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Vlasov Analysis of Microbunching Gain for Magnetized Beams

Cheng-Ying Tsai

September 12, 2016

NAPAC'16 at Chicago

WEA2CO04

Outline

- Introduction: micro-bunching instability (MBI)
- Motivation: early design of JLEIC Circulator Cooling Ring
- Theory of MBI for magnetized beams
- Example
- Summary and Possible future work

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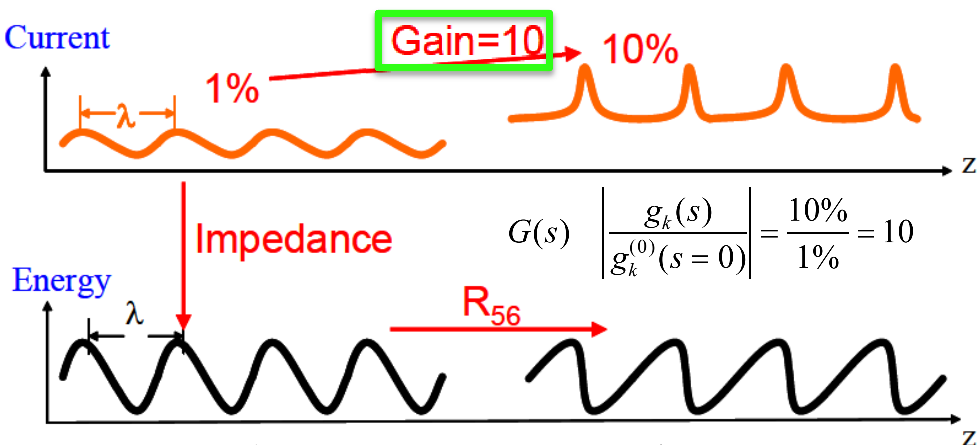
Micro-bunching instability (MBI)

- <u>Physical mechanism</u>:
  - Heifets, Stupakov, and Krinsky, PRST-AB **5**, 064401 (2002)
  - Huang and Kim, PRST-AB **5**, 074401 (2002)

$$G \equiv \left| \frac{b(k_z;s)}{b_0(k_0;0)} \right|$$



Z. Huang and J. Wu, Beam Dynamics Newsletter, No. 38

density modulation
$Z(k)$ R_{56}
energy modulation

- MBI can be an issue for linac-based machines, e.g. FEL light sources.
- MBI can be a more serious issue for **recirculating** machines, because of *many more* dipoles (multi-stage amplification).

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JLEIC Circulator Cooling Ring

- Early design of JLEIC Circulator Cooling Ring (CCR)
  - serious microbunching instability
  - due to coherent synchrotron radiation (CSR) and longitudinal space charge (LSC)

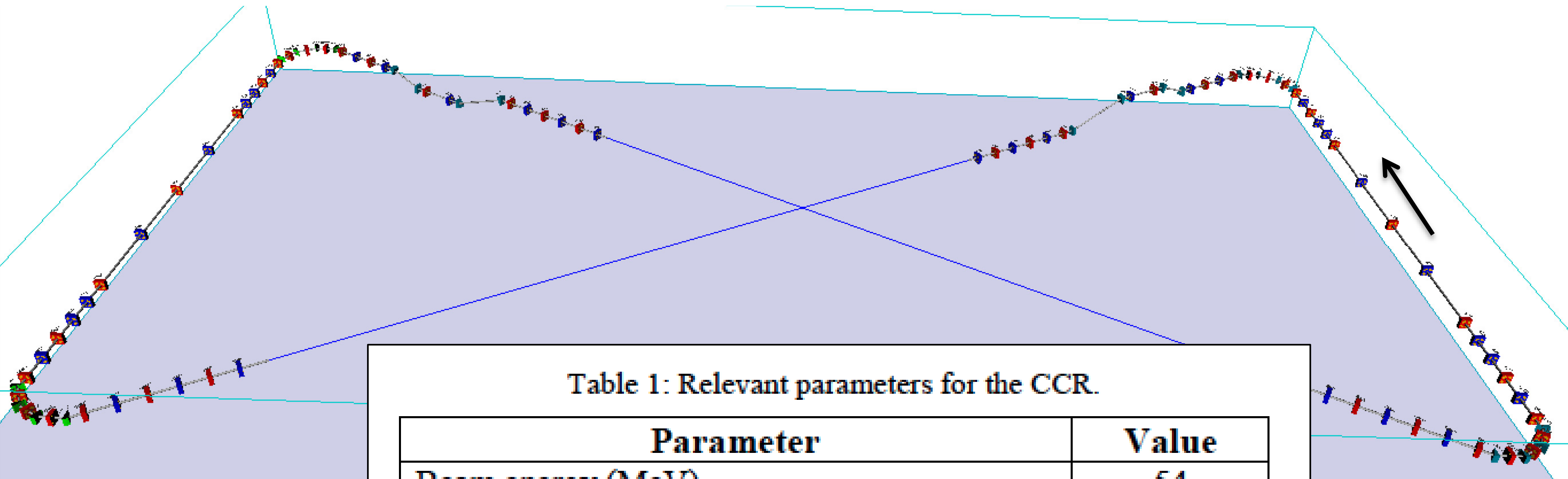


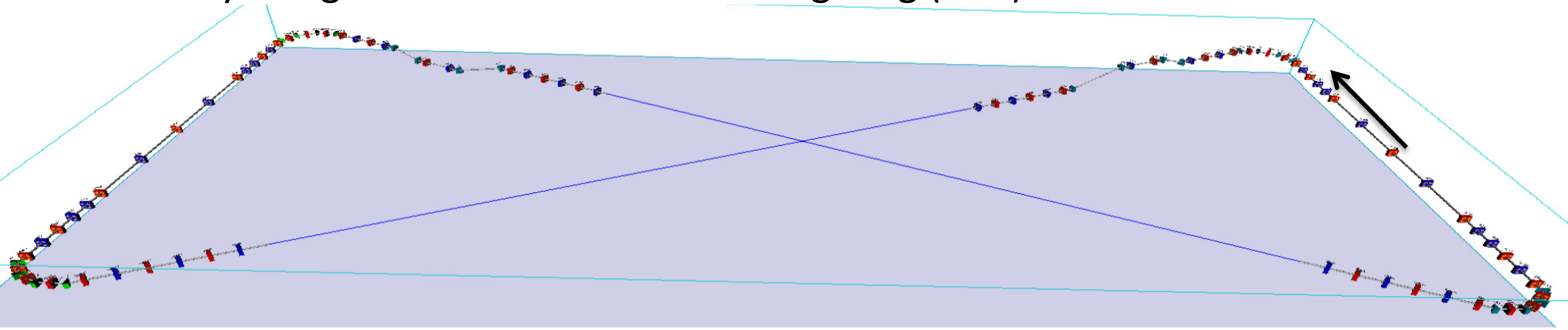
Table 1: Relevant parameters for the CCR.

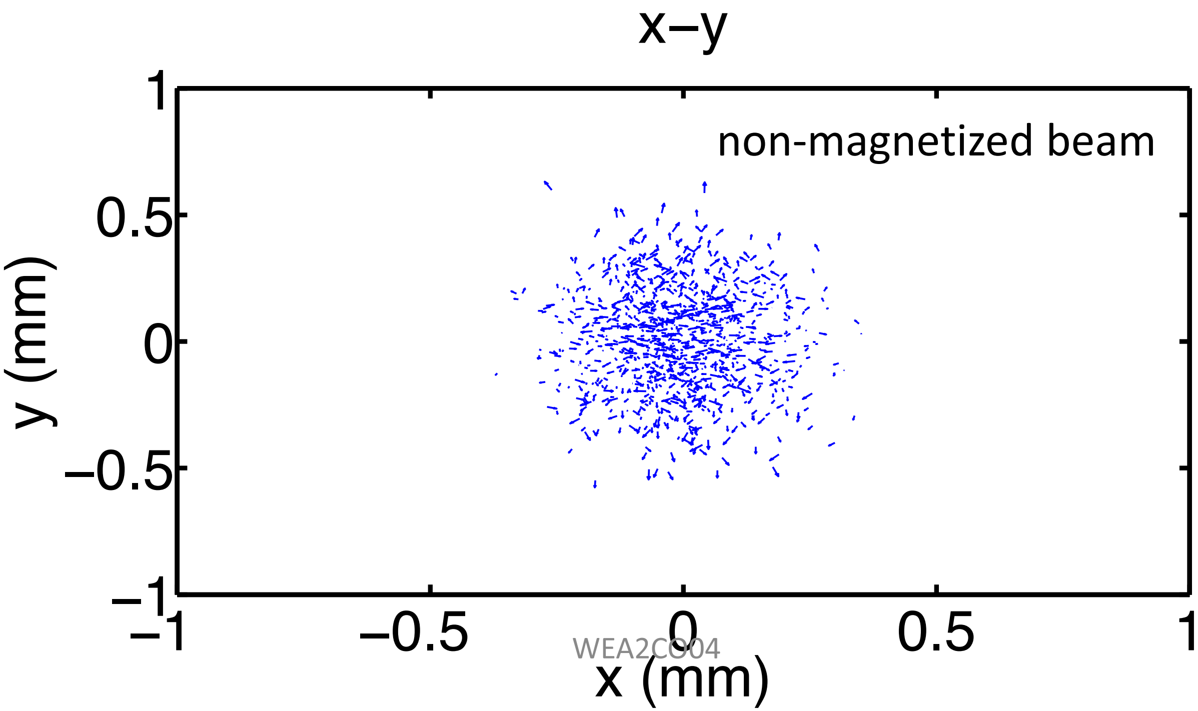
Parameter	Value
Beam energy (MeV)	54
Bunch charge (nC)	2
Repetition rate (MHz)	750
Relative energy spread	10^{-4}
RMS bunch length (ps)	33.33
Longitudinal emittance (keV-psec)	180
Transverse normalized emittance (mm-mrad)	3
Cooling solenoid field (kg)	20

MEIC Design Report (2012)

JLEIC Circulator Cooling Ring

- Early design of JLEIC Circulator Cooling Ring (CCR)





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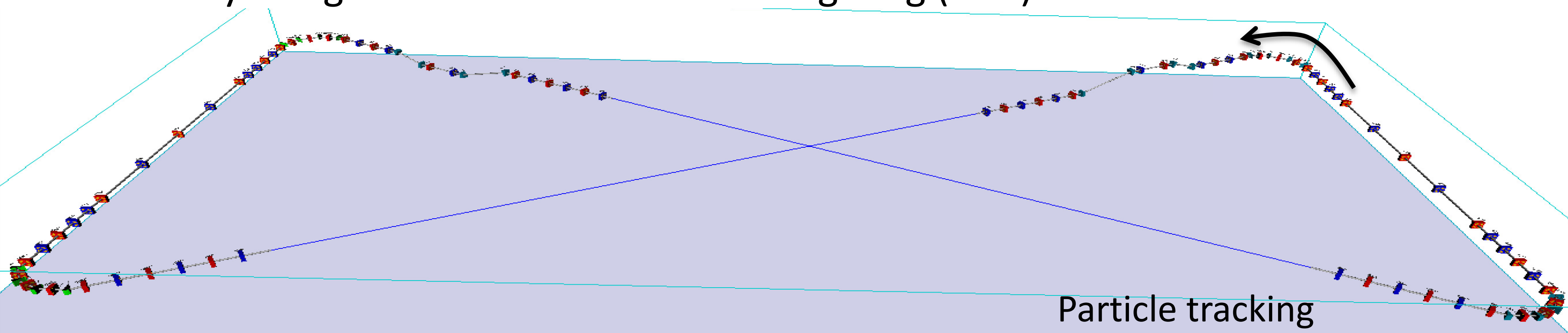
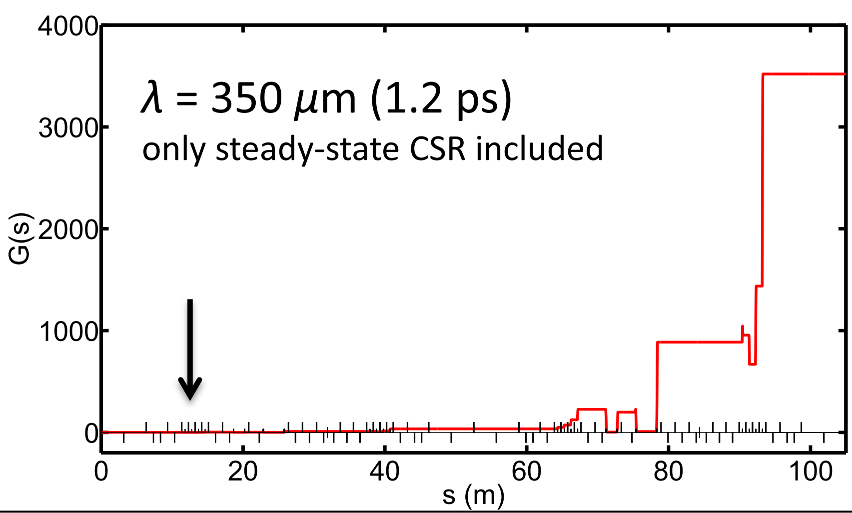
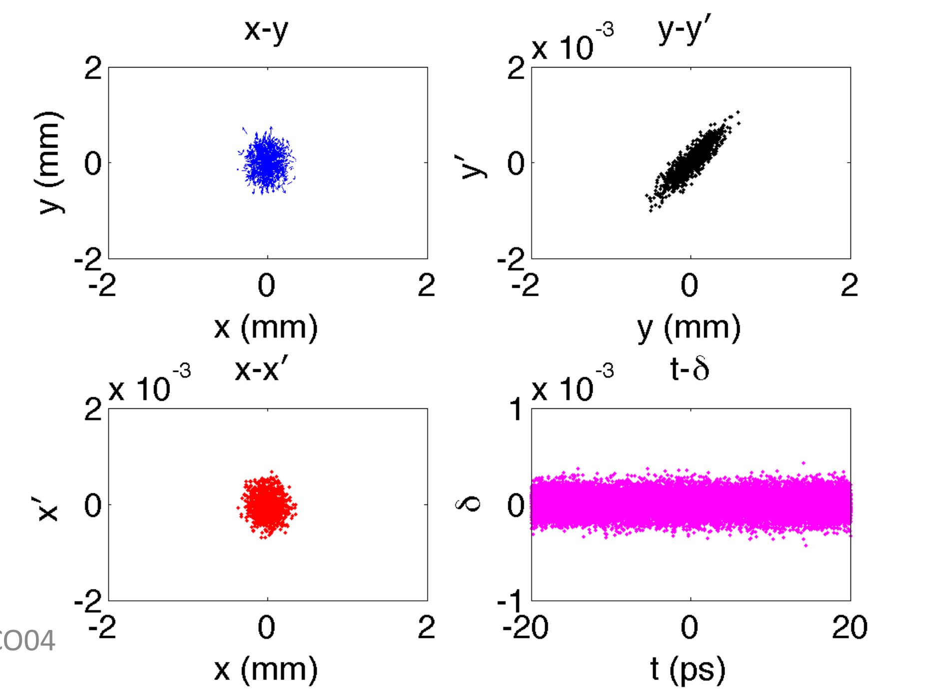


Particle tracking

semi-analytical Vlasov simulation





C. -Y. Tsai et al., ERL2015 (TUICLH2034)

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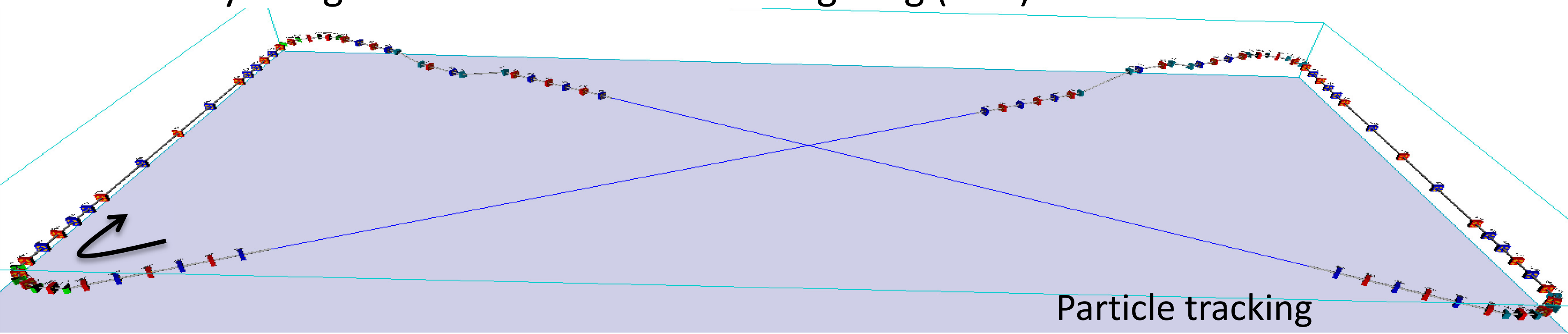

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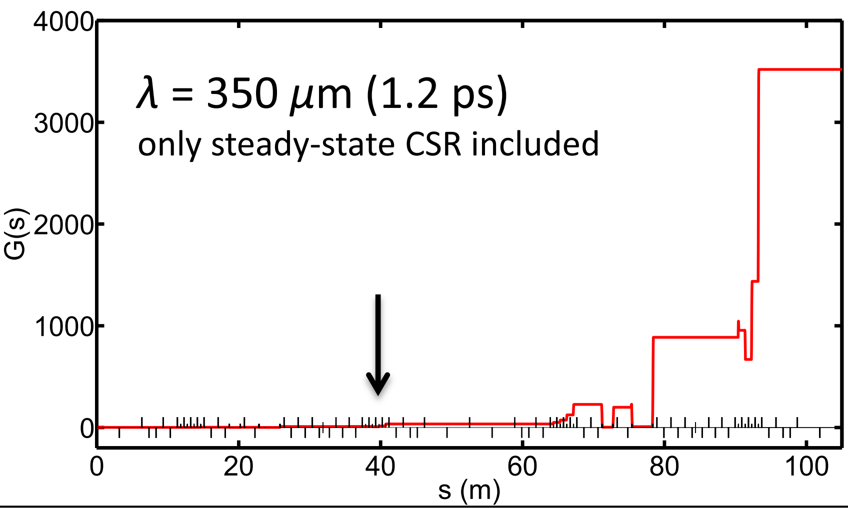

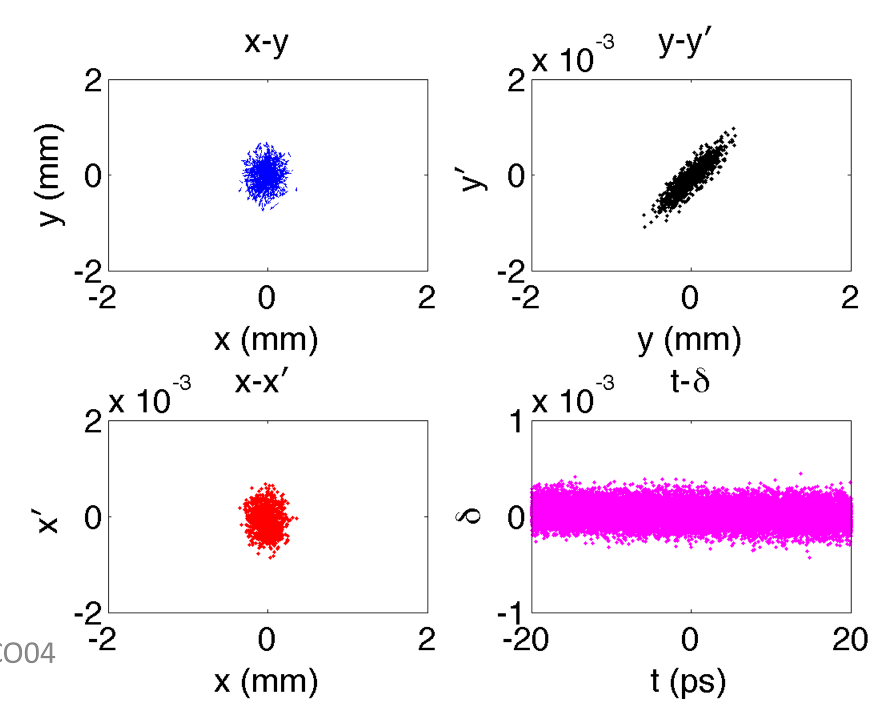

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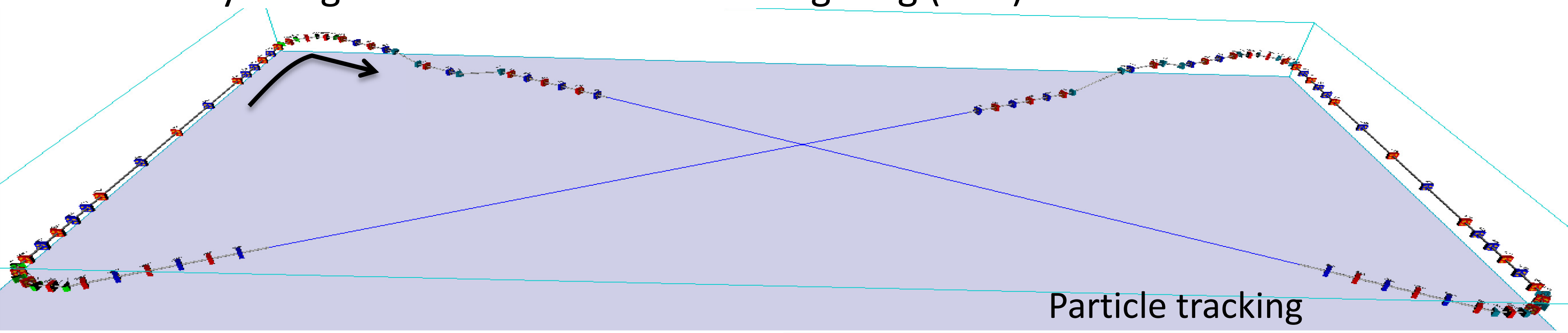

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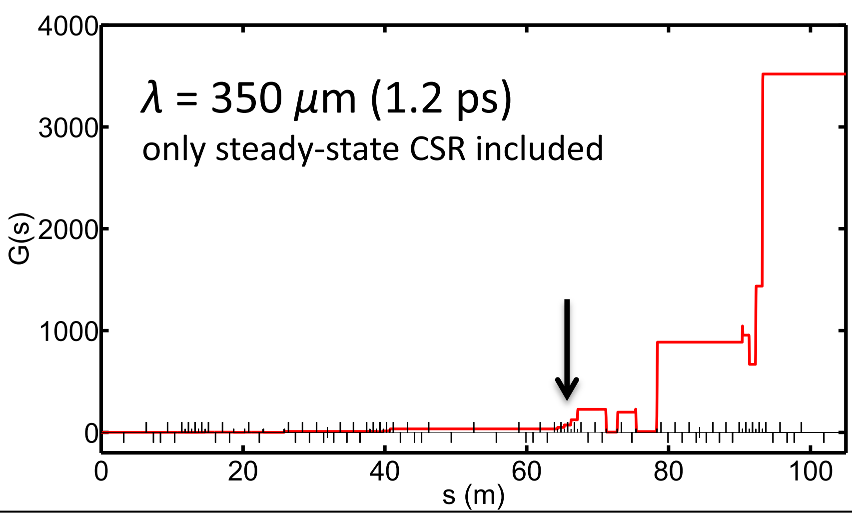

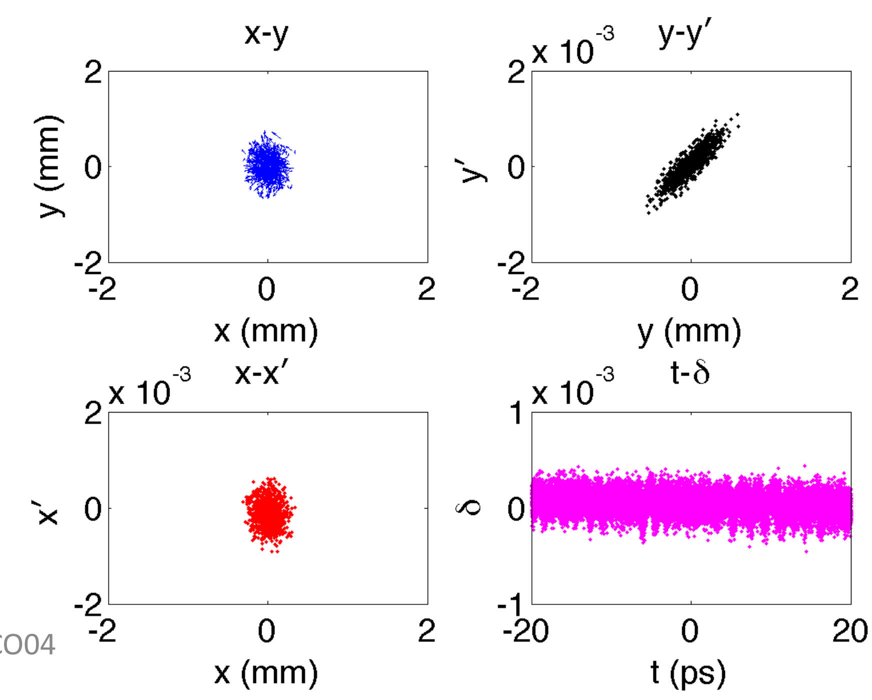

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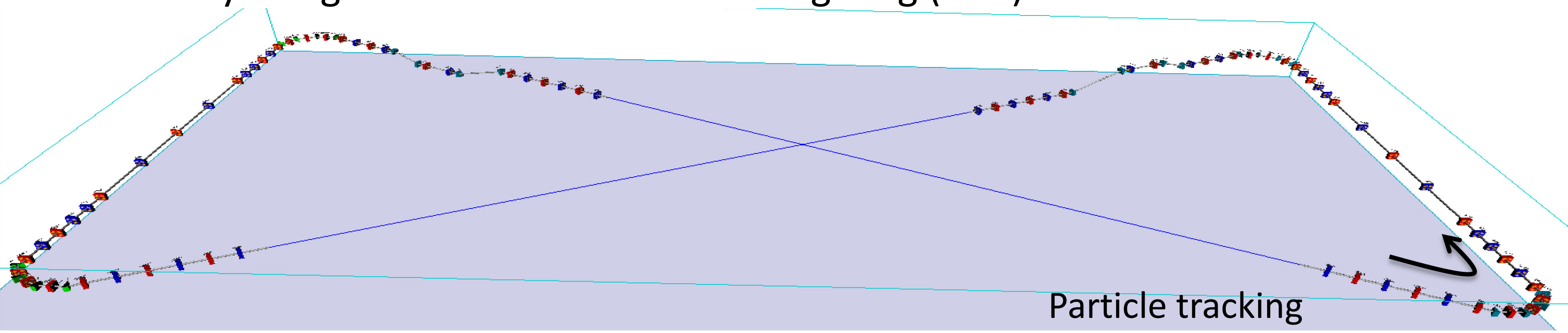

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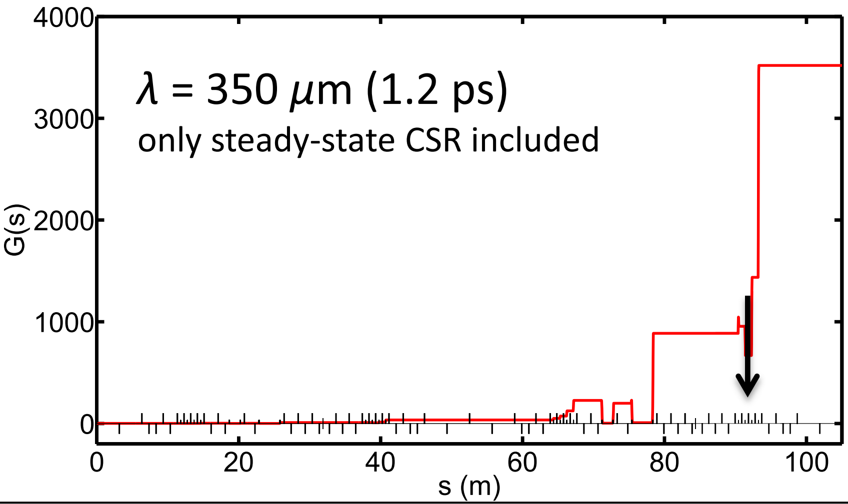

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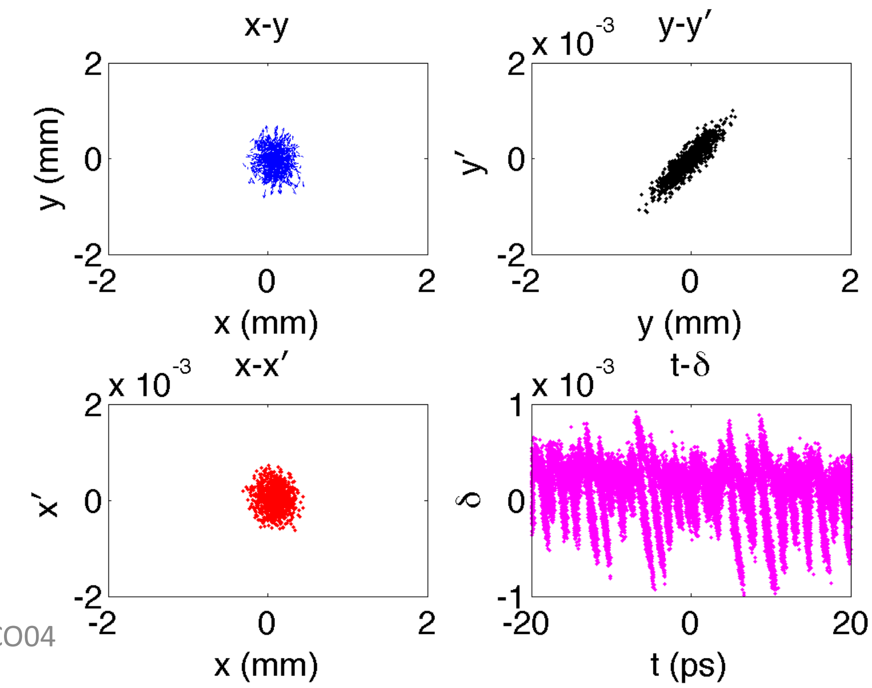

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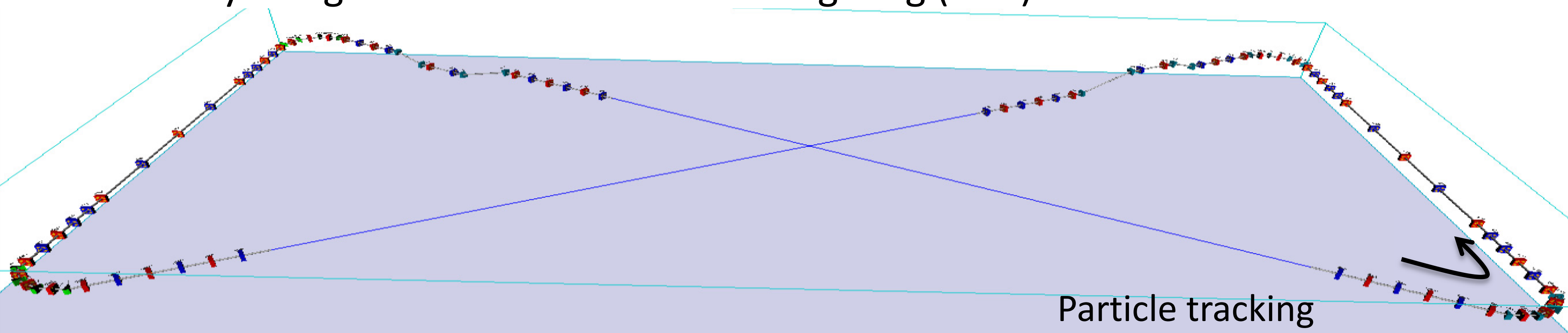

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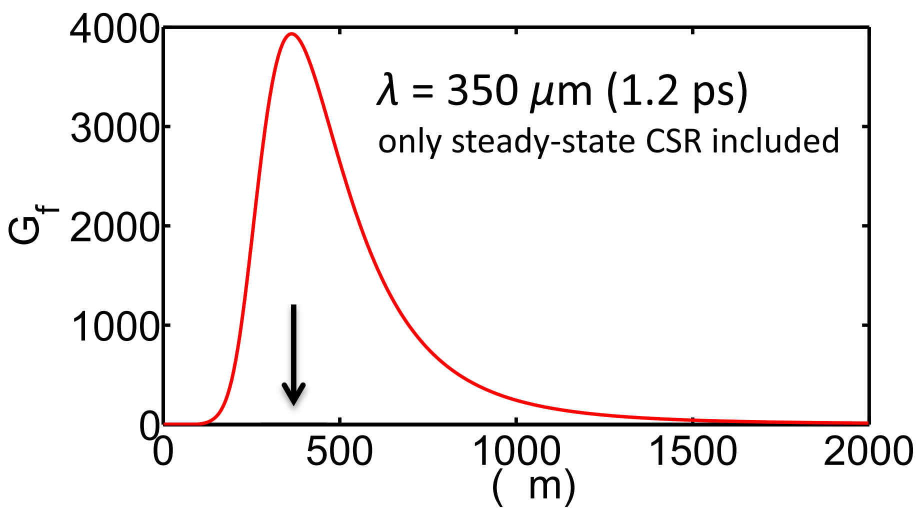

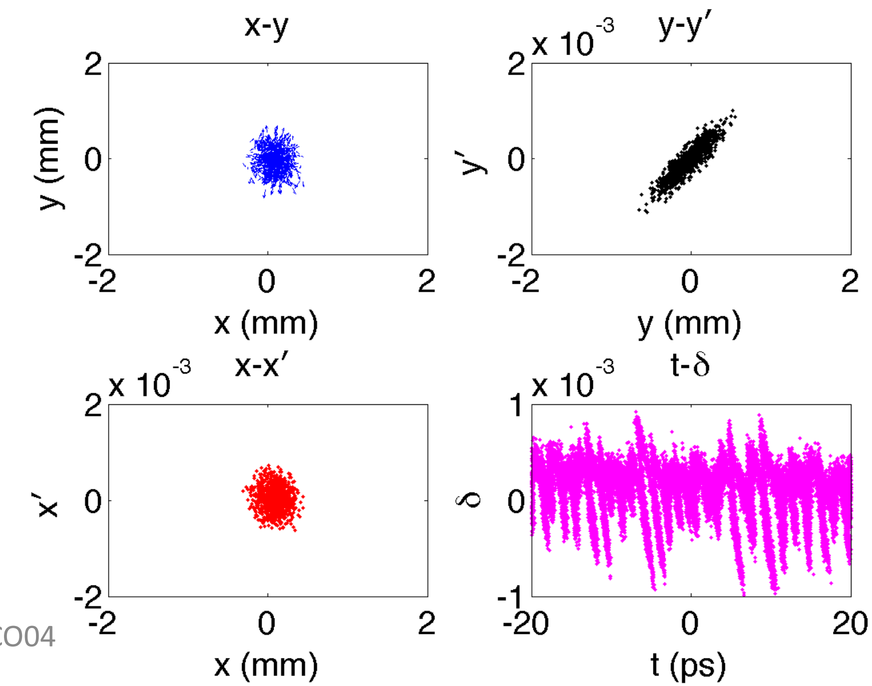

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JLEIC Circulator Cooling Ring

- **Mitigation** of micro-bunching instability (MBI)
  - Our main interest of this talk
  - Larger beam size should help suppress MBI
- **Improvement** of cooling performance
  - Cooling beam: low temperature (emittance), beam size comparable to ion beam
  - For non-magnetized beam, difficult to meet the requirements simultaneously for small emittance (or low temperature)
  - For magnetized beam, easier to satisfy the above requirements while maintaining small Larmor emittance
- Derbenev's Idea: use **magnetized** beam
- Cathode immersed in solenoid field. Beam has non-zero angular momentum.

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Semi-analytical Vlasov formalism

- Theory is not new
  - Heifets, Stupakov, and Krinsky, PRST-AB **5**, 064401 (2002)
  - Huang and Kim, PRST-AB **5**, 074401 (2002)
  - for non-magnetized (transverse **un-coupled**) beam
- We extend to more general case of transverse **coupled** beams

Semi-analytical Vlasov formalism

- Model assumptions
- <u>Beam dynamics:</u>
  - **coasting** beam approximation
  - **linear** beam transport $\mathbf{X}(s) = \mathbf{R}(\mathbf{s})\mathbf{X}(\mathbf{0})$ where $\mathbf{X} = (x, x', y, y', z, \delta)$

Semi-analytical Vlasov formalism

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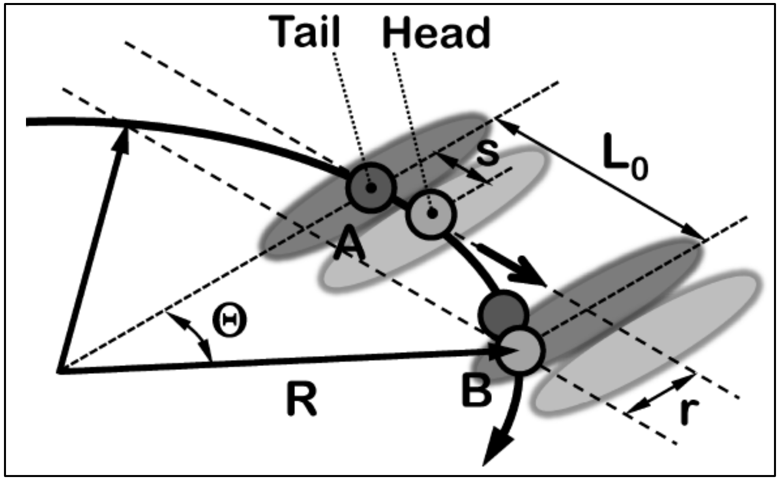
- <u>Beam dynamics:</u>
  - **coasting** beam approximation
  - **linear** beam transport $\mathbf{X}(s) = \mathbf{R}(\mathbf{s})\mathbf{X}(\mathbf{0})$ where $\mathbf{X} = (x, x', y, y', z, \delta)$
- <u>Electrodynamics:</u>
  - **1-D CSR**: assume Derbenev ratio is small
  - Note: this assumption can be violated for typical magnetized beam



$$\kappa \equiv \frac{\sigma_x(s)}{[\lambda^2(s)\rho(s)]^{1/3}}$$

Semi-analytical Vlasov formalism

- Define $\mathbf{X}_{4D} = (x, x', y, y')$
- Phase space distribution function satisfies **Vlasov** equation $\frac{d}{ds} f\left(\mathbf{X}; s\right) = 0$

Semi-analytical Vlasov formalism

- Define $\mathbf{X}_{4D} = (x, x', y, y')$
- Phase space distribution function satisfies **Vlasov** equation $\frac{d}{ds} f\left(\mathbf{X}; s\right) = 0$
- Formulate **perturbed** phase space distribution (to first order)

$$\boxed{f(\mathbf{X}; s) = f_0(\mathbf{X}_0) - \int_0^s d\tau \frac{\partial f_0(\mathbf{X}_\tau)}{\partial \delta_\tau} \frac{d\delta}{d\tau}}$$

Semi-analytical Vlasov formalism

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- Micro-bunching can be quantified by

$$b\left(k_z(s); s\right) \equiv \frac{1}{N} \int d\mathbf{X} f\left(\mathbf{X}; s\right) e^{-ik_z(s) z_s}$$

- Goal: obtain governing equation for $b(k_z; s)$

Semi-analytical Vlasov formalism

- To be more specific, given an initial beam phase space distribution,

$$\bar{f}_0\left(\mathbf{X}_0\right)=\frac{n_0}{(2\pi)^{5/2}\varepsilon_{4D}^2\sigma_\delta}\exp\left\{-\frac{1}{2}\mathbf{X}_{4D,0}^T\mathbf{\Sigma}^{-1}\mathbf{X}_{4D,0}-\frac{(\delta_0-hz_0)^2}{2\sigma_\delta^2}\right\} \quad \varepsilon_{4D}=\sqrt[4]{\det\left(\Sigma\right)}$$

with density modulation on top of *z*,

$$f_0^{(z)}(\mathbf{X}_0)=\left(1+\frac{\Delta n_0(z_0)}{n_0}\right)\bar{f}_0(\mathbf{X}_0)$$

where the beam sigma matrix

$$\mathbf{\Sigma}=\begin{pmatrix}\langle xx\rangle & \langle xx'\rangle & \langle xy\rangle & \langle xy'\rangle\\ \langle x'x\rangle & \langle x'x'\rangle & \langle x'y\rangle & \langle x'y'\rangle\\ \langle yx\rangle & \langle yx'\rangle & \langle yy\rangle & \langle yy'\rangle\\ \langle y'x\rangle & \langle y'x'\rangle & \langle y'y\rangle & \langle y'y'\rangle\end{pmatrix}$$

Semi-analytical Vlasov formalism

- Remember we want to calculate $b\left(k_z(s);s\right) \equiv \frac{1}{N}\int d\mathbf{X} f\left(\mathbf{X};s\right) e^{-ik_z(s)z_s}$
- When we proceed, we will encounter the integration

$$\int d\mathbf{X}_{4D,0}dz_0 d\delta_0 e^{-\frac{1}{2}\mathbf{X}_{4D,0}^T\Sigma^{-1}\mathbf{X}_{4D,0}-\frac{(\delta_0-hz_0)^2}{2\sigma_\delta^2}-ik_z(s)z_s}$$

$$\exp\left\{-\frac{1}{2}\mathbf{X}_{4D,0}^T\Sigma^{-1}\mathbf{X}_{4D,0}\right\}$$

Semi-analytical Vlasov formalism

- Remember we want to calculate $b\left(k_z(s);s\right) \equiv \frac{1}{N}\int d\mathbf{X} f\left(\mathbf{X};s\right)e^{-ik_z(s)z_s}$
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- Assume there exists diagonal matrix such that

$$\boxed{\mathbf{X}_{4D,0}^T\Sigma^{-1}\mathbf{X}_{4D,0} = \mathbf{U}_0^T D^{-1}\mathbf{U}_0}$$

Semi-analytical Vlasov formalism

- Remember we want to calculate $b\left(k_z(s);s\right) \equiv \frac{1}{N}\int d\mathbf{X} f\left(\mathbf{X};s\right) e^{-ik_z(s)z_s}$
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$$\int d\mathbf{X}_{4D,0}\,dz_0\,d\delta_0\,e^{-\frac{1}{2}\mathbf{X}^T_{4D,0}\Sigma^{-1}\mathbf{X}_{4D,0}-\frac{(\delta_0-hz_0)^2}{2\sigma_\delta^2}-ik_z(s)z_s}$$

- Assume there exists diagonal matrix such that

$$\boxed{\mathbf{X}^T_{4D,0}\Sigma^{-1}\mathbf{X}_{4D,0} = \mathbf{U}_0^T D^{-1}\mathbf{U}_0}$$

All dynamics information is contained in *D*.

where $\mathbf{U}_0 = (u_1, u_2, u_3, u_4)$

$$\mathbf{U}_0 = \mathbf{V}\mathbf{X}_{4D,0} \qquad \det\left(\mathbf{V}\right) = 1, \mathbf{V}^T = \mathbf{V}^{-1}$$

- Then

$$z_s = \sum_{j=1}^{4} R_{5j}(s) X^j_{4D,0} + R_{55}(s) z_0 + R_{56}(s)\delta_0$$
$$= \sum_{i=1}^{4}\sum_{j=1}^{4} R_{5j}(s) V^{-1}_{ji} u_i + R_{55}(s) z_0 + R_{56}(s)\delta_0$$

Courant-Snyder parameters for the (coupled) system are contained in **V**.

Semi-analytical Vlasov formalism

- Putting all together,

$$\boxed{b(k_z;s) = b_0(k_z;s) + \int_0^s d\tau K^{MAG}(\tau,s) b(k_z;\tau)} \qquad G \equiv \left| \frac{b(k_z;s)}{b_0(k_0;0)} \right|$$

where $b_0(k_z;s) = b_0(k_0;0)\left\{L.D.;s,0\right\}$

$$K^{MAG}(\tau,s) = \frac{ik_z(s)I(\tau)}{\gamma I_A} R_{56}(\tau \to s) Z(k_z;\tau)\left\{L.D.;s,\tau\right\}$$

$$\left\{L.D.;s,\tau\right\} = \exp\left\{-\frac{k_0^2}{2}\sum_{i=1}^{4}\frac{1}{D_{ii}^{-1}}\left(\sum_{j=1}^{4}\Re_{5j}(s,\tau)V_{ji}^{-1}\right)^2 - \frac{k_0^2}{2}\Re_{56}^2(s,\tau)\sigma_\delta^2\right\}$$

$$\Re_{5j}(s,\tau) = C(s)R_{5j}(s) - C(\tau)R_{5j}(\tau), j = 1,2,3,4,6$$

$$C(s) = \left[R_{55}(s) + hR_{56}(s)\right]^{-1}$$

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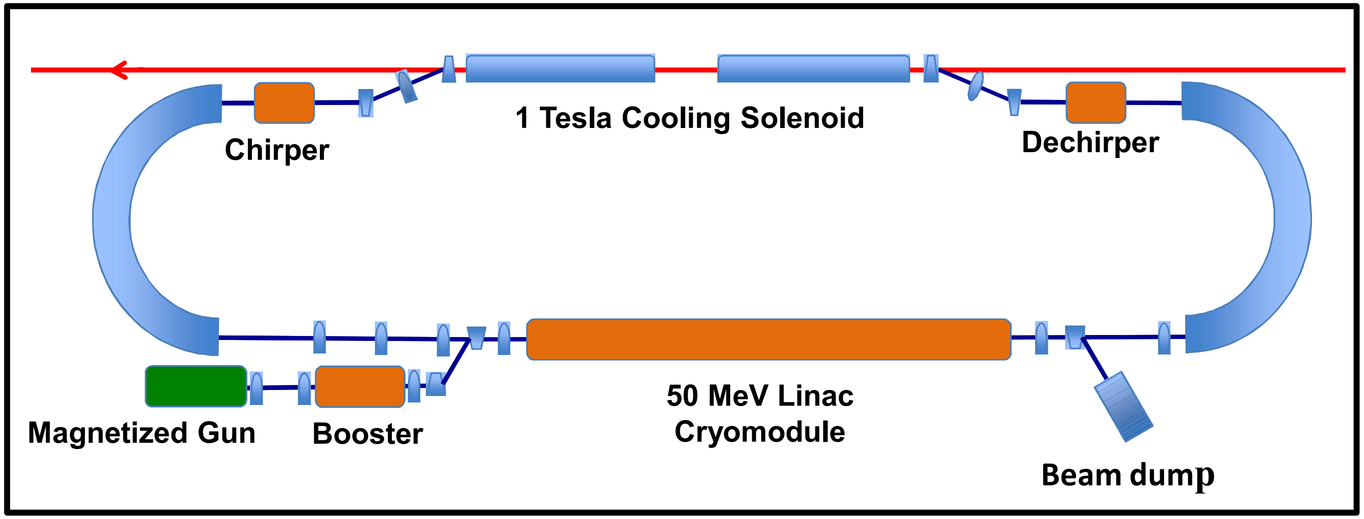
- ❑ **Example**
- ❑ Summary and Possible future work

ERL cooler design [S. Benson, JLEIC Collaboration Meeting, Spring 2016]



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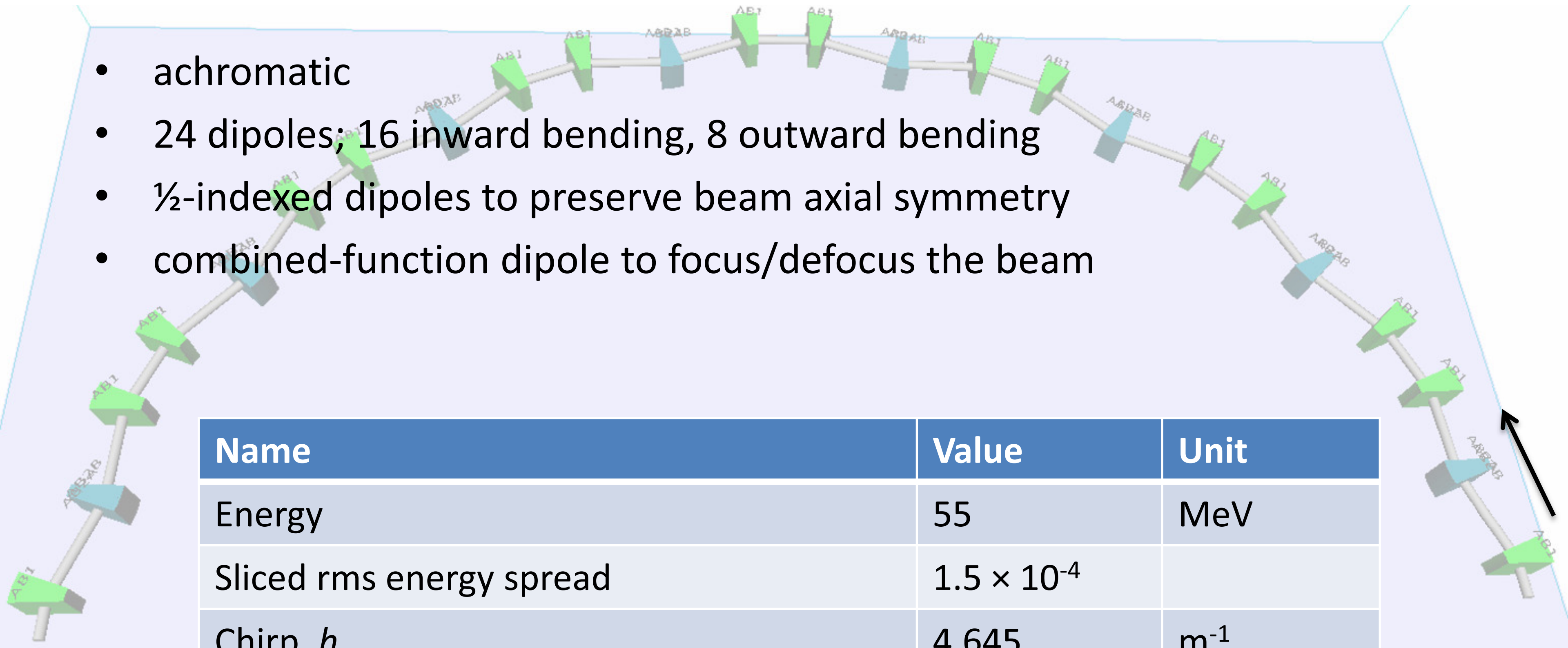

- achromatic
- 24 dipoles; 16 inward bending, 8 outward bending
- ½-indexed dipoles to preserve beam axial symmetry
- combined-function dipole to focus/defocus the beam

Name	Value	Unit
Energy	55	MeV
Sliced rms energy spread	1.5×10^{-4}	
Chirp, *h*	4.645	m^{-1}
Bunch charge	420	pC
Initial/final bunch (peak) current	22.5/6.3	A
Compression/Decompression factor	0.28/3.57	
4-D geometric emittance ε_{4D}	1.11×10^{-5}	cm

Particle tracking

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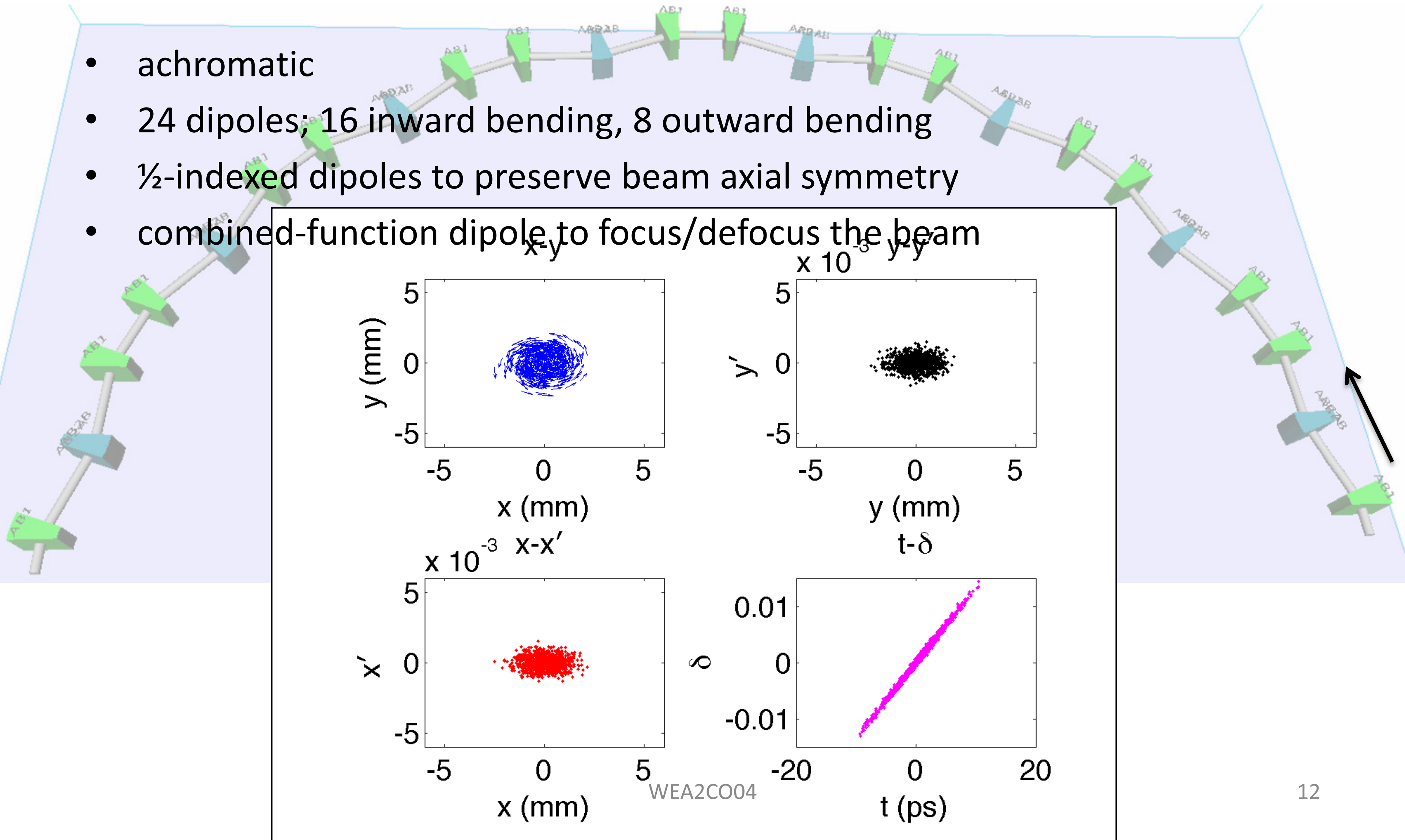

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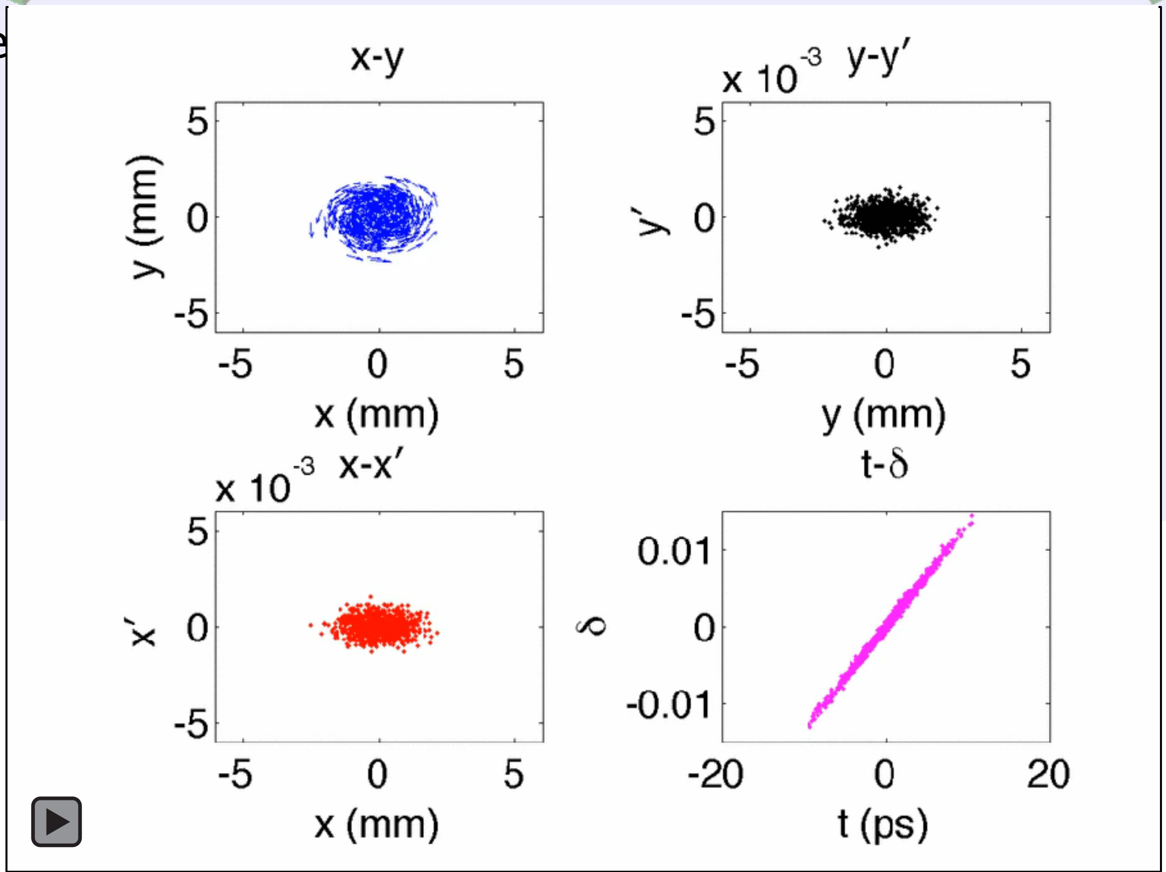

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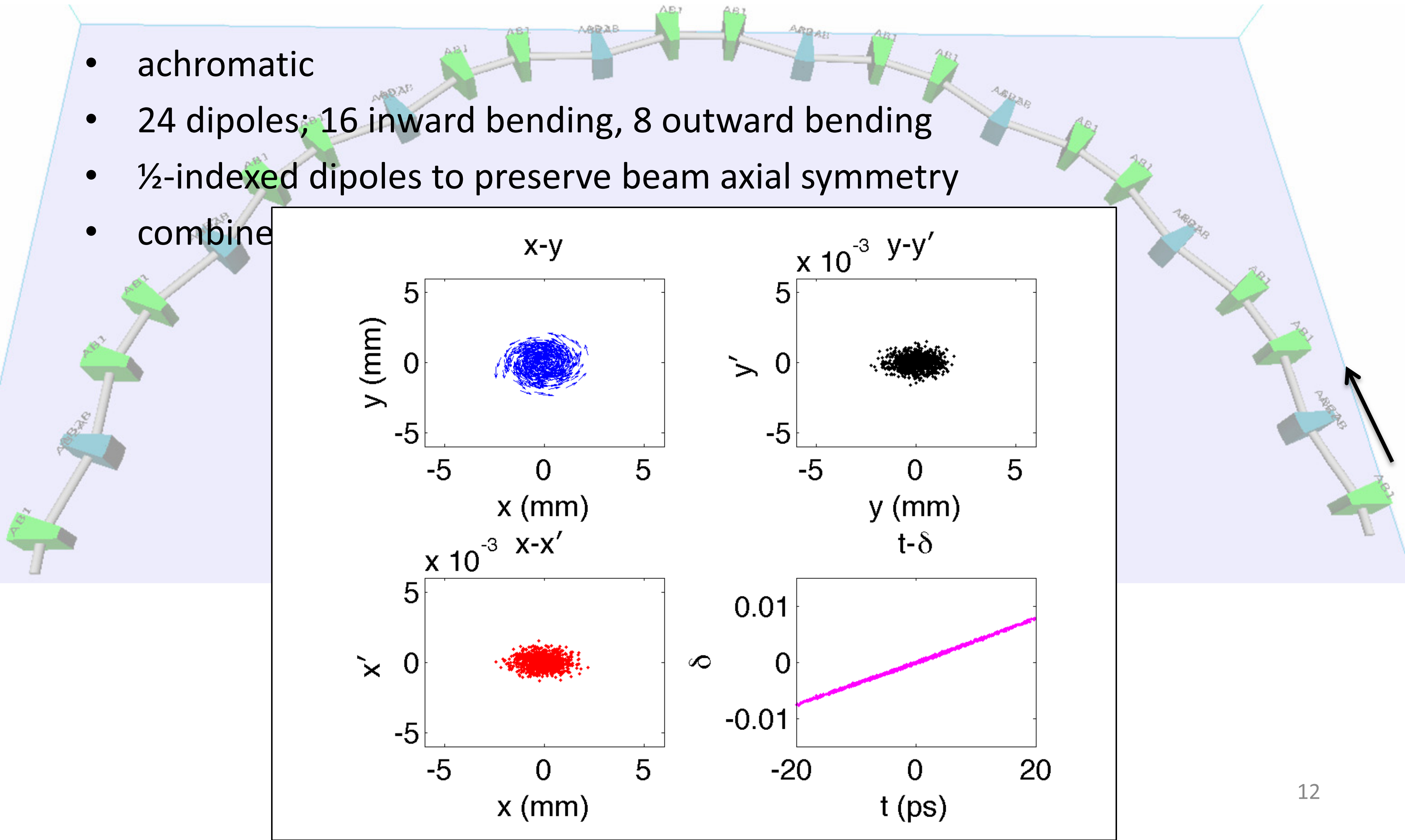

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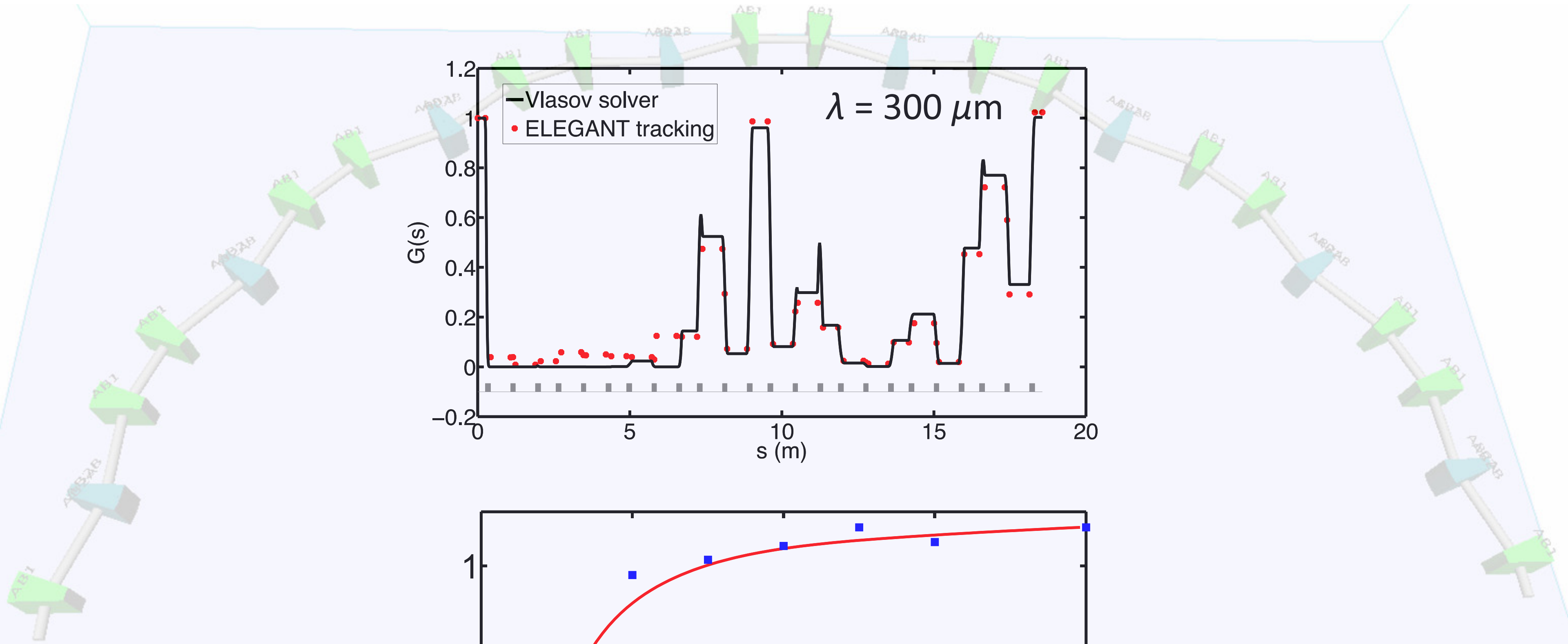
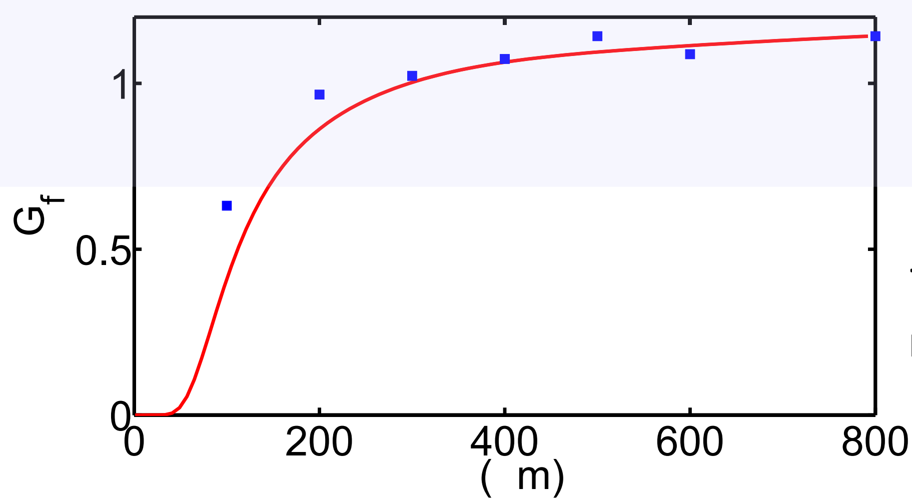


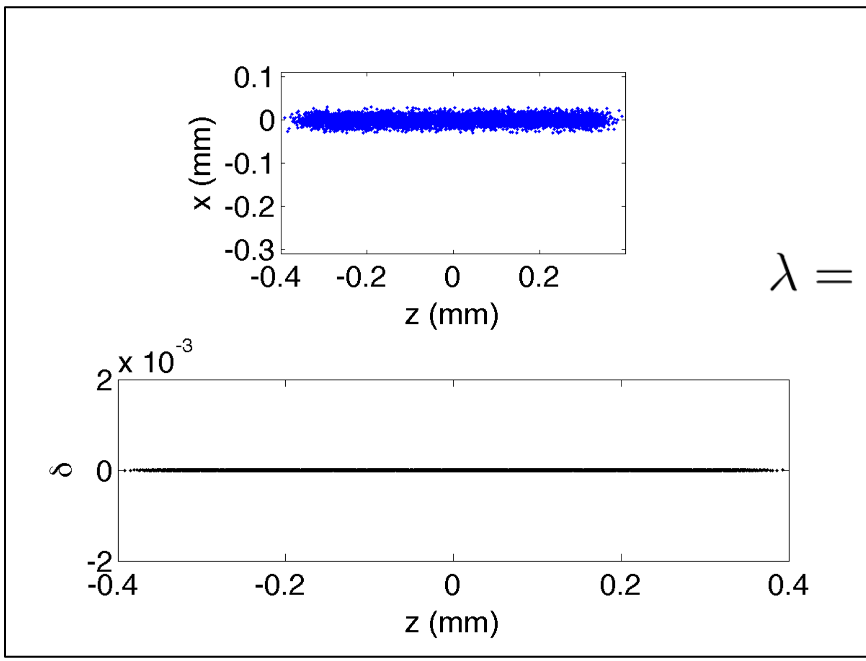


16M macroparticles
700 bins set to minimum resolution 50 μm

Discussion

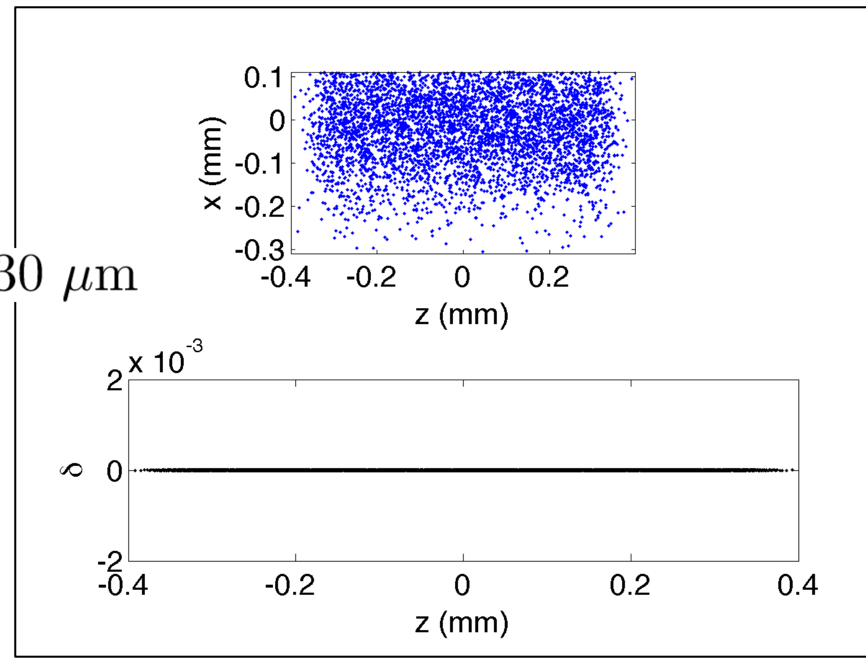
- Smearing effect due to R_{51}
- Effective phase mixing when $R_{51}\sigma_x > \lambda$

$R_{51}\sigma_x \approx 10\ \mu\text{m}$



$\lambda = 30\ \mu\text{m}$

$R_{51}\sigma_x \approx 100\ \mu\text{m}$



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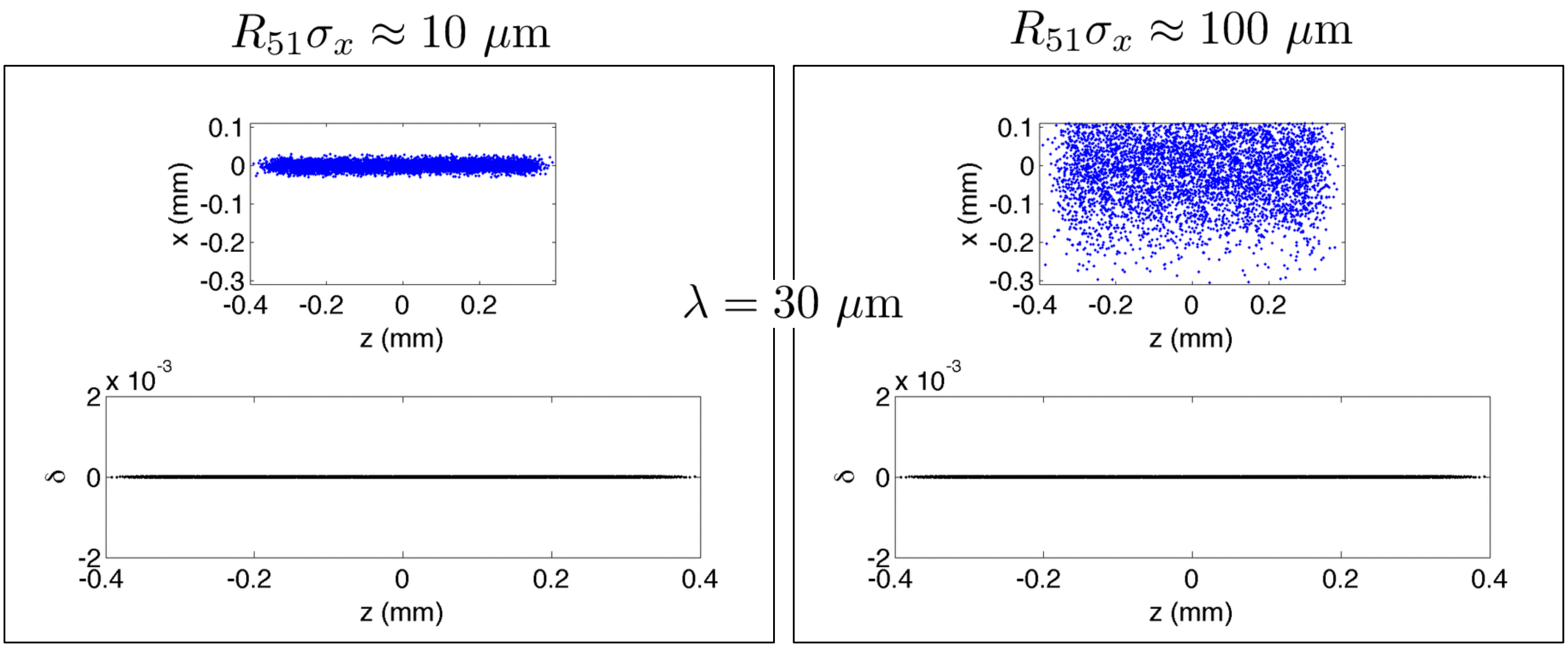

- Our Example: $R_{51}\sigma_x \approx 2\ \mathrm{mm} > 0.8\ \mathrm{mm}$
- Early design of CCR: $R_{51}\sigma_x \approx 7\ \mu\mathrm{m} \ll 350\ \mu\mathrm{m}$

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Summary and Possible future work

- ✓ **Extend** Vlasov approach to micro-bunching for magnetized beams
- ✓ **Demonstrate** MBI gains for a magnetized-beam transport arc
- ✓ Results **benchmarked** against particle tracking simulation
- ✓ The arc design does **not** show MBI gain growth (gain ≈ 1)
- ✓ Larger beam size provides effective suppression of MBI

- ➢ Parametric dependencies of MBI for magnetized beams, e.g. angular momentum
- ➢ More aspects of MBI under study (see also THPOA35)
- ➢ More complete analysis can be done when full-ring lattice is available
- ➢ Extend the formalism to nonlinear beam transport

Thank you for your attention

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