Emittance Growth from Modulated Focusing in Bunched Beam Cooling

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This work follows work by Gang Wang and Vladimir Litvinenko

Outline:

Introduction

Theory of Emittance Growth

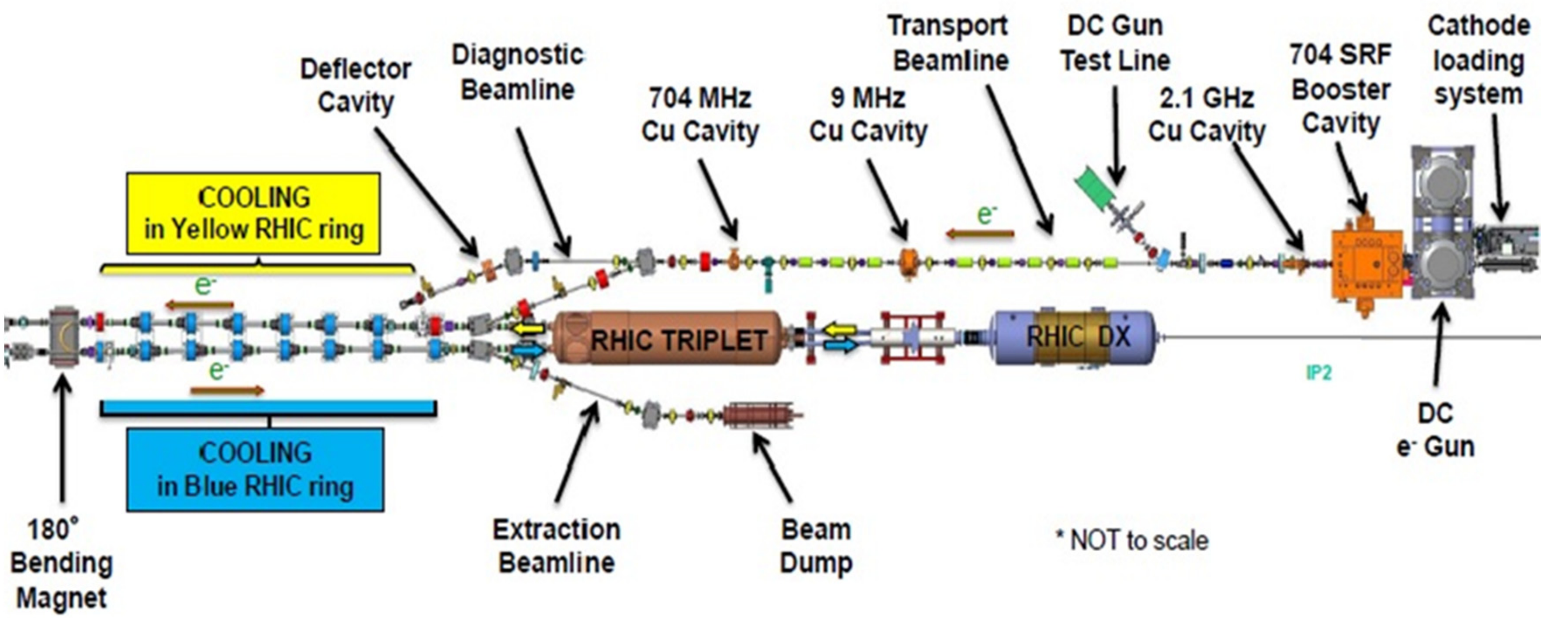
Comparison of Theory and Simulations

Introduction

The Low Energy RHIC electron Cooling (LEReC) project is currently under construction at BNL (see WEA4CO05).

A linac supplies electrons with kinetic energies from 1.6 to 2.6 MeV which cool the ions over a 20 meter drift (C=3.8km).

The goal is to improve Au-Au luminosity, aiding in the study of the quark-gluon plasma near its phase boundary.



The electron bunches are short compared to the ion bunches, 3cm .vs. 4m.

When timing is stable, most heating is due to electron space charge augmented by longitudinal Intra-Beam Scattering (IBS).

These simulations used 10 times nominal e bunch charge.

Nominal IBS growth time is 650 sec. Saturation occurs for nominal and faster IBS.

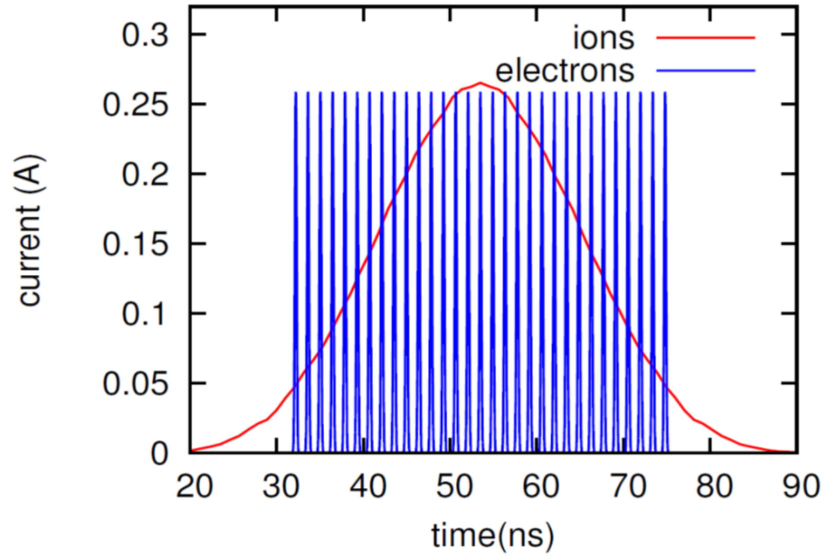
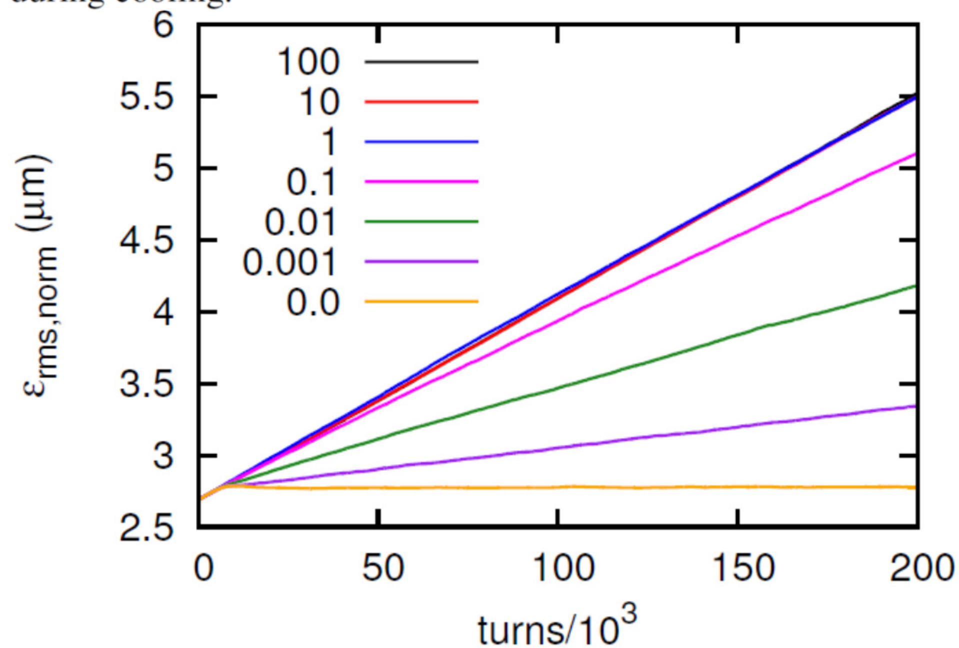


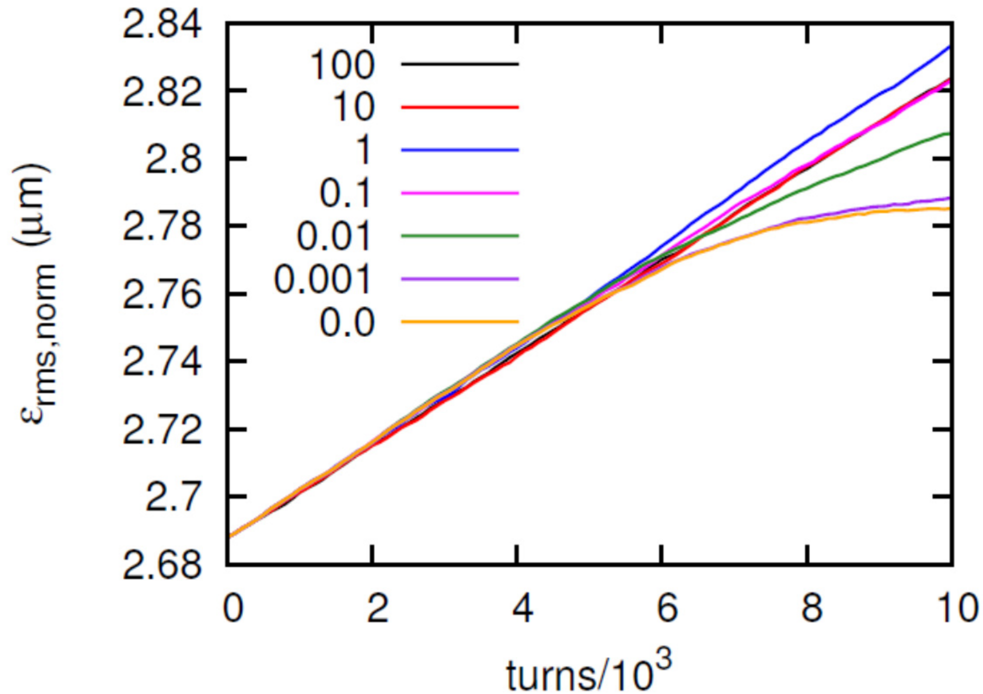
Figure 1: Longitudinal bunch structure for ions and electrons during cooling.



Basic idea is that emittance growth is due to synchrobetatron resonance.

Individual resonances saturate at relatively low levels.

Longitudinal IBS causes J_s and subsequently Q_s to vary, replenishing the resonant reservoir, leading to steady rms growth.



This effect, as well as the dependence of the growth rate on various dynamical parameters can be explained analytically.

Theory

Consider the Hamiltonian for the ions

$$H_0(x,p) = \frac{Q_x}{2}\left(p^2 + x^2\right) + 2\pi a^2 \delta_p(\theta)\Delta Q_x(\tau)\ln\left(1 + \frac{x^2}{a^2}\right)$$

where θ is the update variable and $\Delta Q_x(\tau)$ is the space charge tune shift from the electrons as a function of the longitudinal coordinate. $\delta_p(\theta)$ is the periodic delta function.

Switch to action-angle variables

$$x = \sqrt{2J}\cos\psi, \qquad p = -\sqrt{2J}\sin\psi$$

$$H_0(\psi,J) = Q_x J + 2\pi a^2 \delta_p(\theta)\Delta Q_x(\tau)\ln\left(1 + \frac{2J\cos^2\psi}{a^2}\right)$$

$$= Q_x J + 2\pi\delta_p(\theta)\Delta Q_x(\tau)\sum_{n=0}^{\infty} a_n(J)\cos(2n\psi)$$

For n>0 $$a_n(J) = \frac{-2a^2}{n}\left(\frac{-J/a^2}{1+J/a^2+\sqrt{1+2J/a^2}}\right)^n$$

We consider small ΔQ_x hence only a single betatron sideband

$$H_1(\psi, J) = Q_x J + 2\pi\delta_p(\theta)\Delta Q_x(\tau) a_n(J)\cos(2n\psi)$$

H_1 is easy to simulate but difficult analytically owing to the delta function. To proceed take the slow approximation

$$\cos(2n\psi)\left[\frac{1}{2\pi} + \sum_{p=1}^{\infty}\frac{\cos(p\theta)}{\pi}\right] \approx \frac{\cos(2n\psi - p\theta)}{2\pi}$$

With $|p-2nQ_x|$ minimized. We also make a scale change with

$$\Psi = 2n\psi - p\theta$$

In these variables we have

$$H_2(\Psi, J) = (2nQ_x - p)J + 2n\Delta Q_x(\tau)a_n(J)\cos\Psi$$

Next go back to Cartesian coordinates

$$\widetilde{x} = \sqrt{2J}\cos\Psi, \qquad \widetilde{p} = -\sqrt{2J}\sin\Psi, \qquad \delta = 2nQ_x - p$$

$$\Delta Q_x(\tau)/\Delta Q_x = C(\tau) \le 1, \qquad \varepsilon(J) = 2n\Delta Q_x\, a_n(J)/\sqrt{2J}$$

In these variables we get the easy looking Hamiltonian

$$H_2(\widetilde{x}, \widetilde{p}) = \frac{\delta}{2}\left(\widetilde{x}^2 + \widetilde{p}^2\right) + \widetilde{x}C(\tau)\varepsilon(J)$$

Of course we need to take J as a function of the other two variables.

The equations of motion are

$$\frac{d\tilde{x}}{d\theta} = \frac{\partial H_2}{\partial \tilde{p}} = \delta\tilde{p} + \tilde{x}\tilde{p}\varepsilon'(J)C(\tau)$$

$$\frac{d\tilde{p}}{d\theta} = -\frac{\partial H_2}{\partial \tilde{x}} = -\delta\tilde{x} - \varepsilon(J)C(\tau) - \tilde{x}^2\varepsilon'(J)C(\tau)$$

These are solved approximately.

Define $u = \tilde{p} + i\tilde{x}$

$$\frac{du}{d\theta} = i\delta u - \alpha_q C(\tau)$$

where $\alpha_0 = \varepsilon(J_0), \qquad \alpha_1 = \varepsilon(J_0) + J_0\varepsilon'(J_0)$

The second expression is just the θ average of the driving term.

All the random stuff is in $\tau = \tau(\theta)$.

Integrating the solution one finds

$$\Delta u(\theta) = u(\theta) - u_0 e^{i\delta\theta} = -\alpha_q \int_0^{\theta} d\theta_1 C(\tau(\theta_1)) \exp(i\delta(\theta-\theta_1))$$

The second moment determines emittance growth ($\langle \Delta u^* u_0 \rangle = 0$)

$$\left\langle |\Delta u(\theta)|^2 \right\rangle = \alpha_q^2 \int_0^{\theta} d\theta_1 \int_0^{\theta} d\theta_2 \exp(i\delta(\theta_2-\theta_1)) \left\langle C(\tau(\theta_1)) C(\tau(\theta_2)) \right\rangle$$

Now define $$C(\tau) = \sum_{m=0}^{\infty} C_m(\tau_0) \cos(m\psi_s(\theta))$$

It is clear that IBS driven variations in τ_0 lead to changes in Q_s making ψ_s a random variable. The phase variations are bound to swamp any variations in the C_ms so we take τ_0 constant and treat ψ_s as a random variable.

The second moment becomes

$$\left\langle C(\tau_1)C(\tau_2)\right\rangle = \sum_{m,k} C_m(\tau_0)C_k(\tau_0)\left\langle \cos(m\psi_{s,1})\cos(k\psi_{s,2})\right\rangle$$

Now we have $$\psi_s(\theta) = \psi_s(0) + \int_0^{\theta} d\theta_1 Q_s(\theta_1)$$

Average $\psi_s(0)$ over initial phases.

$$\left\langle C(\tau_1)C(\tau_2)\right\rangle = \sum_{m=0}^{\infty} \frac{C_m^2(\tau_0)}{2-\delta_{m,0}} \left\langle \cos(m\int_{\theta_1}^{\theta_2} Q_s(\chi)d\chi) \right\rangle$$

Substitute this into the emittance growth equation and assume $Q_s(\theta)$ is a stationary random variable, take $\chi=\theta_2-\theta_1$.

$$\left\langle |\Delta u(\theta)|^2 \right\rangle = \alpha_q^2 \int_{-\theta}^{\theta} d\chi \exp(i\delta\chi)(\theta - |\chi|) \sum_{m=0}^{\infty} \frac{C_m^2(\tau_0)}{2-\delta_{m,0}} \left\langle \cos[m\int_0^{\chi} Q_s(\phi)d\phi] \right\rangle$$

Repeating

$$\left\langle |\Delta u(\theta)|^2 \right\rangle = \alpha_q^2 \int_{-\theta}^{\theta} d\chi \exp(i\delta\chi)(\theta - |\chi|) \sum_{m=0}^{\infty} \frac{C_m^2(\tau_0)}{2-\delta_{m,0}} \left\langle \cos[m\int_0^{\chi} Q_s(\phi)d\phi] \right\rangle$$

We need to evaluate the expectation value of the cosine.

Assume the initial variation in Q_s swamps any change on [0,χ].

Suppose Q_s is a Gaussian random variable with mean Q_{s0} and standard deviation σ_s. Then

$$\left\langle \cos[m\int_0^{\chi} Q_s(\phi)d\phi] \right\rangle = \cos[mQ_{s0}\chi]\exp(-m^2\sigma_s^2\chi^2/2)$$

We typically have $\sigma_s >$ Q_s /100 and IBS time scales of several minutes or more. This justifies our earlier assumption and explains the weak dependence of growth rate on IBS rate.

For θ>> $1/\sigma_s$ and $\delta\neq 0$ the growth is nearly linear in θ.

$$\left\langle |\Delta u(\theta)|^2 \right\rangle' = \frac{\alpha_q^2}{2} \int_{-\infty}^{\infty} d\chi \sum_{m=1}^{\infty} C_m^2(\tau_0) \cos[mQ_{s0}\chi] \exp(i\delta\chi - m^2\sigma_s^2\chi^2/2)$$

The integral is straightforward naturally breaking into two parts

$$\frac{d}{d\theta}\left\langle |\Delta u(\theta)|^2 \right\rangle = \left\langle |\Delta u(\theta)|^2 \right\rangle' = \left\langle |\Delta u(\theta)|_+^2 \right\rangle' + \left\langle |\Delta u(\theta)|_-^2 \right\rangle'$$

With

$$\left\langle |\Delta u(\theta)|_\pm^2 \right\rangle' = \sum_{m=1}^{\infty} C_m^2(\tau_0) \frac{\sqrt{2\pi}\alpha_q^2}{4m\sigma_s} \exp\left(-\frac{1}{2}\left[\frac{mQ_{s0} \pm \delta}{m\sigma_s} \right]^2 \right)$$

This is the main result. We go on to compare this formula with simulations using the thin lens kick and single betatron sideband.

We take $C(\tau) = 1/(1+\tau_0^2 \sin^2 \psi_s)$ $\quad z = \tau_0^2/(2+\tau_0^2)$

$$C_{2m}(\tau_0) = \frac{2}{(1+\tau_0^2/2)\sqrt{1-z^2}} \left(\frac{1-\sqrt{1-z^2}}{z} \right)^{2m}$$

For the random process we take

$$\overline{\psi}_s = \psi_s + 2\pi\left[\overline{Q}_s + q\right] \qquad \overline{q} = rq + \sigma_s \sqrt{1-r^2}\, x$$

The bars denote updated values, x is a zero mean unit standard deviation deviate and 0<r<1.

Space charge update done with a canonical transformation

$$F(J,\overline{\psi}) = 2\pi \Delta Q_x(\tau) a_n(J) \cos(2n\overline{\psi})$$

$$\overline{\psi} = \psi + \frac{\partial F}{\partial J}, \qquad \overline{J} = J - \frac{\partial F}{\partial \overline{\psi}}$$

Simulations with

$\Delta Q_x = 10^{-4}$, $\tau_0 = 3$, $Q_s = 0.01$, $\sigma_s = 0.001$

$Q_x = 0.011$, $n = 1$, $r = 0.999$, $x/a = 2$

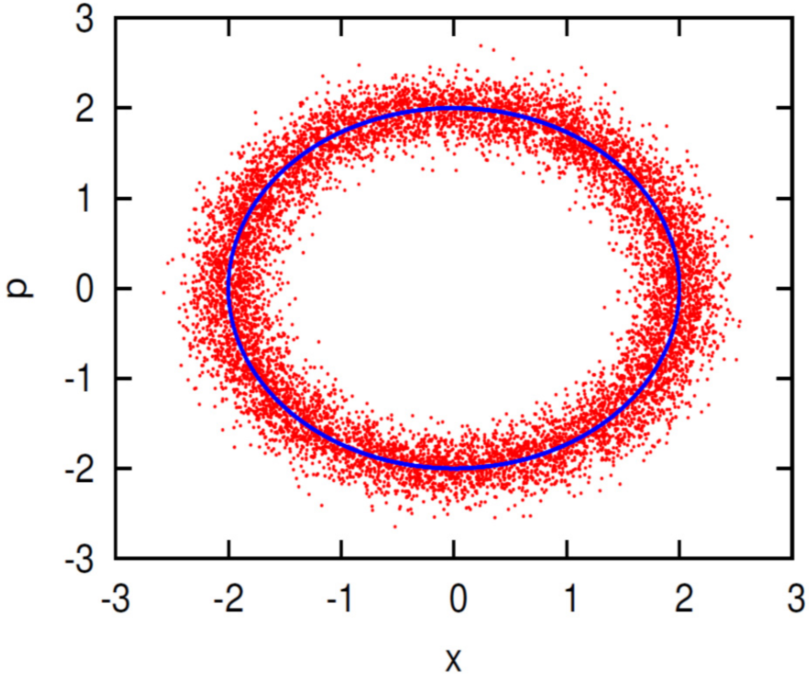


Figure 4: Initial (blue) and final (red) particle coordinates for a simulation with 10^4 particles.

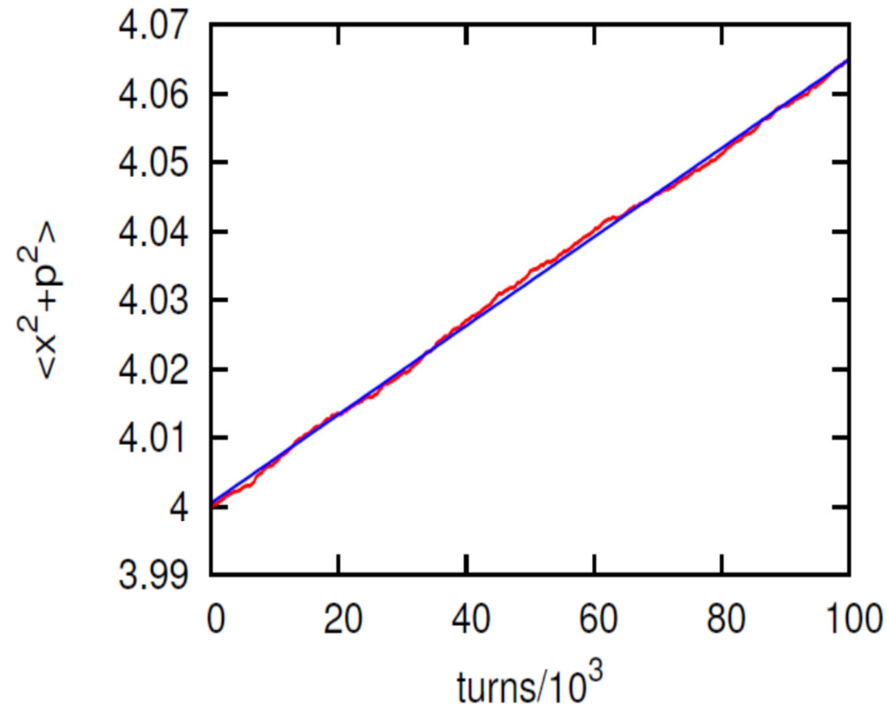


Figure 5: $< |u|^2 >$ versus time for a simulation with 10^5 particles and the same beam parameters as in Figure 4. The simulation data are in red and the blue line is a least squares fit. The slope of the line is the emittance growth and is to be compared with equations (16) and (17).

Single sideband and full thin lens

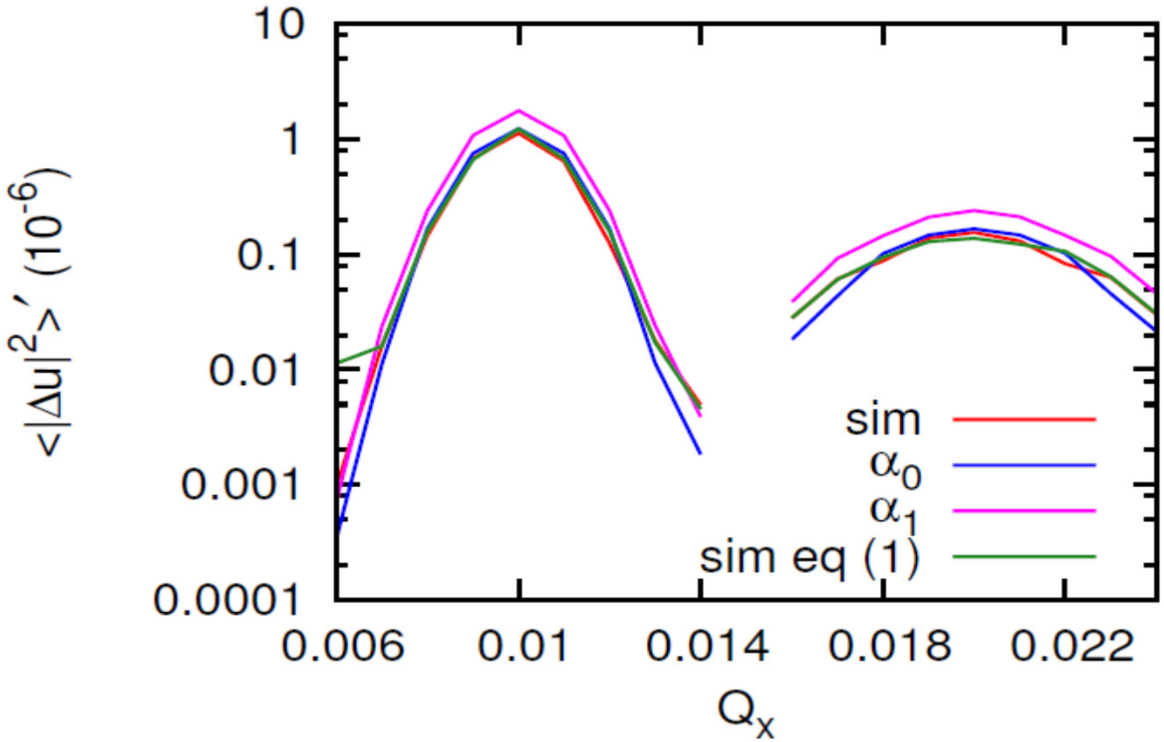


Figure 4: Emittance growth rates for $n = 1$, $J = 2a^2$, $\hat{\tau} = 3$ as a function of betatron tune. The formulas for α_0, α_1 and a simulation using the full Hamiltonian in eq (1) are also shown.

Different synchrotron amplitudes, single sideband simulations

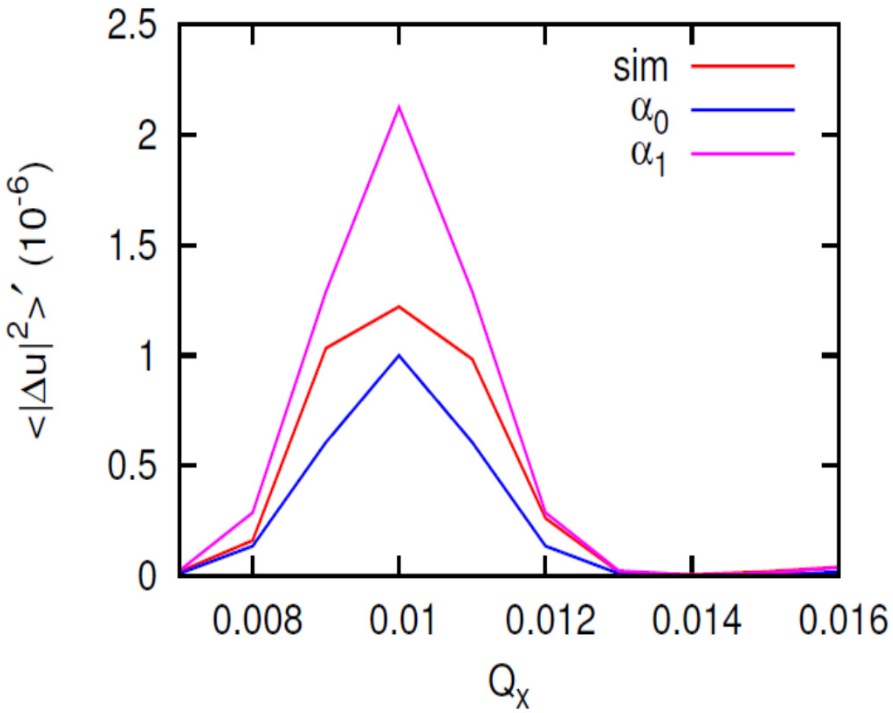


Figure 5: Emittance growth rates for $n = 1$, $J = a^2/2$, $\hat{\tau} = 3$ as a function of betatron tune. The formula for α_0 and α_1 bracket the simulation.

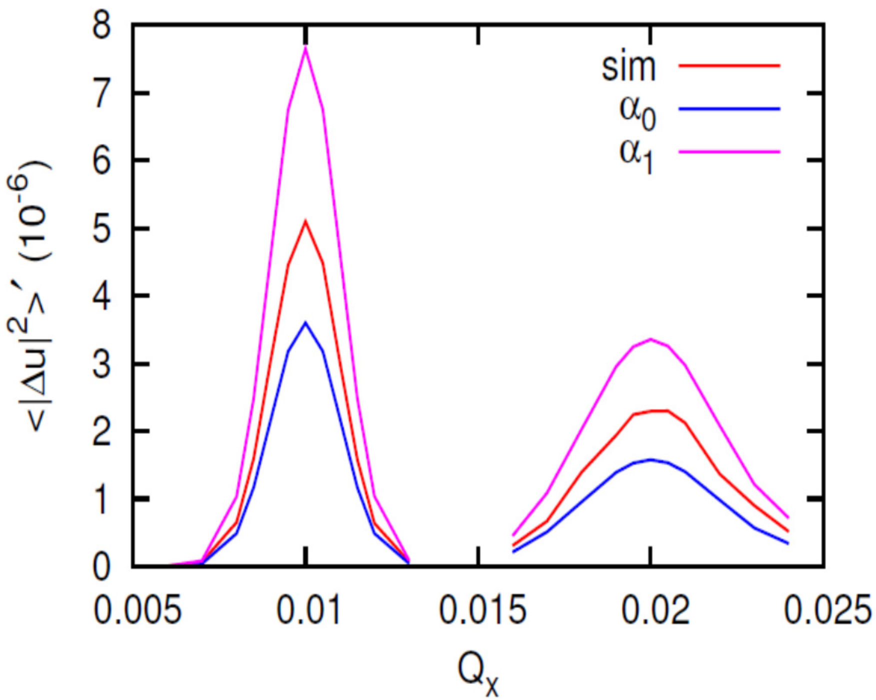


Figure 6: Emittance growth rates for $n = 1$, $J = a^2/2$, $\hat{\tau} = 30$ as a function of betatron tune. The formula for α_0 and α_1 bracket the simulation.

For small betatron amplitude all looks pretty good.

Why are there problems at large J?

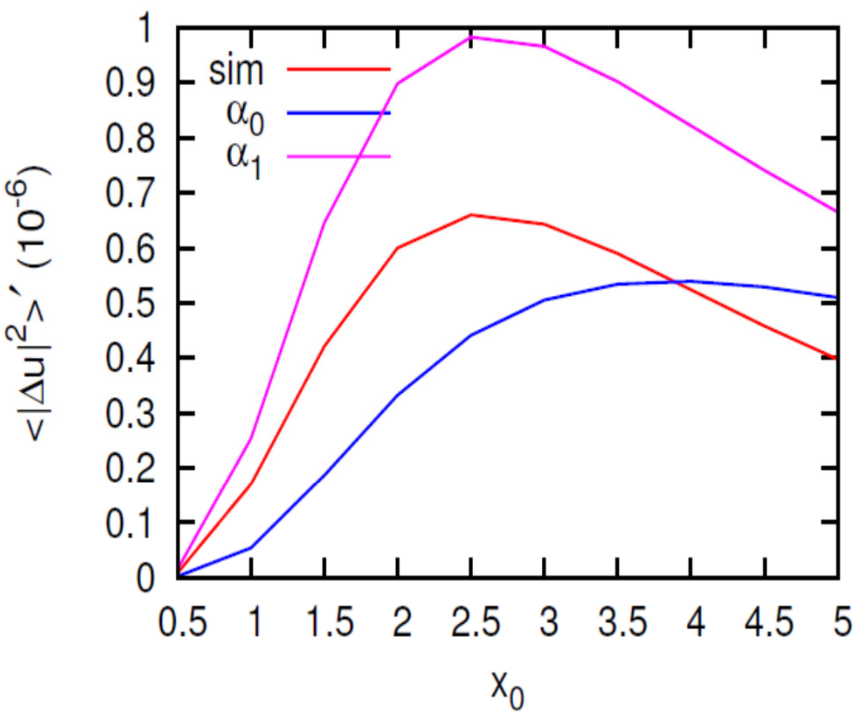


Figure 8: Emittance growth rates for $n = 2$, $\hat{\tau} = 30$ as a function of $x_0 = \sqrt{2J}/a$. For large betatron amplitudes the formulas overestimate the growth.

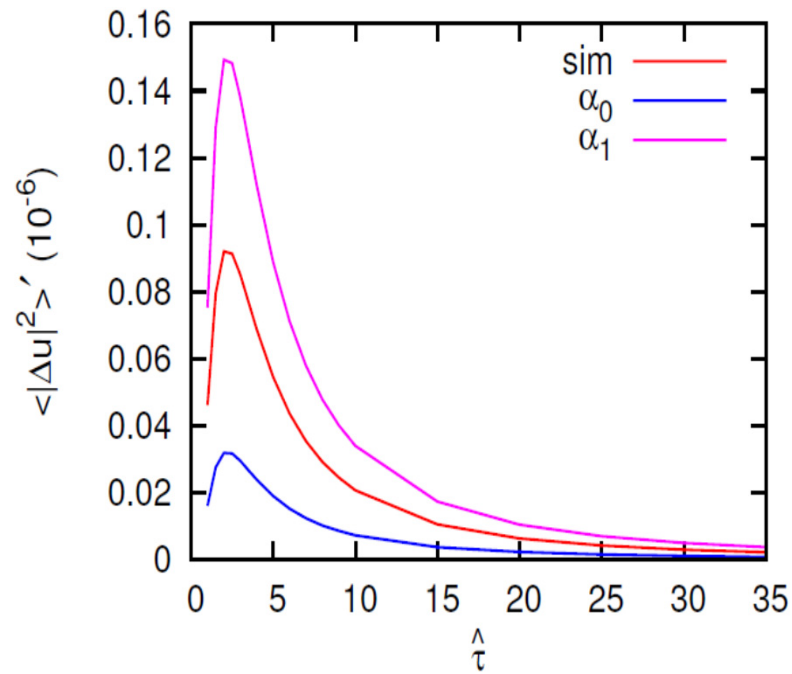


Figure 7: Emittance growth rates for $n = 2$, $J = a^2/2$ versus τ_0

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Actual LEReC is OK

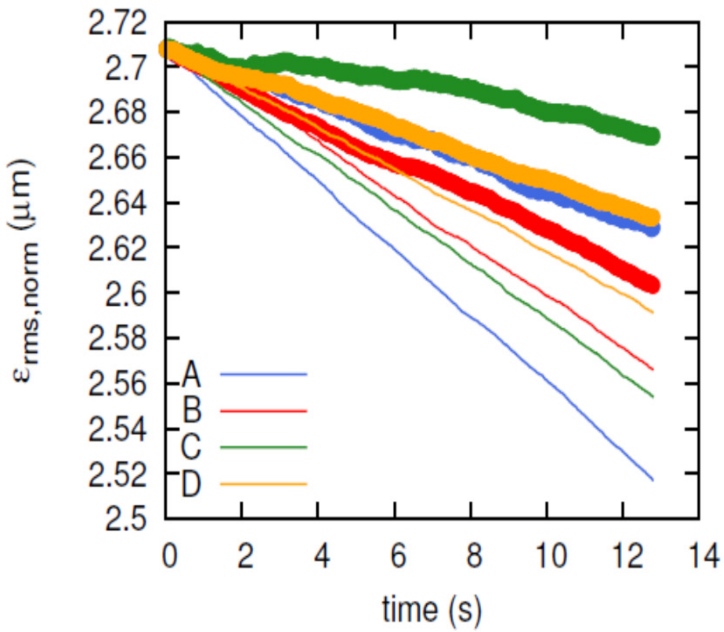


Figure 9: Ion emittance versus time for $\gamma = 4.1$. The thick lines show the predicted behavior while the thin lines neglect the coherent kicks from the electrons. There are four cases with different rms momentum spread σ_p, rms emittance ϵ and electron bunch charge Q_e. The stated electron bunch charge was used for the coherent kick while the charge used for cooling was 30% less.
A, $\sigma_p = 4 \times 10^{-4}$, $\epsilon = 2\ \mu$m, $Q_e = 130$ pC;
B, $\sigma_p = 4 \times 10^{-4}$, $\epsilon = 1\ \mu$m, $Q_e = 65$ pC;
C, $\sigma_p = 8 \times 10^{-4}$, $\epsilon = 2\ \mu$m, $Q_e = 130$ pC;
D, $\sigma_p = 8 \times 10^{-4}$, $\epsilon = 1\ \mu$m, $Q_e = 65$ pC.

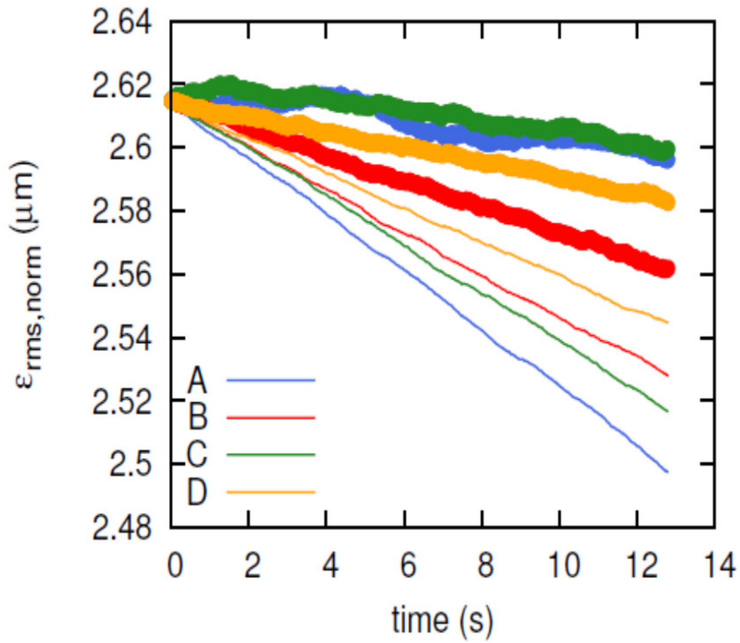
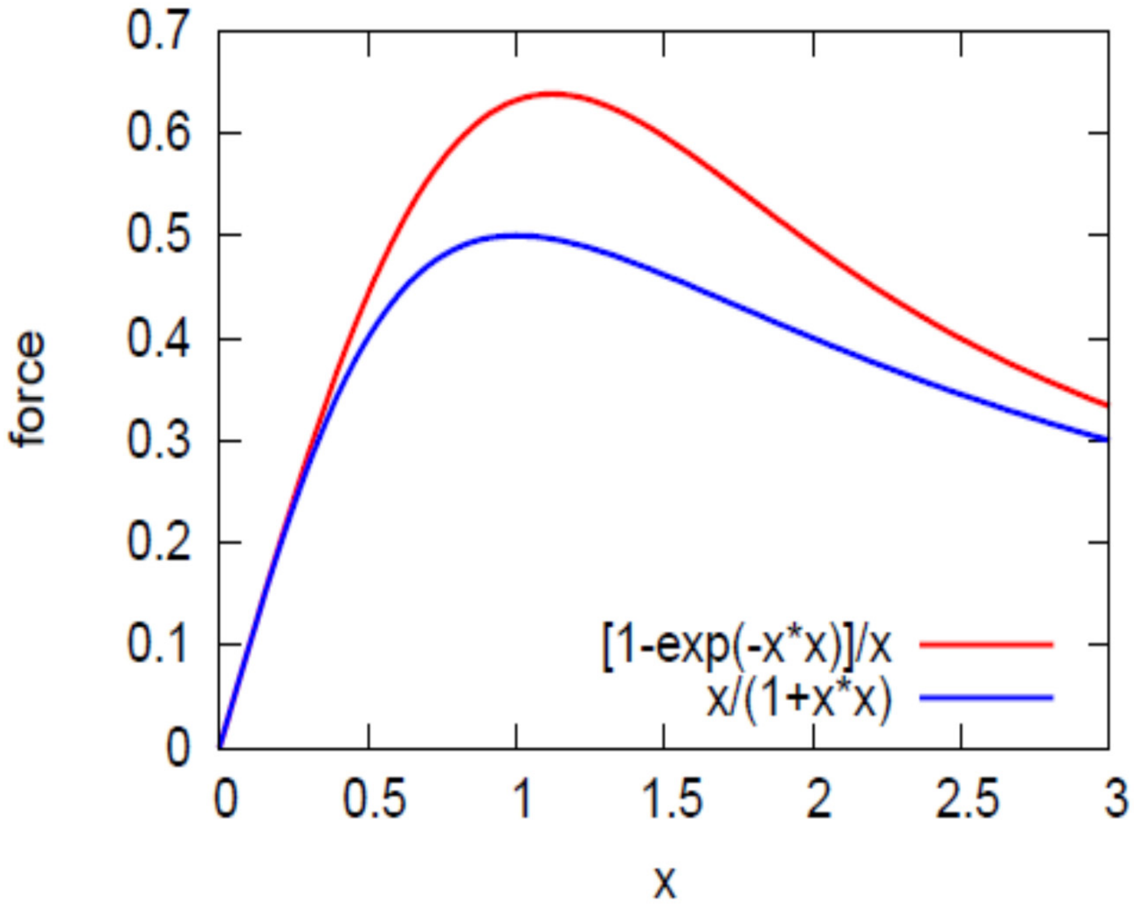


Figure 10: Ion emittance versus time for $\gamma = 6.$. The thick lines show the predicted behavior while the thin lines neglect the coherent kicks from the electrons. There are four cases with different rms momentum spread σ_p, rms emittance ϵ and electron bunch charge Q_e. The stated electron bunch charge was used for the coherent kick while the charge used for cooling was 30% less.
A, $\sigma_p = 4 \times 10^{-4}$, $\epsilon = 2\ \mu$m, $Q_e = 156$ pC;
B, $\sigma_p = 4 \times 10^{-4}$, $\epsilon = 1\ \mu$m, $Q_e = 78$ pC;
C, $\sigma_p = 8 \times 10^{-4}$, $\epsilon = 2\ \mu$m, $Q_e = 156$ pC;
D, $\sigma_p = 8 \times 10^{-4}$, $\epsilon = 1\ \mu$m, $Q_e = 78$ pC.

Summary and Conclusions

- The analytic theory of emittance growth from noise driven synchrobetatron resonances is pretty good.
- Factor of 2 discrepancies exist and are left for future work.
- The actual LEReC is OK but the effect is clearly visible.
- Reducing intensity and transverse emittance of the electron bunches might be beneficial.

Comparing a Gaussian and this model



Tune shift from electrons

$$\Delta Q_x = \frac{\beta_L Z_0 I_e \Delta s}{8\pi^2 (\beta\gamma)^3 a^2 E_T / q}$$

Typically 5.e-4