Progress on Convergence Map Based on Square Matrix for Nonlinear Lattice Optimization

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Abstract

We report progress on applying the square matrix method to obtain in high speed a "convergence map", which is similar but different from a frequency map. We give an example of applying the method to optimize nonlinear lattice for NSLS-II. The convergence map is obtained from solving nonlinear dynamic equations by iteration of the perturbation method and studying the convergence. The map provides information about the stability border of the dynamic aperture. We compare the map with the frequency map from tracking. The result in our example of nonlinear optimization of NSLS-II lattice shows the new method may be applied in nonlinear lattice optimization, taking the advantage of the high speed (about 30 to 300 times faster) to explore horizontal, vertical, and the off-momentum phase space.

Introduction

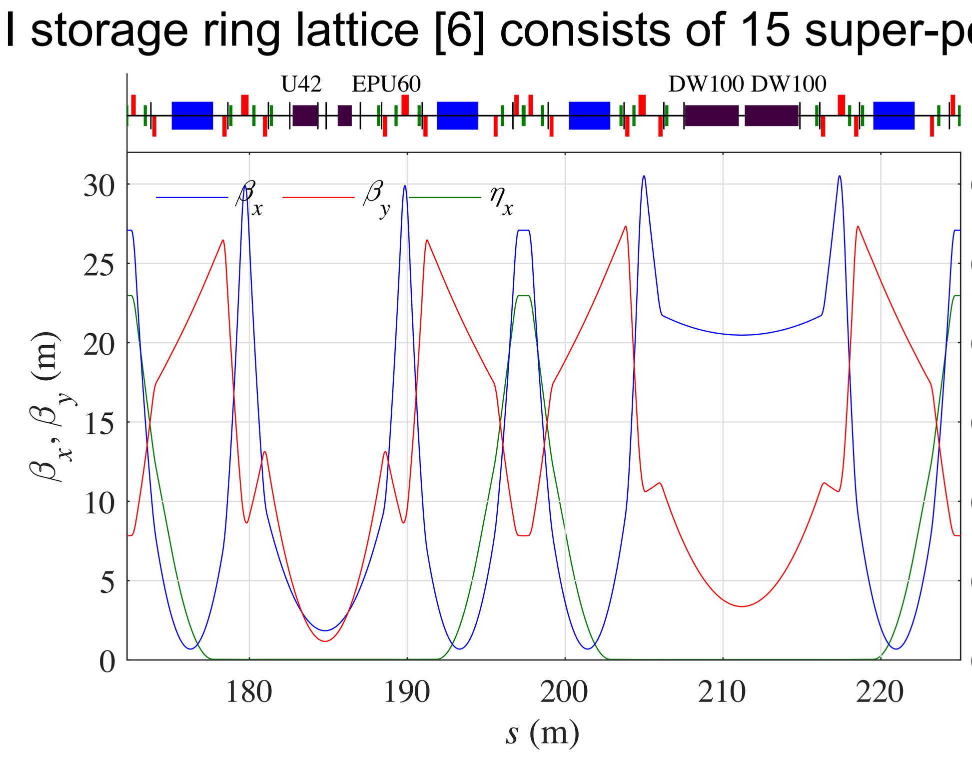
- A study of the long-term behavior of charged particles in storage rings is based on many iterations of the particle phase space transformation by the one-turn map representing the storage ring.
- The most accurate and reliable numerical approach is particle tracking in a magnet lattice model with appropriate integration methods.
- A more compact representation of the one-turn map for fast analysis: canonical perturbation theory, Lie operators, power series, normal form, etc. [1–3].
- On a basis of the square matrix [4], we developed a novel technique of "convergence map", which is a much faster alternative to the tracking-based frequency map [5].
- The convergence map provides information about the dynamic aperture and can be applied to nonlinear lattice optimization, taking the advantage of the high speed.

Square Matrix Method for Analysis of Nonlinear Dynamics

- The analysis of a nonlinear dynamic system can be greatly simplified using linear algebra. A novel method to optimize the nonlinear dynamic system using the square matrix has been developed at NSLS-II a few years ago [4].
- The main feature of this method is that we can achieve high order in one step. This is a significant advantage when compared with canonical perturbation theory and normal form, where the calculation is carried out order-to-order by a complicated iteration process.
- The developed theory shows good potential in theoretical understanding of a complex dynamic system to guide the optimization of dynamic aperture in circular accelerators.
- The square matrix method is general and may be applied to other areas, for example, nonlinear dynamics in physics and astronomy.

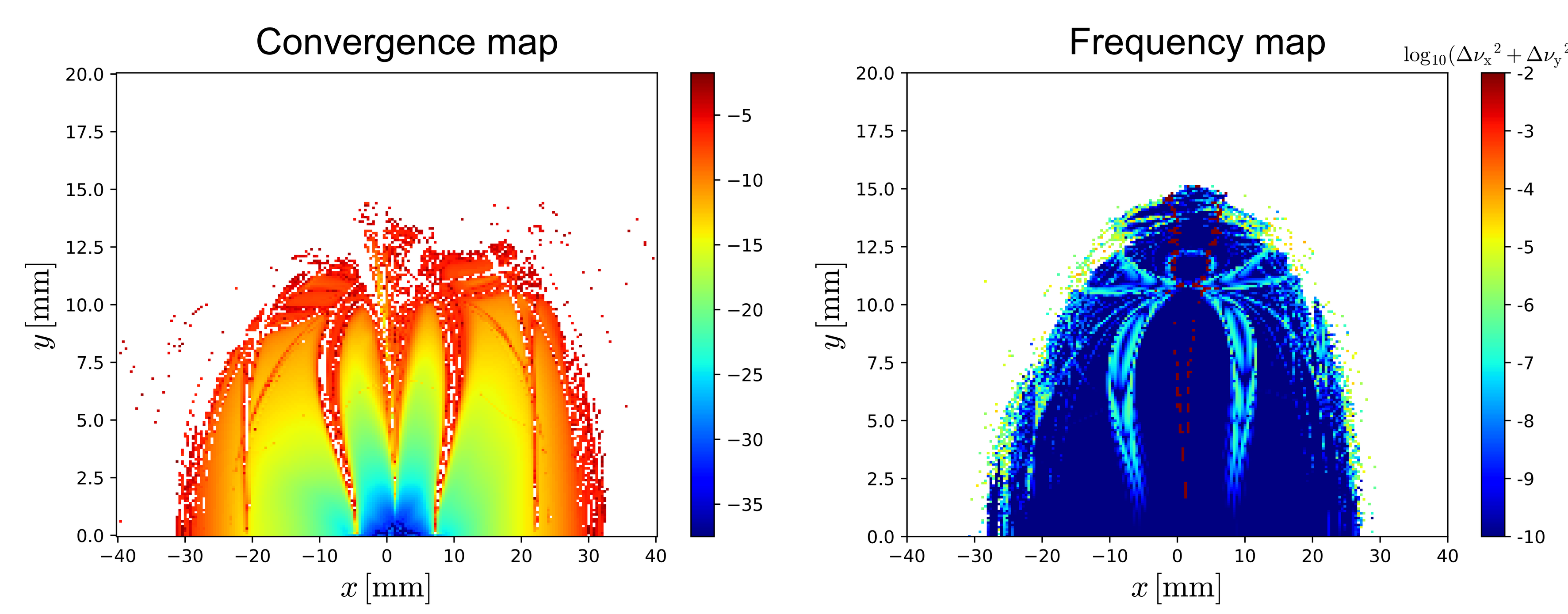
Example: NSLS-II

- The NSLS-II storage ring lattice [6] consists of 15 super-periods.



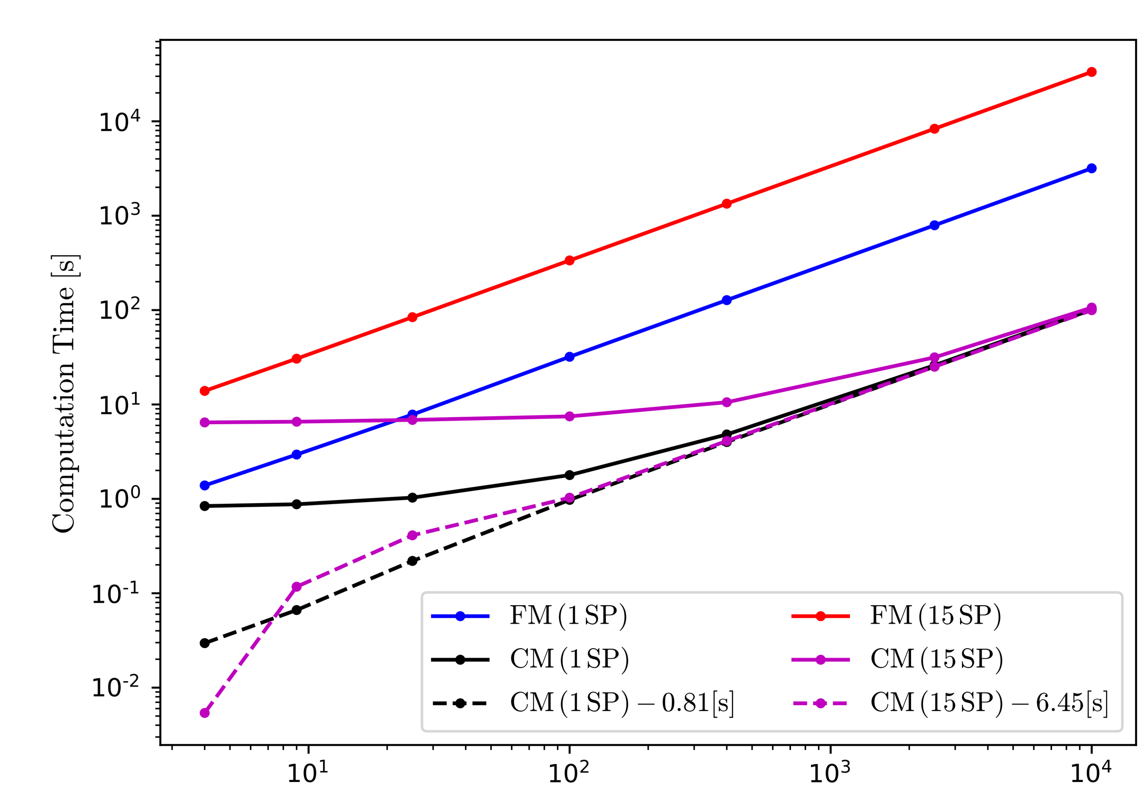
Convergence Map vs Particle Tracking

- We introduce a convergence map calculated using action-angle variables in the form of polynomials provided by a square matrix, which is derived from the one-turn map for an accelerator lattice.
- For the nominal NSLS-II lattice, we compared the convergence map with the frequency map [5] calculated by particle tracking using ELEGANT tracking code [17].
- The same number of points (1024) in horizontal (x) and vertical (y) planes were used for both the convergence map and the frequency map.
- Since the iterations leading to the solution of the nonlinear dynamic equations expressed by these action-angle variables can be carried out by Fourier transform, the computation speed is very high.
- Both diagrams show the same stable area and major resonances. Thus, the convergence map provides the same information about the nonlinear motion and dynamic aperture.



Benchmarking of Computation Time

- For both types of maps, an initial coordinate region of $+10 \leq x$[mm] $\leq +11$ and $+1 \leq y$[mm] $\leq +2$ was selected as particles launched from this region are very stable and can last at least 1024 turns specified for frequency map analysis.
- This square region was divided into 2×2, 3×3, 5×5, 10×10, 50×50, 100×100 grid points. Each grid point is used as an initial transverse coordinate for both maps. The momentum offset was zero.
- Frequency map computations: ELEGANT's "frequency map" command [7] to compute the diffusion defined by the tune changes between the first 512 and the latter 512 turns.
- Convergence map computations: PyTPSA [8] + symplectic integration method from TRACY [9] .
- All the computations were performed using a single core of Intel Xeon Gold 6252 CPU at 2.10 GHz (hyper-threading enabled).



Conclusion

- We introduce a convergence map calculated using action-angle variables in the form of polynomials provided by a square matrix, which is derived from the one-turn map for an accelerator lattice.
- The convergence map looks similar to but is different from the frequency map calculated by particle tracking.
- The dynamic aperture, tune footprint, phase space trajectory, and frequency spectrum calculated using the convergence map agree with the tracking to high precision.
- The convergence map method is much faster than tracking. For the NSLS-II lattice as an example, it is 30 (1 super-period) to 300 (whole ring) times faster.
- The convergence map is an efficient tool for nonlinear optimization, especially for complex lattices with low or no periodicity.

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