Impact of Space Charge on Beam Dynamics and Integrability in the IOTA Ring

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Outline

• Description of IOTA

• Requirements for integrability with nonlinear optics

• Adjusting the lattice for space charge

• Behavior of the invariants in simulation with space charge

• Conclusion
Integrable Optics Test Accelerator

Experimental initiative to test nonlinear integrable optics: Danilov & Nagaitsev “Nonlinear accelerator lattices with one and two analytic invariants,” PRSTAB 13, 084002 (2010)

Use of a special nonlinear magnet can result in dynamics that remain integrable and thus still maintain invariants of motion.

• Single particle trajectories are regular and bounded.
• Mitigates parametric resonances via nonlinear decoherence.
Lattice Requirements for Integrability

- $\beta_x(s) = \beta_y(s), \ D(s)=0$ through underlying drift region*.
- Must have $n\pi$ phase advance around the ring between the exit and entrance of the nonlinear magnet region.

Very tricky in the presence of space charge and the tune depression that results.

Work with KV distribution in simulation due to the uniform and easily calculable tune depression.

See also: N. Cook et al., “Nonlinear Dynamics, Dispersive, and Chromatic Considerations in the IOTA Lattice”, WEP0A31

*Danilov & Nagaitsev “Nonlinear accelerator lattices with one and two analytic invariants,” PRSTAB 13, 084002 (2010)
Initial Efforts to Include Space Charge

- Initially used 2D particle-in-cell (PIC) to model transverse space charge of the proton beam.
- Found invariants to be poorly maintained.

For a KV bunch the distribution in H starts as a delta function but quickly spreads.
Adjusting for Space Charge Tune Depression

Need an $\pi n\pi$ phase advance around the ring between the exit and entrance of the nonlinear magnet with space charge.

An iterative rematching method is used to correct the lattice for the appropriate tune depression and preserve the phase advance.

Can then find an optimal current setting to nearly zero the phase advance in both planes.

-Results shown are optimized for modest depression of $dQ_{SC} = 0.03$

Results with the Compensated Lattice

- Phase advance around the ring is now corrected with the new compensated lattice.
- This leads to much better behavior of the first invariant (the Hamiltonian).
Changing to a Simplified Model

• Will use lattice is modified to account for tune depression of the beam to preserve integer phase advance between the end and beginning of the nonlinear magnets.

In Simulation:

• Lattice of the ring is modeled using first-order maps to preserve linearity.
  • Nonlinear magnet elements still include full higher-order treatment.

• Space charge is implemented using a ’frozen’ KV model that assumes completely linear space charge forces for all particles.

• Goal is to reduce system complexity to find a baseline working point.
New Behavior of the First Invariant

With these fixes/modifications behavior of the invariants improves:

- Average of H over the ensemble of particles remains steady over 1000 turns.
- Increasing macroparticle number does show reduction in amplitude of fluctuations of the average.
New Behavior of the First Invariant Cont.

Even with analytic SC there is still a spread that develops in the distribution of the first invariant.

- Though the average remains fixed there is still significant diffusion of the invariant.
- This diffusion appears to be largely numerical.
- Even for this 'frozen' space charge model there is still a strong dependence on macroparticle number for the invariant behavior.
Amplitude Oscillations in the First Invariant

- Oscillations of the invariant seem to primarily stem from oscillations of the beam envelope in x.
- These oscillations occur near the depressed tune of the lattice.
**Future Work**

- Continue to study numerical diffusion effects in simulation
  - Quantify numerical diffusion
  - Find a convergence point with macroparticle number
  - Further study of amplitude of oscillations

- Use new working point for current and the space charge compensated lattice to return to a self-consistent space charge model
Conclusions

• IOTA will provide an important testbed for integrable nonlinear optics which may provide a path to very high intensity hadron beam accelerators.

• Correction of the linear lattice for space charge tune depression is essential for proper operation.

• Simulations of IOTA with space charge show growth in the spread of the invariants with both PIC and analytic models. It is certain that there is some significant numerical contribution here from the space charge calculation.