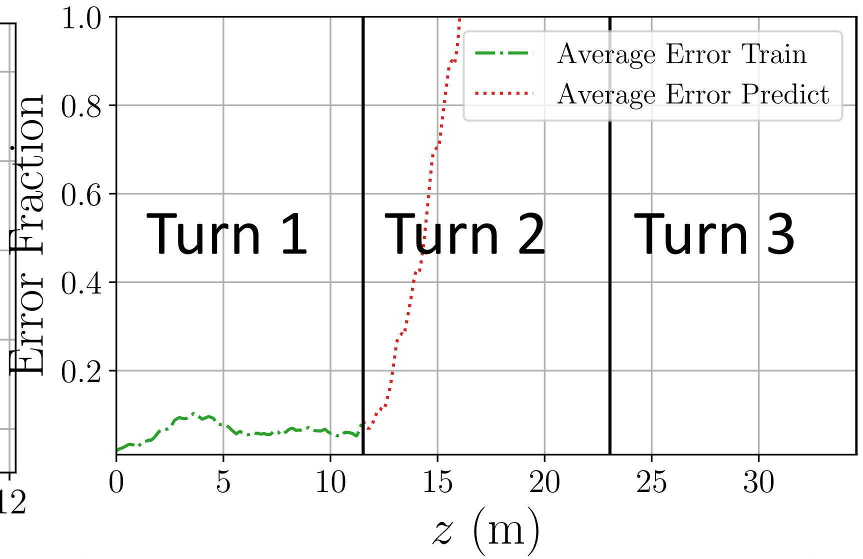


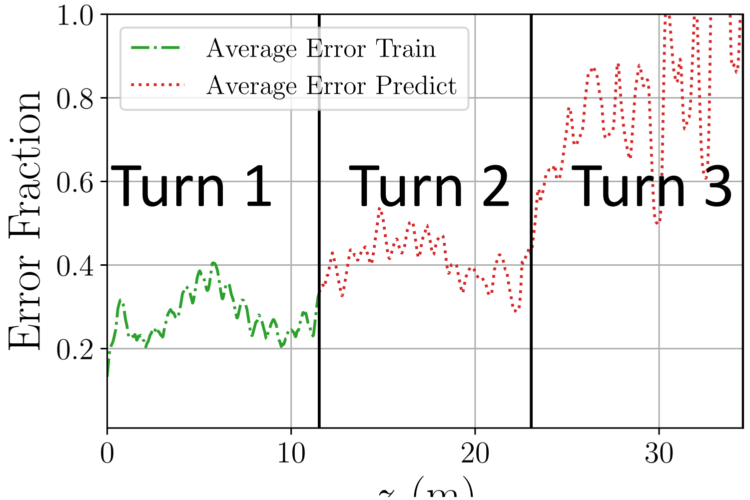
WARP
SINDy
0.002
0.001
0.000
−0.001
−0.002
y_c (m)
−0.002 −0.001 0.000 0.001 0.002
x_c (m)
0
1
2
3
4
5
z (m)
0
1
2
3
4
z (m)

Comparison to Machine Learning

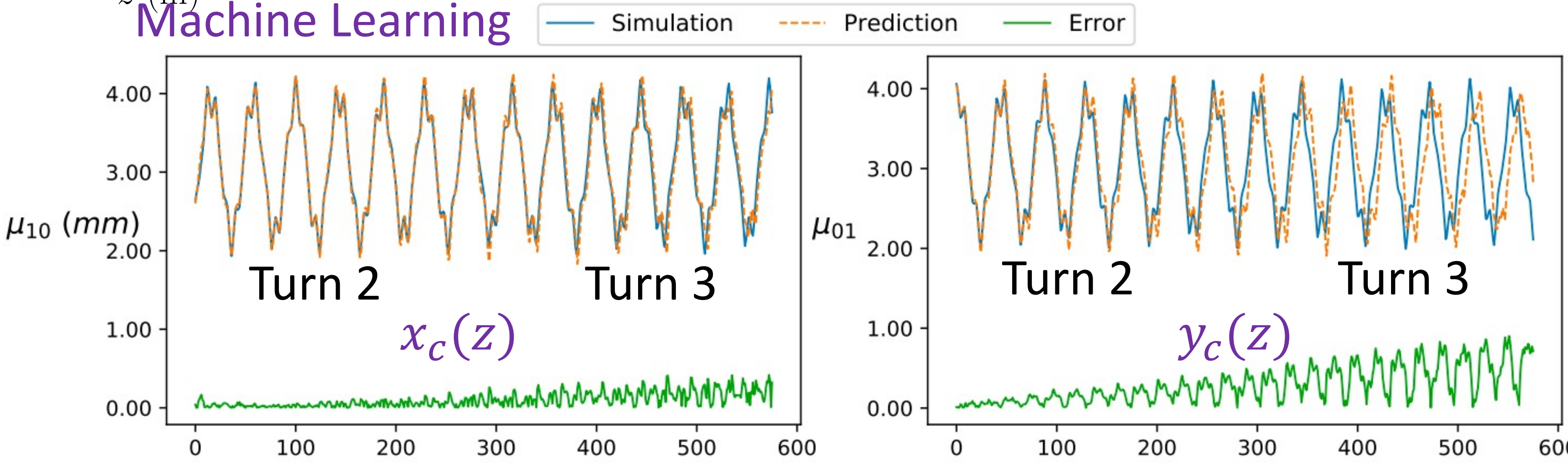
3rd Try with SINDy





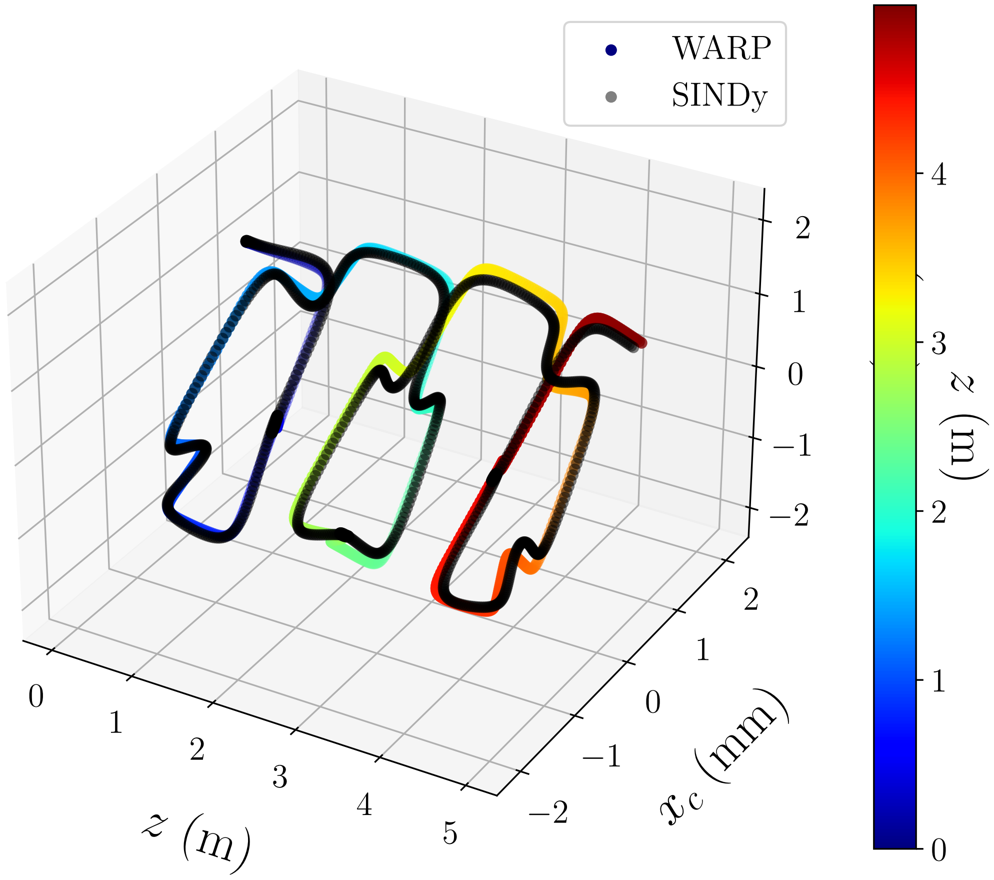


Machine Learning



SINDy is a Promising Approach

- We aim to develop a *Predictive* and *Productive* framework for beam dynamics with high fidelity.
- We desire to apply SINDy in areas of interest to the broader community.
- Many thanks to David Sutter for collaborative disscusions as well as the SINDy community enabling this work.
- Work supported by US DOE-HEP grants: DE-SC0010301 and DE-SC0022009



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