Calculating Spin Lifetime

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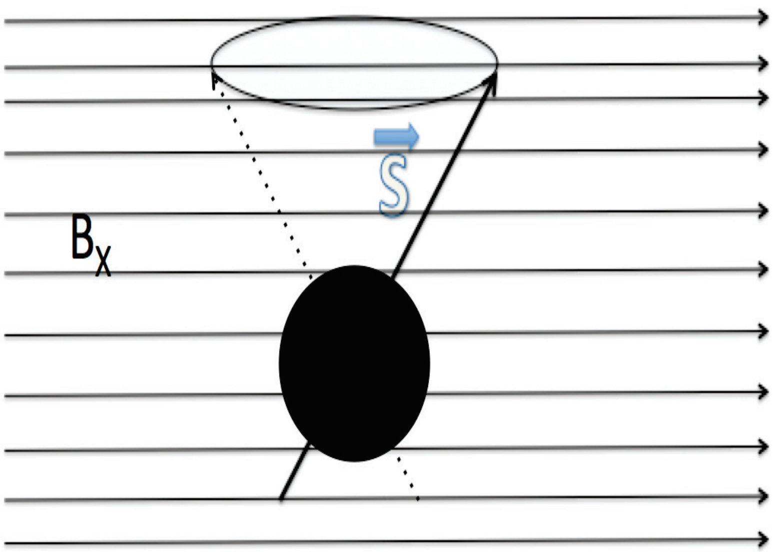
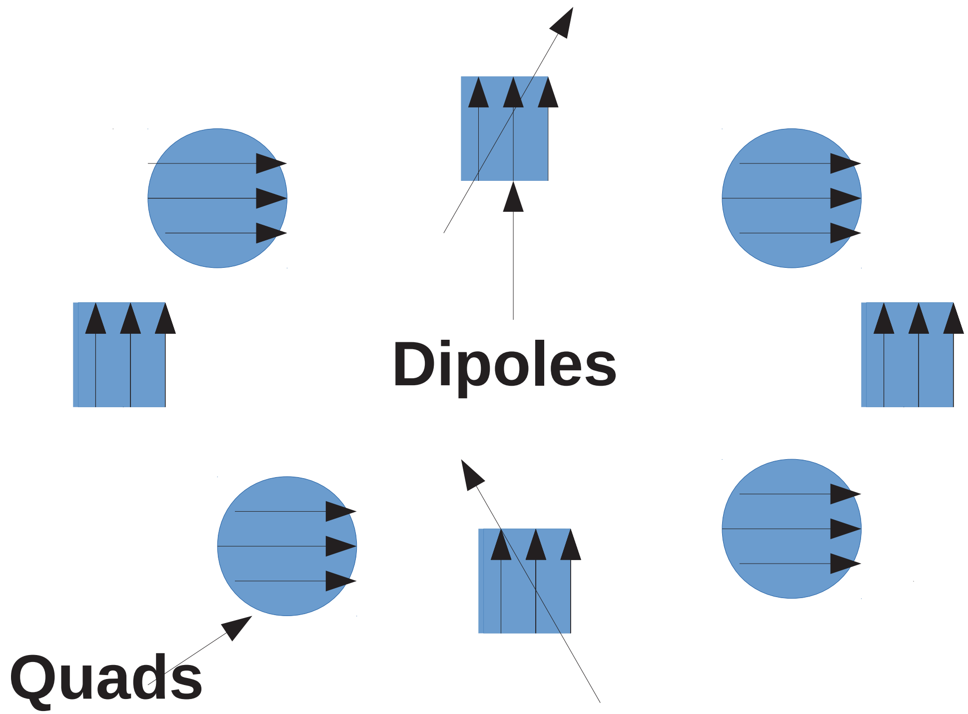
my LITTLE PONY
Polarization lifetime
is
MAGIC

Spin Dynamics Review

T-BMT Equation: $$\frac{d\vec{S}}{dt} = \frac{q}{\gamma m}\vec{S} \times \left((1+G\gamma)\vec{B}_\perp + (1+G)\vec{B}_\parallel\right)$$





Spin Resonance: Spin Tune = Rate of Horizontal field kicks
(vertical motion through Quads)

Gγ = N +/- Qz Intrinsic or **Gγ = N** Imperfection
Due to vertical betatron Due to vertical closed orbit error

Can be cured
Using Snakes
Flip spin by PI
Spin tune = 1/2

Problem of Polarization Lifetime

- Contributes between 0.5 to 2 % of polarization loss per hour.
- We don't understand this at all. (Its MAGIC)
  - We have good analytical theory for single spin resonance model even with snakes.
  - We have some analytical theory for overlapping spin resonances but not with snakes
  - Numerical : With direct spin tracking to simulate 1 hour of beam time would take 300 million turns in RHIC
    - Even if we could do 100K turns in 1 hour (which is about 4 times faster than what I can do) it would take 125 days to do this! Even if we would allocate the time we don't have the compute resources to do this for any kind of realistic distribution

Lattice Independent Integration of T-BMT equation

$$\frac{d\Psi}{d\theta} = -\frac{i}{2}\begin{pmatrix} f_3 & -\xi \\ -\xi^* & -f_3 \end{pmatrix}\Psi.$$

Here $f_3 \cong G\gamma$. Expanding the driving terms in a Fourier series in the standard way:

$$\xi(\theta) = F_1 - iF_2 = \sum_K \varepsilon_K e^{-iK\theta}$$

Integrating this equation requires a lot of care. Especially since the integrator needs to be both accurate and preserve unitarity over about a billion integration Steps . To accomplish this we turned to a Magnus based unitary 4^{th} order gaussian quadrature integrator.

Speed up with lattice independent tracking:

- Now with a new C++ version of this code we can now track 300 million turns in about 2 ½ hours! (originally I wrote this in python and then it took me 2 weeks of cpu time to do 1 hour.)
- However tracking with only spin resonances driven by betatron motion and even with closed orbit motion yielded no mechanism for polarization loss
- So we added longitudinal motion effects:

Adding Longitudinal effects to Spin Tracker:

To understand the effect of longitudinal motion on spin we need to first understand its effect on betatron motion. The betatron equation of motion with the addition of longitudinal dynamics can be approximated:

$$\frac{d^2Y(s,\delta,z)}{ds^2} + \omega_\beta^2(\delta)/c^2Y(s,\delta,z) = 0.$$

$$\omega_\beta(\delta) = \omega_0 Q + \xi\omega_0\delta.$$

$$\delta(s) = \frac{-\omega_s}{\eta c} r\sin(\omega_s s/c + \phi)$$

Approximated solution entail the addition of a modulating phase:

$$Y(s, \delta, z) \approx Ae^{\pm i\Phi(s,\delta,z)},$$

where A is the constant amplitude and

$$\Phi(s, \delta, z) = \int_0^s ds'[\omega_0 Q/c + \omega_0 \xi \delta(s)/c]$$

$$\Phi(s, \delta, z) = \omega_0 Qs/c + \frac{\xi \omega_0}{\eta c}[z(s) - z(0)]$$

Thus we can approximate the effects by adding a phase modulation to the Betatron phase part of the spin resonance: So for a given spin resonance we Have:

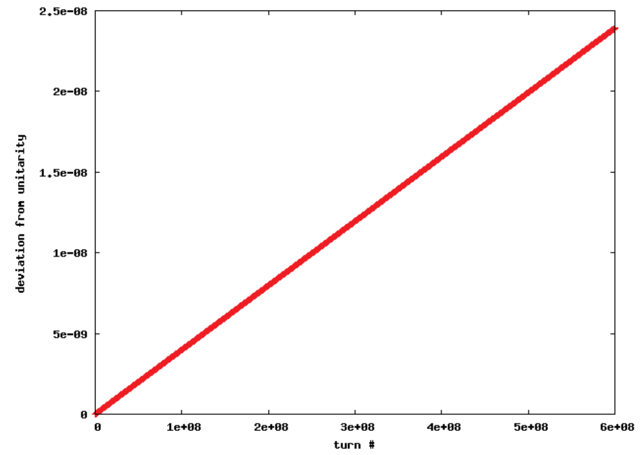
$$\sum_K \epsilon_K e^{-iK\theta + i\frac{\xi\omega_0\tau_0}{\eta}(1 - cos(\omega_s s/c))}$$

In addition the energy modulation effect can be modeled using:

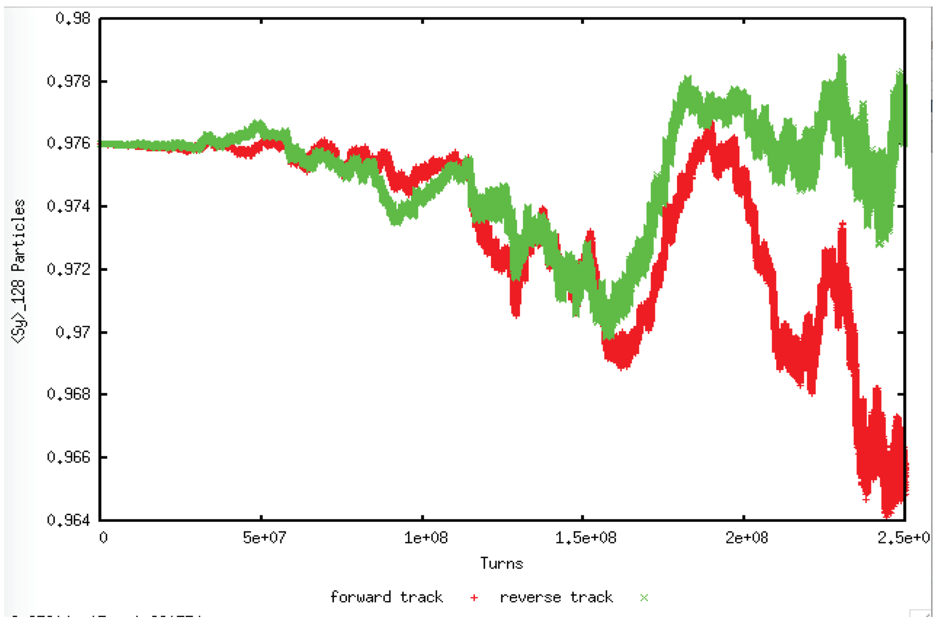
$$G\gamma(s) = (G\gamma_0 + \alpha\theta)(1 + \delta(\theta))$$

Error Estimation

- Hard to check but we can check unitarity and do reverse tracking:

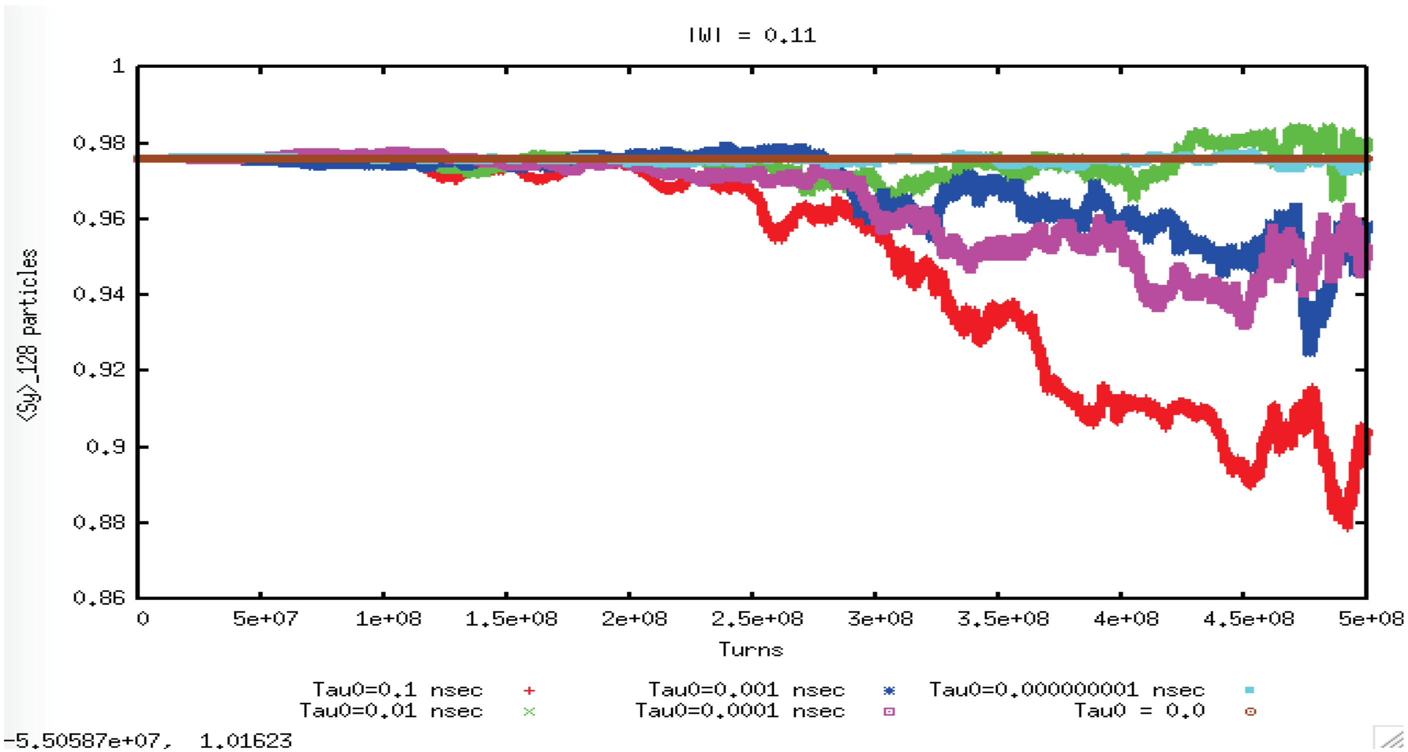


It seems violation of unitarity grow at
a nice linear rate per turn out at
2 hours of beam time it is only violated at
1e-8 level.



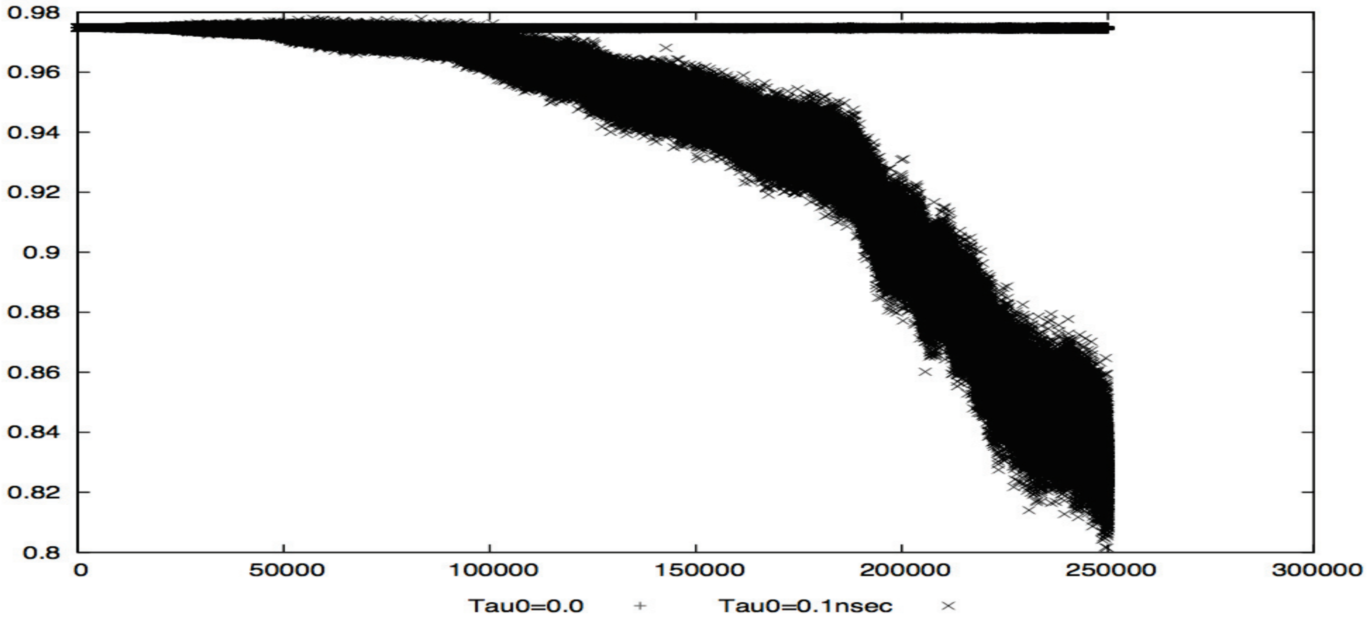
We also performed reverse tracking. Here we took 128 particles out to 2 hours (5e8 turns) and then reverse tracked to recover our starting spin values to within 5e-6.

Longitudinal dynamics Key Ingredient for Polarization decay



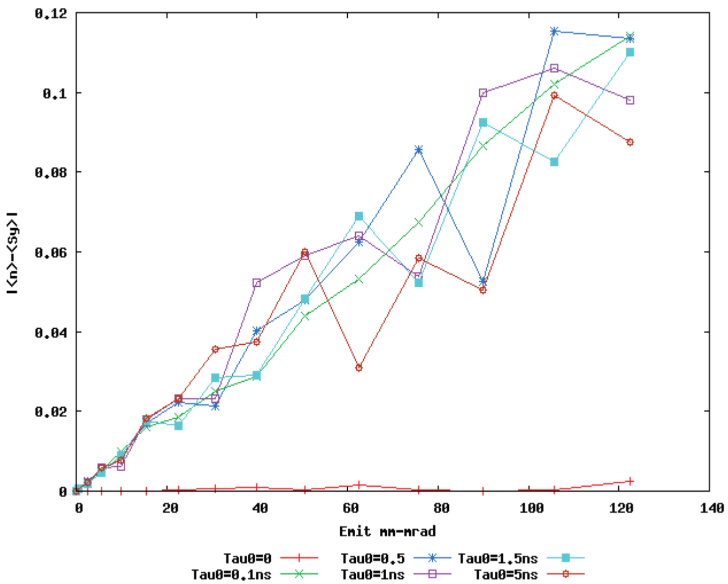
With only a single intrinsic spin resonance, the introduction of a longitudinal amplitude of 1e-16 sec can generate polarization decay.

What about overlapping Intrinsic resonances?



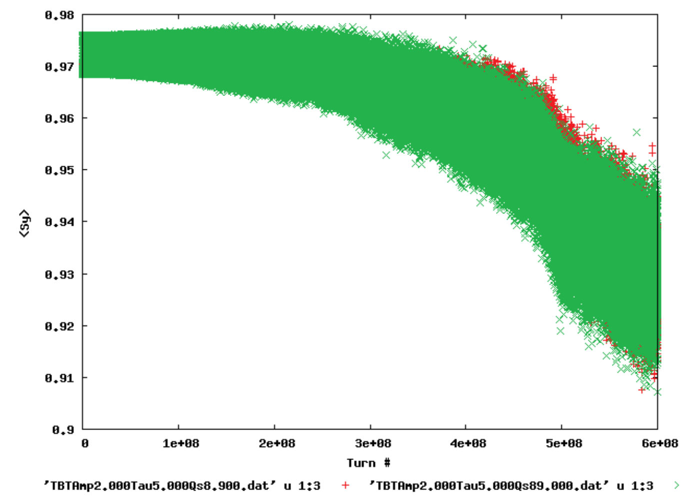
Even considering the effects of up to five nearby intrinsic resonance without longitudinal motion no decay is observed. This is actually what motivated my consideration of Longitudinal motion.

Effect of Longitudinal parameters

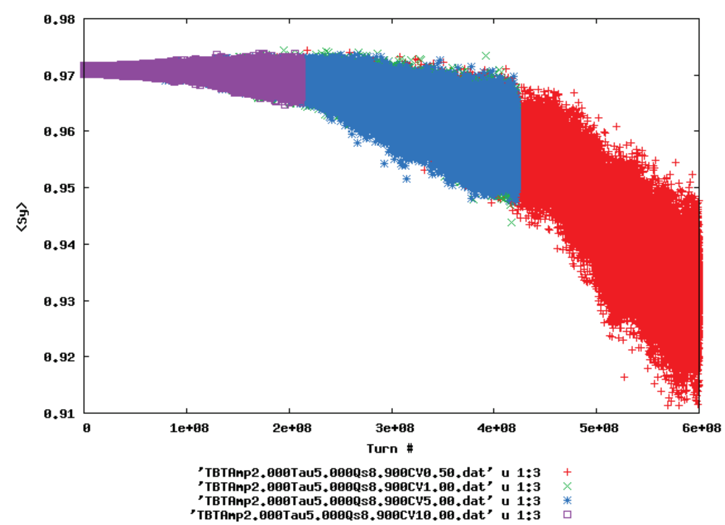


Effect of time offset (tau)

Effect of Qs
Synchrotron tune
From 8.8e-4 to
89.0e-4



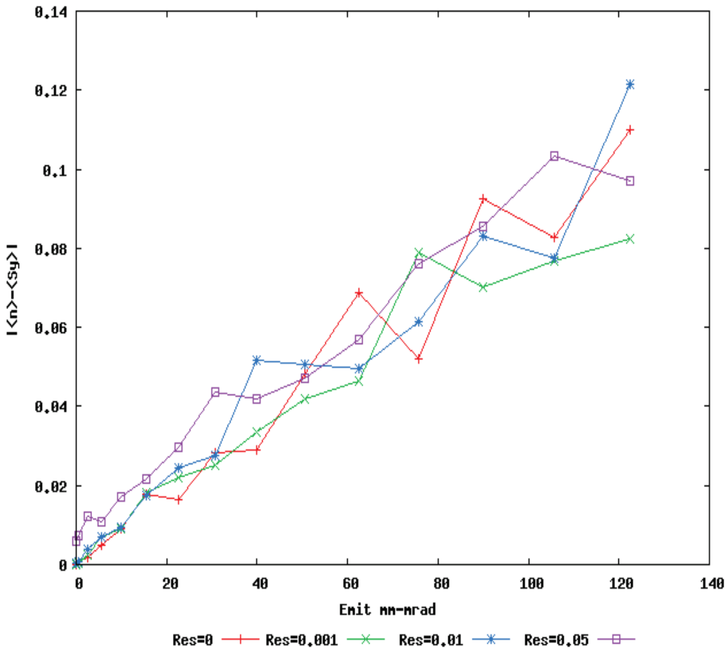
Effect of Chromaticity
From 0.5 to 10 units



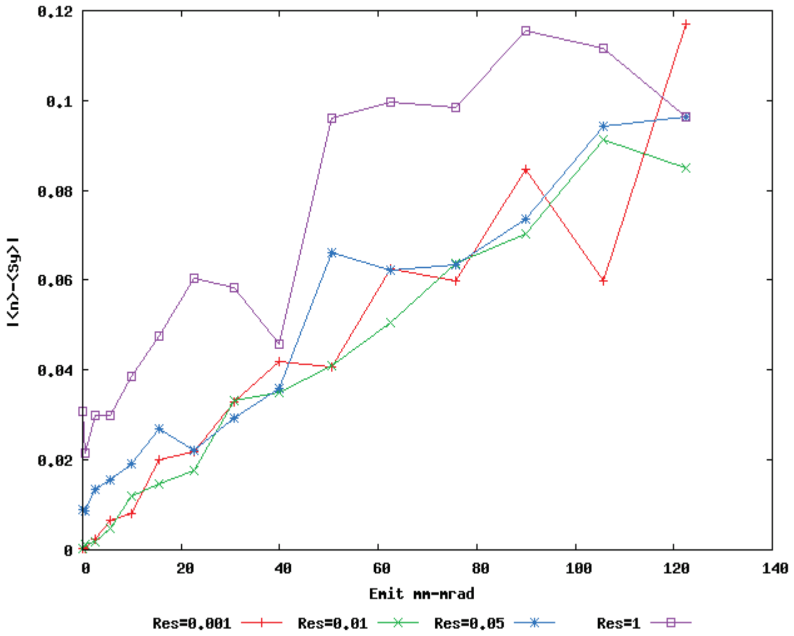
Drivers of Polarization Lifetime

- Magnitude of nearby Intrinsic resonances
  - May have noticed decay increases with emittance which is directly calculated from intrinsic resonance for our model.
- Magnitude of nearby imperfection resonances
  - As we will see on next slide: threshold is about 0.05 magnitude for both closest integer.
  - Indirectly this means snake and rotator imperfections matter. For example de-tuning a snake by $\Delta\varphi = \pi - \varphi$ will give an imperfection resonance $= \Delta\varphi/\pi$ a 0.05 Imperfection = 9 degree de-tuning from a single snake. For 0.1 Imperfection this becomes 18 degrees.
- Value of tune
  - This still remains to be studied in detail (I have no good plots) but some preliminary results indicate (unsurprisingly) that tune matters.

Introduction of Imperfections



Ggamma = 191



Ggamma=192

Polarization Lifetime Estimates for FY15 100 GeV pp lattice (no Imperfections)

```
rho[emit_, emit0_] := Exp[-emit/(2*emit0)]/(2*emit0);

NIntegrate[rho[emit, 10/6]*ifun[emit], {emit, 0, 100}]
0.00284495

NIntegrate[rho[emit, 20/6]*ifun[emit], {emit, 0, 100}]
0.00597461

NIntegrate[rho[emit, 25/6]*ifun[emit], {emit, 0, 100}]
0.00748009

NIntegrate[rho[emit, 30/6]*ifun[emit], {emit, 0, 100}]
0.00895748
```

Loss per hour ranges between ~ 0.14 to 0.47 % per hour for 10-30 pi mm-mrad Emittance beam.

Two Hour Loss with Imperfection of 0.05 at 192

```
rho[emit_, emit0_] := Exp[-emit / (2 * emit0)] / (2 * emit0);

NIntegrate[rho[emit, 10 / 6] * ifun[emit], {emit, 0, 100}]
0.0132551

NIntegrate[rho[emit, 20 / 6] * ifun[emit], {emit, 0, 100}]
0.0161864

NIntegrate[rho[emit, 25 / 6] * ifun[emit], {emit, 0, 100}]
0.0174108

NIntegrate[rho[emit, 30 / 6] * ifun[emit], {emit, 0, 100}]
0.0186034
```

Loss per hour ranges between ~ 0.65 to 0.9 % per hour for 10-30 pi mm-mrad Emittance beam.

Two Hour Loss with Imperfection of 0.1 at 192

```
rho[emit_, emit0_] := Exp[-emit / (2 * emit0)] / (2 * emit0);

NIntegrate[rho[emit, 10 / 6] * ifun[emit], {emit, 0, 100}]

0.0303993

NIntegrate[rho[emit, 20 / 6] * ifun[emit], {emit, 0, 100}]

0.0350187

NIntegrate[rho[emit, 25 / 6] * ifun[emit], {emit, 0, 100}]

0.0369987

NIntegrate[rho[emit, 30 / 6] * ifun[emit], {emit, 0, 100}]

0.0388505
```

Loss per hour ranges between ~ 1.5 to 1.9 % per hour for 10-30 pi mm-mrad Emittance beam.

Conjecture of Mechanism for Polarization Lifetime

If we consider only a single spin resonances with synchrotron motion then Eq. 24 becomes,

$$g_0 = \frac{\xi\omega_0\tau_0 q_s}{\eta}. \qquad \xi(s) = a_1 e^{-iK_1\theta - i\phi_1 + i\frac{g_0}{q_s}(\cos(q_s\theta)-1)}, \qquad (22)$$

In the Interaction Fram for Psi we can get:

$$\Delta = K_0 - K_1$$

$$\frac{d^2\Psi_I^{\pm}}{d\theta^2} + (\pm i g_0 \sin(q_s\theta) \mp i\Delta)\frac{d\Psi_I^{\pm}}{d\theta} + \frac{a_1^2}{4}\Psi_I^{\pm} = 0.$$

Using a standard parametric transformation we can obtain:

$$a_1^2/4 + \Delta^2/4 = (nqs)^2/4$$

$$\frac{d^2 q}{d\theta^2} = \Omega^2(\theta) q$$

$$\Omega^{\pm 2} = W_0^2 - \frac{g_0\Delta}{2}\sin q_s\theta + \frac{g_0^2}{4}\sin^2 q_s\theta \mp i\frac{q_s g_o}{2}\cos q_s\theta$$

$$W_0^2 = \frac{a_1^2}{4} + \frac{\Delta^2}{4} \qquad (28)$$

Introduces infinite number of Parametric resonances which Get driven after enough turns.

Problem:

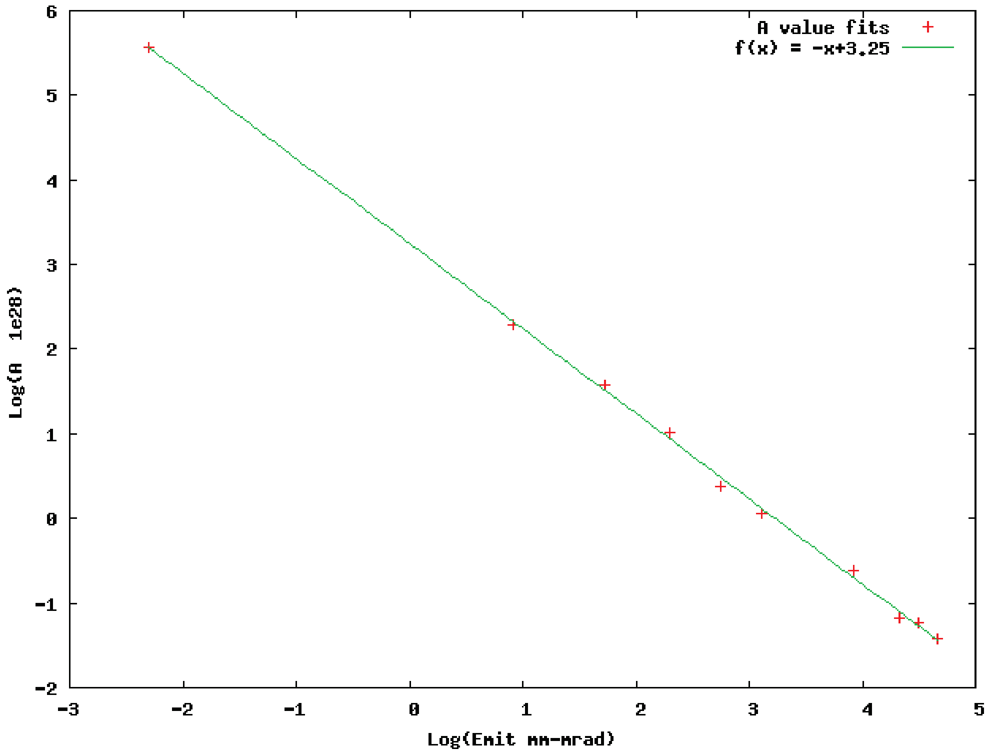
Similar argument for overlapping Resonances but no Pol. Decay? Due to how Snakes act?

Future Plans

- Problem: Beyond 2 hours this model seems to over estimate Polarization lifetime.
  - Could be due couple factors:
    - How we approximate longitudinal effects
    - Beam loss and emittance migration not modeled. We loose the largest amplitude particles first.
- More systematic scans of Tunes
- Include possible detuning effects of rotators and snakes.

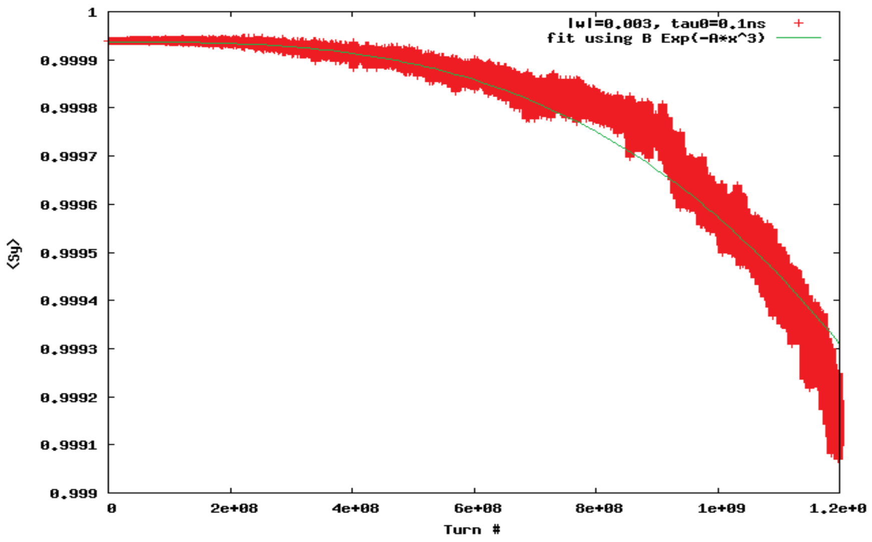

OBEY!
ONWARDS INTO A BRIGHT3R FUTUR3!
HAIL CELESTIA!
mlp.reactor.cc

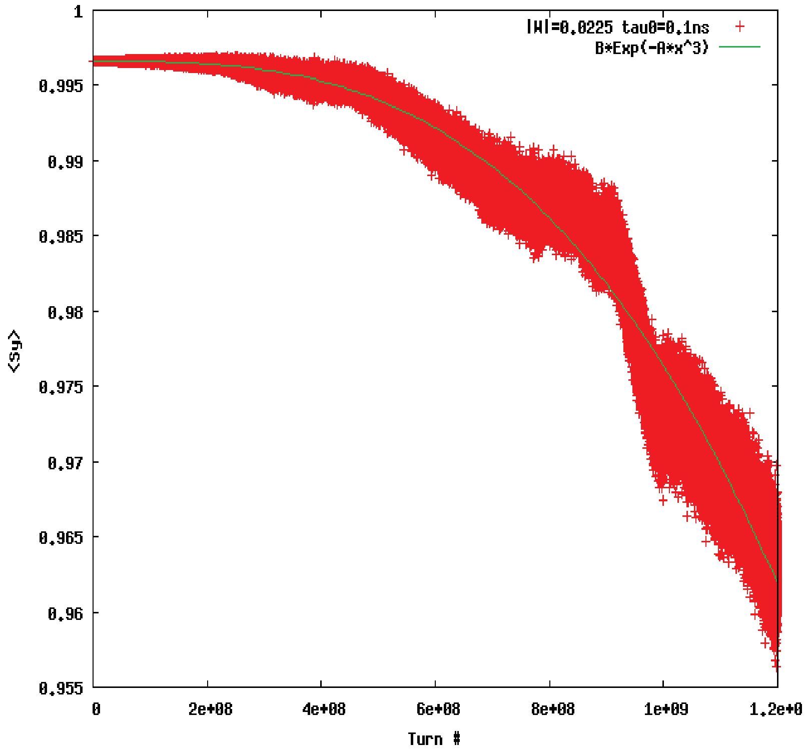
Fit to Beam evolution Fits



Fitting Exp(-Turn^3/(a*1e28))*b fitting the 'a' term find that a(x) = Exp(-x+3.25) were x
Is the emittance

Possible fitting of decay rate.





Simulating 4 hours of Beam time at various Resonance amplitudes decay curve seems To fit a B*Exp(-A*x^3) form

Lifetime Calculations for ATS Lattice at Store Energy Ggam=191.5 including five nearest Intrinsic resonances as calculated via DEPOL.