Intrinsic Landau Damping of Bunched Beams at Coupling Resonance

Alexandru Macridin

Collaborators:

A. Burov, J. Amundson, E. Stern, P. Spentzouris

Fermilab

COMPASS - SciDAC

U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING

Introduction

- **Incoherent tune spread caused by space charge contributes to Landau damping.**
- **The damping mechanism caused by space charge is not completely understood.**
- **Numerical investigations are necessary.**
- **Developing tools for modal analysis in particle tracking simulations is necessary.**
    - Single value decomposition is not so good (appropriate for stationary problems).
    - Dynamic mode decomposition works (provides damping/growing rates).

Investigating the Landau damping of space charge modes at coupling resonance we found a novel damping mechanism.

Synergia simulations

Synergia simulation package

- **Single-particle physics**
  - direct symplectic tracking
  - arbitrary-order polynomial maps
- **Apertures** (different shapes)
- **Collective effects** (single and multiple bunches)
  - space charge (different solvers)
  - wake fields (arbitrary wakes)

https://cdcvs.fnal.gov/redmine/projects/synergia2

Space charge modes simulation:

- **10 x OFORODO lattice**
- **10^8 macroparticles**
- **3D space charge Poisson solver with open boundary conditions** (Hockney)
- **Gaussian beams with equal transverse emittance**
- **Beam initially excited with an approximate (guessed) mode shape**

Dynamic Mode Decomposition

P.J. Schmid, Journal of Fluid Mechanics, 656, 2010
C.W. Rowley et al., Journal of Fluid Mechanics, 641, 2009
A. Macridin et al., PRSTAB, 074402, 2015

- **Store** $X(z,\delta, t_n)$ **at every turn** $X(z,\delta,t_0), X(z,\delta,t_1),\ldots.., X(z,\delta,t_N)$

$$X(z,\delta,t)=\frac{\int dx\, dy\, x\, \rho(x,y,z,\delta,t)}{\rho(z,\delta,t)}$$ **dipole density**

- **DMD assumption: A linear operator** A **exists such as:**

$$X(z,\delta,t_{k+1})=A\,X(z,\delta,t_k) \quad \textbf{for all} \quad k=\overline{1,N-1}$$

- **the eigenvalues and the eigenvectors of** A **describe the dynamics**

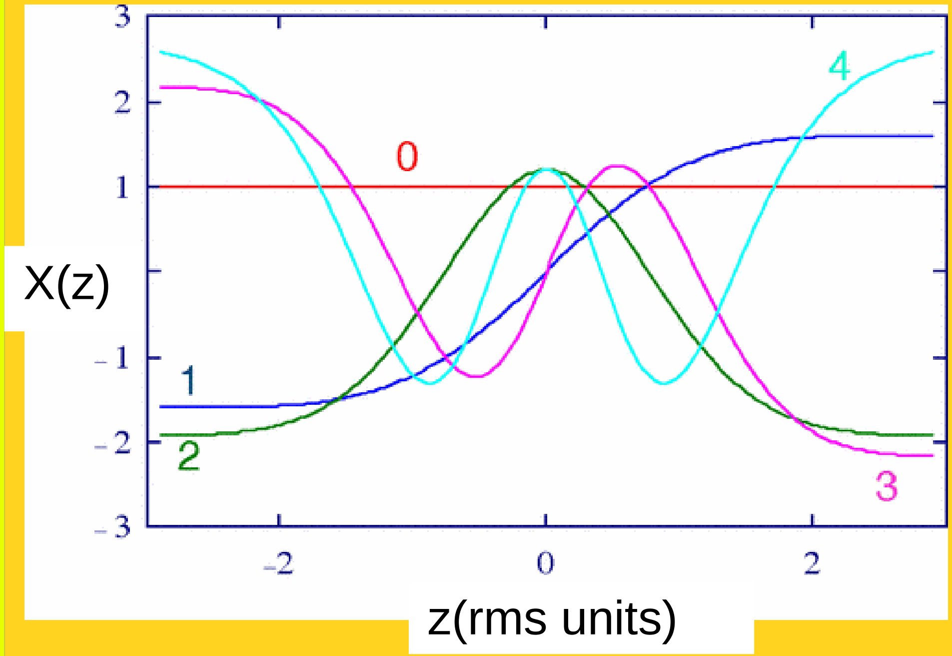
$$A\phi_j(z,\delta)=\mu_j\phi_j(z,\delta)$$

$$X(t_k)=A^k X(t_0)=\sum_j \mu_j^k \alpha_j \phi_j=\sum_j e^{-\lambda_j k} e^{-i\omega_j k}\psi_j \qquad X(t_0)=\sum_j \alpha_j \phi_j$$

- **DMD finds the best approximation for the eigenvalues and eigenvectors of** A **for a given set of data**

Strong space charge

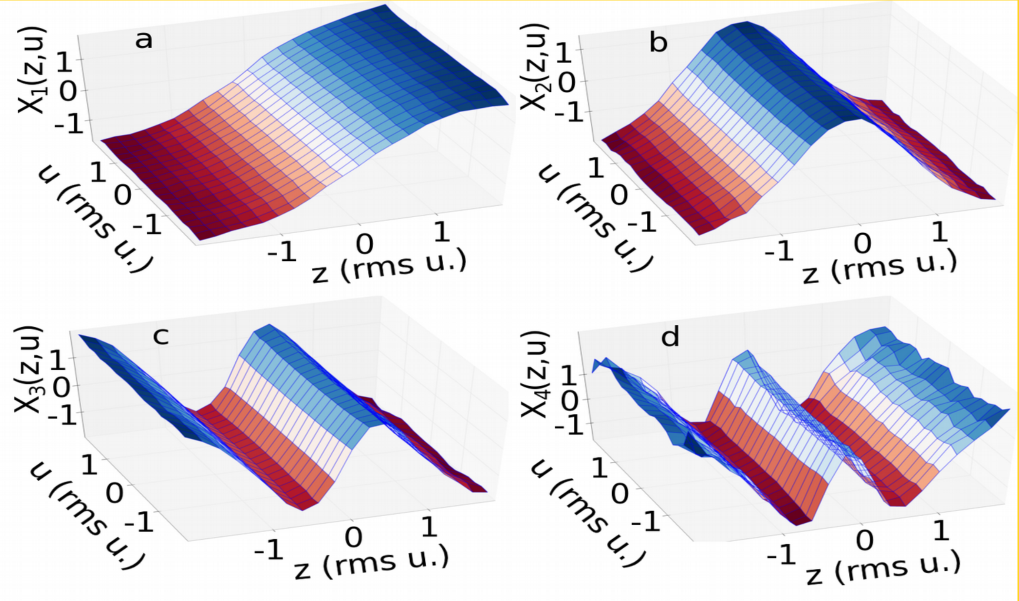
Analytical results, transverse modes



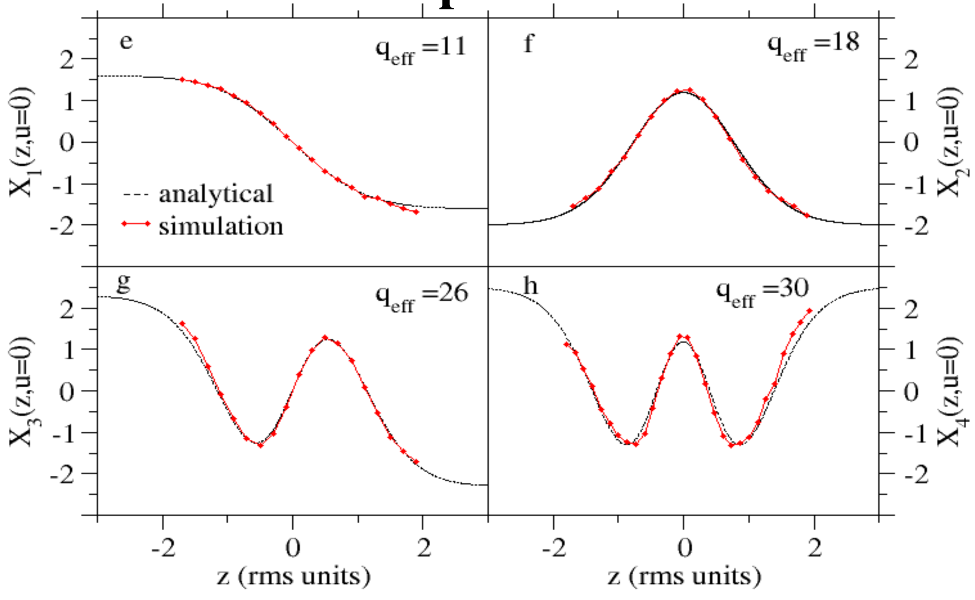
A. Burov, PRSTAB 12, 109901, 2009.
V. Balbekov. PRSTAB12, 124402, 2009.

- **Good agreement between the simulation and the analytical predictions**

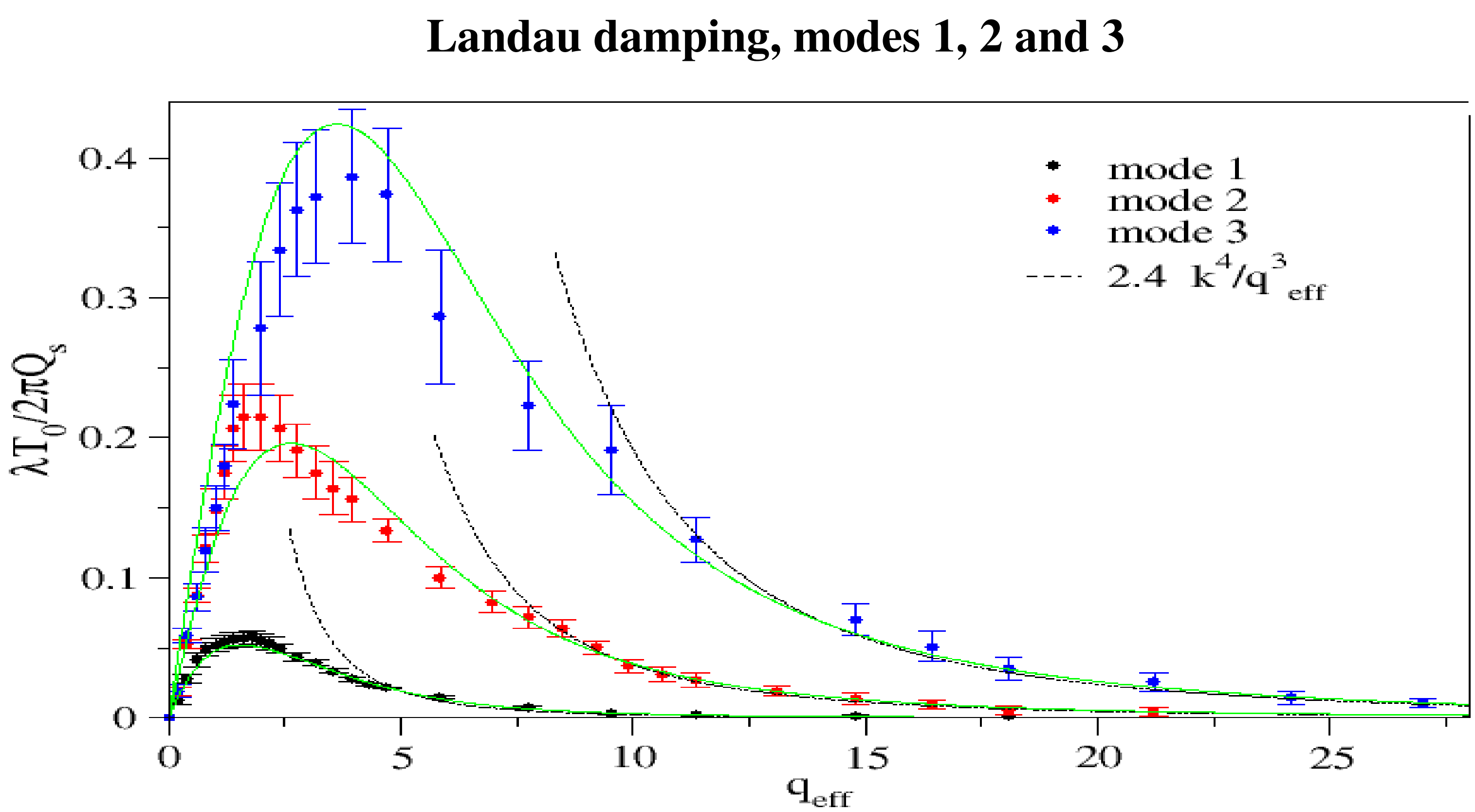
Simulation



Comparison



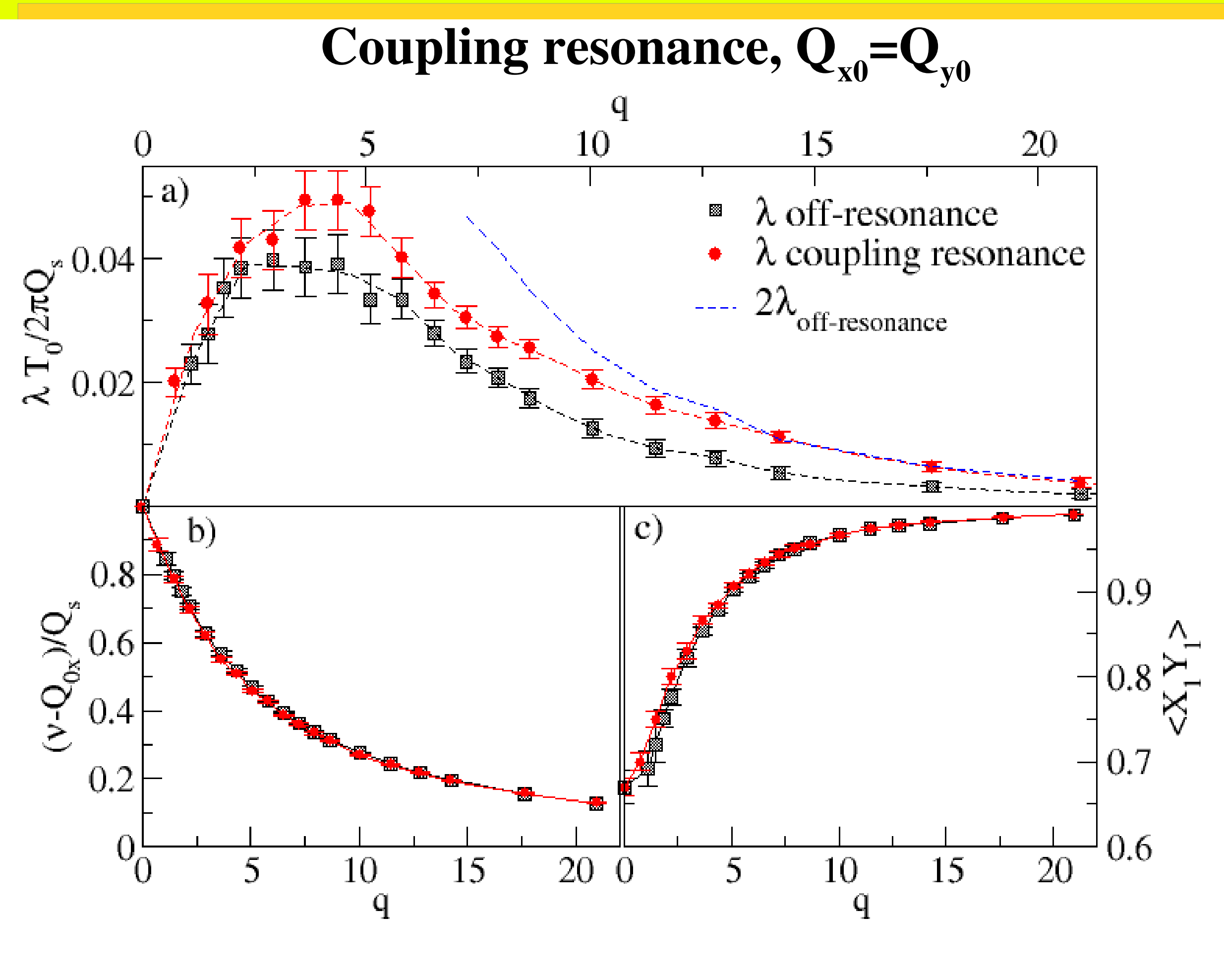
Landau damping versus space charge strength



A. Macridin et al., PRSTAB, 074402, 2015

- **In the strong interaction regime the damping rate is proportional to k^4/q^3,** k **is the mode number,** q **is the space charge parameter** (agreement with A. Burov, *PRSTAB 12, 109901, 2009*)

Mode 1 at coupling resonance



- Nonlinear coupling resulting from the term proportional to x^2y^2 in space charge potential
- Montague's resonance, $2Q_x-2Q_y=0$

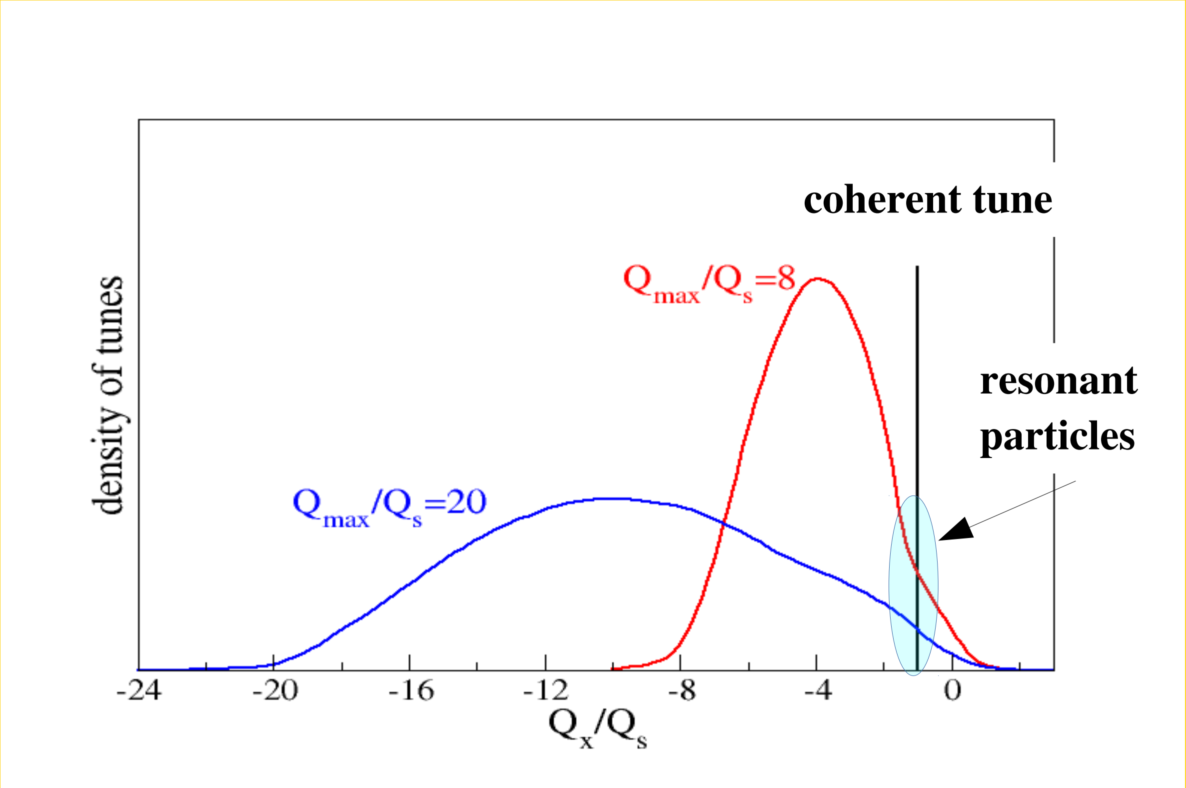
2 times larger damping at CR in strong space charge regime.

Why?

Landau damping mechanism

$$\ddot{x}_i + \omega_0^2 (Q_{0x} - \delta Q_i)^2 x_i = -2\,\omega_0^2 Q_{0x}\,\delta Q_i\,\bar{x}$$

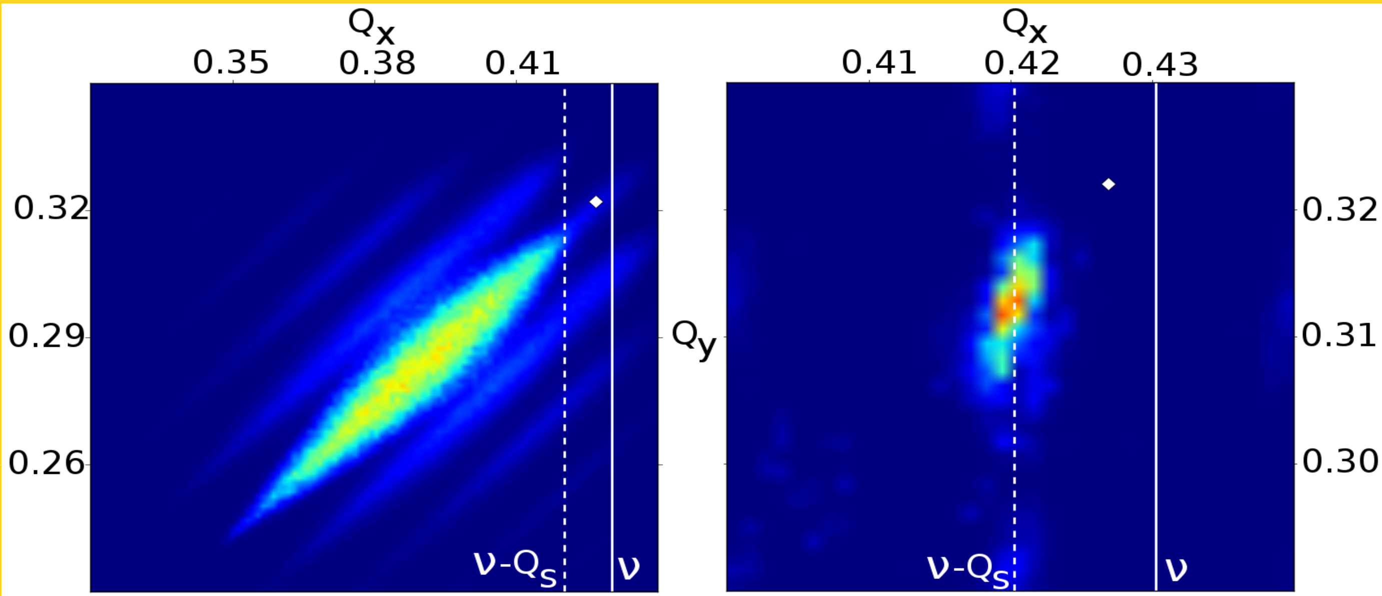
mode-particle coupling



- **The mode energy is transferred to the resonant particles.**
- **The resonant particles tune equal the coherent tune Q_c.**

Landau damping off-resonance

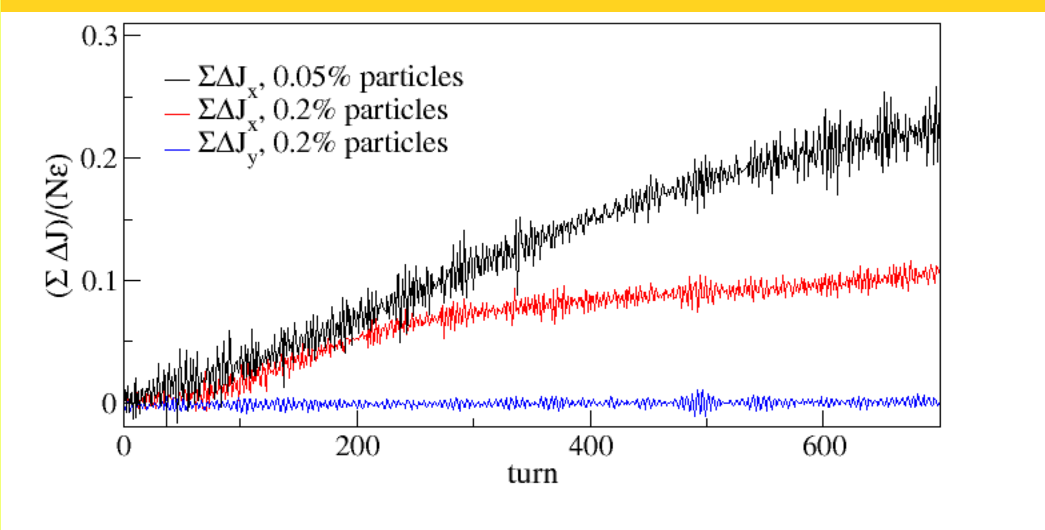
$$\ddot{x}+\omega_0^2(Q_{0x}-\delta Q)^2 x=-2\omega_0^2 Q_{0x}\delta Q\,\bar{x},\quad \delta Q(z,J_x,J_y),\quad \bar{x}\propto e^{i\omega_0(\nu-Q_s)t}$$



bunch tune footprint

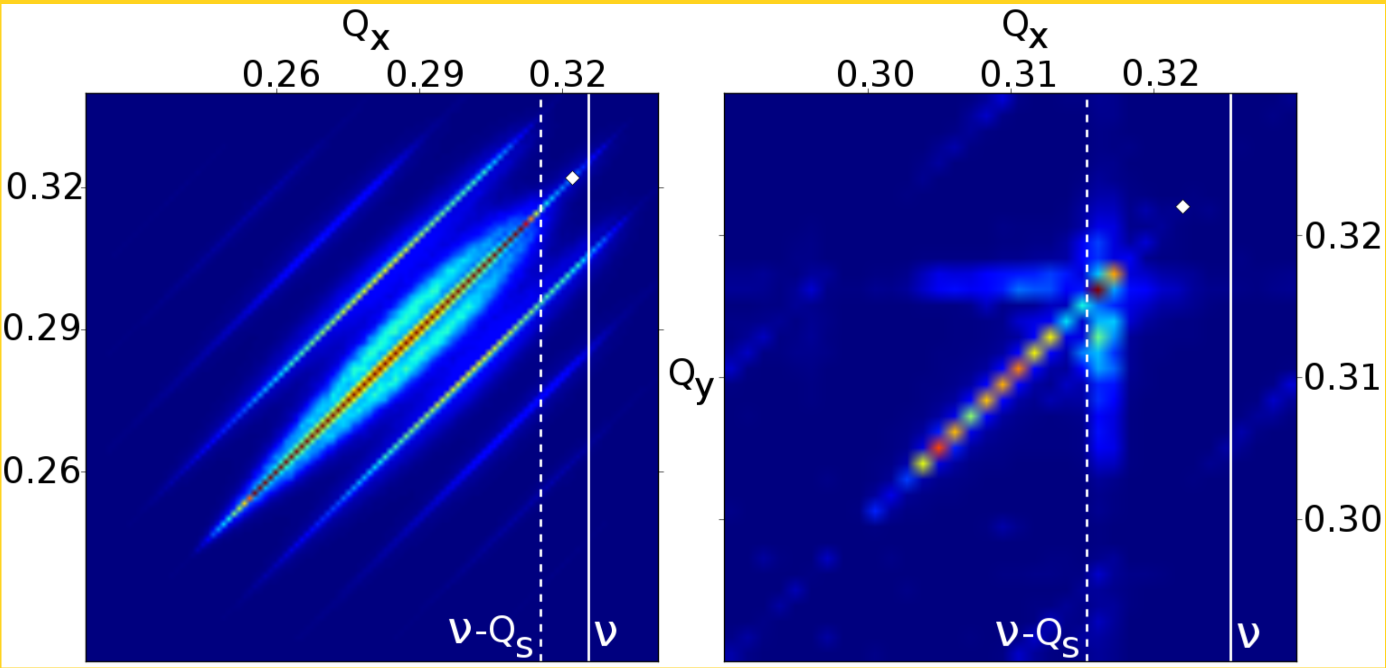
resonant particles

- Conventional LD mechanism.
- The tune of the LD responsible particles is at the coherent tune.



- LD responsible particles increase their energy with time

Landau damping at coupling resonance, $Q_{x0} = Q_{y0}$



bunch tune footprint **resonant particles**

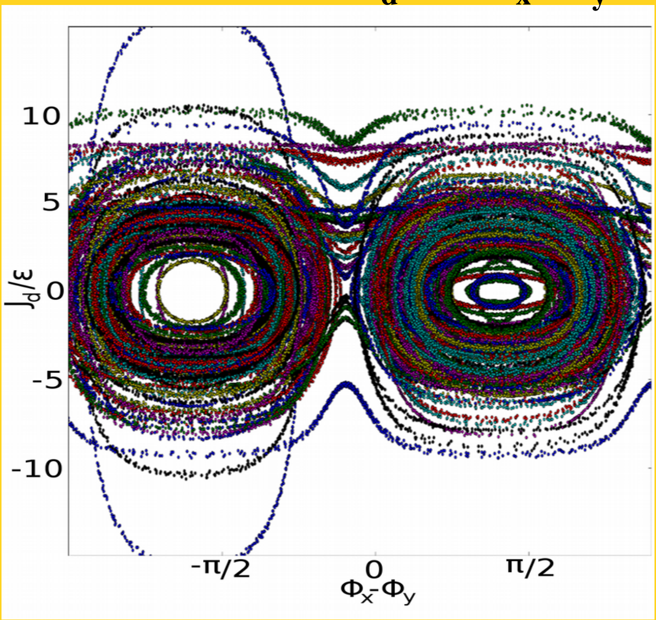
- **The tune of most of the LD responsible particles is not in the vicinity of coherent tune. Why do these particles absorb the mode's energy?**
- **The picture does not fit the conventional LD paradigm.**

Particles dynamics at coupling resonance

- Montague's resonance, simple model for round beams $H_{sc} = \alpha\left(x^2 + y^2\right)^2$
- Stable point: $\cos 2(\Phi_x - \Phi_y) = -1$, $J^*_d \mu\ Q_x - Q_y$
- Trapped particles properties:
  - $J_s = J_x + J_y$, constant of motion
  - $J_d = J_x - J_y$, oscillates around the stabled point
  - the trapping frequency is particle dependent $Q_t \propto \alpha \sqrt{J_s - J_d^{\star}}$

Synergia simulations

Poincaré plots, J_d vs Φ_x-Φ_y



LD responsible particles

- Most of the LD responsible particles are trapped in resonance islands.
- Their actions oscillates with particle dependent frequency.

Parametric Landau damping

$$\ddot{x}_i+\omega_0^2\left(Q_{0x}-\delta Q_i\right)^2 x_i=-2\,\omega_0^2 Q_{0x}\,\delta Q_i\left(z_i,J_{si},J_{di}\right)\bar{x}$$

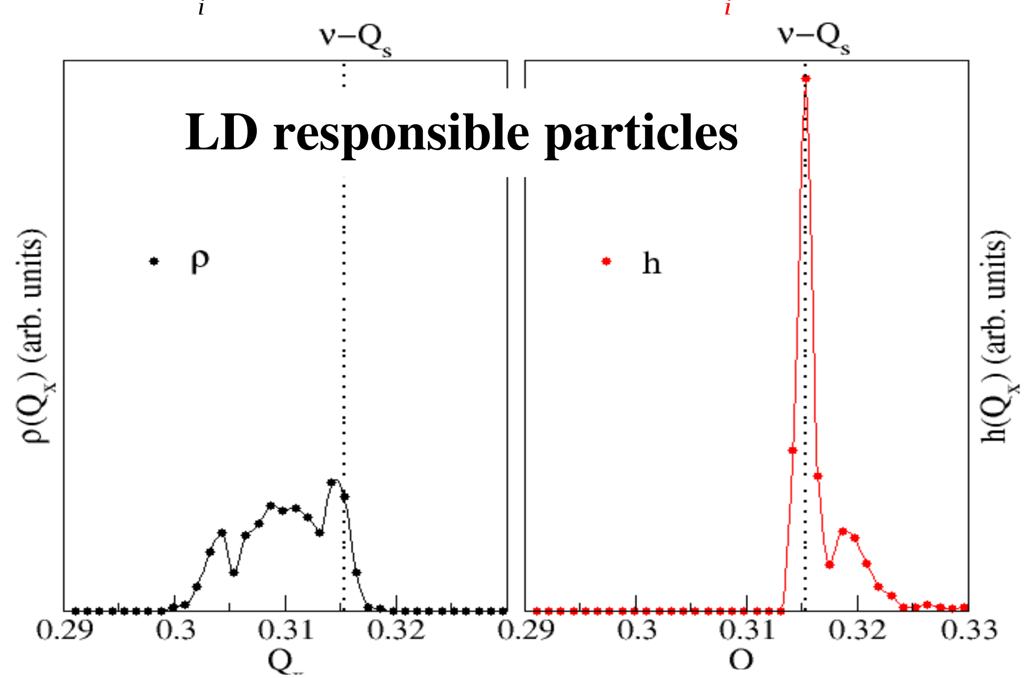
mode-particle coupling

$$\ddot{x}_i+\omega_0^2 Q\left(z,J_{si},J_{di}\right)^2 x=-A\left(z,J_{si}\right)\bar{x}-B\left(z,J_{si}\right)J_{di}\bar{x}\,,\qquad \bar{x}\propto e^{i\omega_0(\nu-Q_s)t},\ J_{di}\propto e^{i\omega_0 Q_{ti}t}$$

resonance condition:

- $A\bar{x}$ **coupling** $\quad Q_i=\nu-Q_s$
  - → **conventional LD**
- $BJ_d\bar{x}$ **coupling** $\quad Q_i+Q_{ti}=\nu-Q_s$
  - → **parametric LD**
  - → **mode-particle coupling modulated by Q_t**
  - → **the tune of the LD resonant particles is not at the coherent tune**

$$\rho(Q)=\sum_i \delta(Q_i-Q) \qquad h(Q)=\sum_i \delta(Q_i+Q_{ti}-Q)$$



conventional LD, $A\bar{x}$ **coupling**

parametric LD, $BJ_d\bar{x}$ **coupling**

Conclusions

- **We employed Synergia and DMD techniques to analyze the transverse space charge modes in bunched beams.**

- **The simulations reveal a novel Landau damping mechanism driven by the modulation of mode-particle interaction.**

- **The amplitude oscillations of the trapped particles at the coupling resonance enhance the Landau damping rate in bunched beams.**