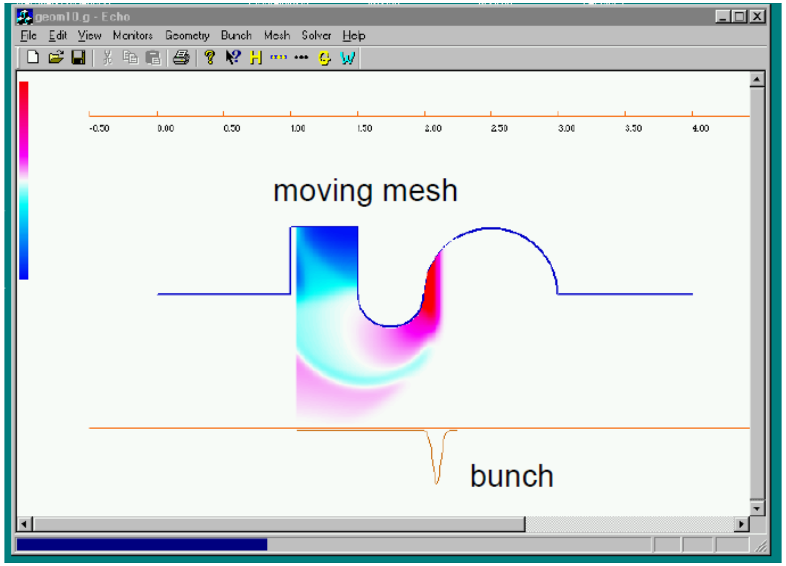
Computation of Electromagnetic Fields Generated by Relativistic Beams in Complicated Structures

Algorithms and Applications



Igor Zagorodnov

2016 North American Particle Accelerator Conference

Chicago, USA
12. October 2016

HELMHOLTZ GEMEINSCHAFT

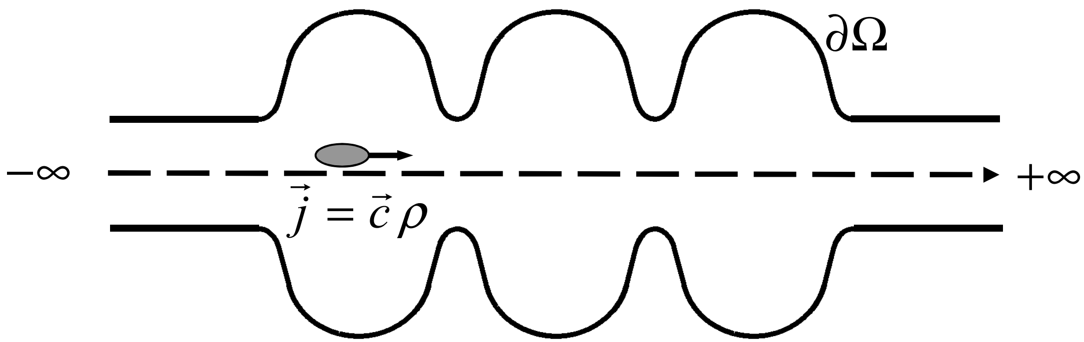
DESY

Overview

- Motivation
- Numerical Methods
  - low-dispersive schemes
  - boundary approximation
  - indirect integration algorithm
  - modelling of conductive walls
- Code Status and Applications
  - geometries of revolution (ECHOz1, ECHOz2)
  - rectangular structures (ECHO2D)
  - fully 3D geometries (ECHO3D)
  - Particle-In-Cell code

Motivation

Wake field calculation – estimation of the effect of the geometry variations on the bunch



Wake potential

$$W_{\parallel}(\mathbf{r}_0, \mathbf{r}, s) = \frac{1}{Q}\int_{-\infty}^{\infty} E_z\left(\mathbf{r}_0, \mathbf{r}, z, \frac{z-s}{c}\right) dz$$

$$\frac{\partial}{\partial s}\mathbf{W}_{\perp}(\mathbf{r}_0, \mathbf{r}, s) = \nabla_{\perp} W_{\parallel}(\mathbf{r}_0, \mathbf{r}, s)$$

First codes in **time domain** ~ 1980

A. Novokhatski (BINP),

T. Weiland (CERN)

Motivation

MAFIA* (the Year 2000)

- rotationally symmetric and 3D
- longitudinal and transverse wakes
- triangular geometry approximation;
- arbitrary materials
- moving window
- dispersion error

NOVO (the Year 2000)**

- rotationally symmetric
- only longitudinal wake (scalar wave equation)
- "staircase" geometry approximation;
- only PEC
- moving mesh
- low dispersion error

* MAFIA Collaboration, ***MAFIA Manual***, CST GmbH, Darmstadt, 1997.

** A. Novokhatski, M. Timm, T.Weiland, ***Transition Dynamics of the Wake Fields of Ultra Short Bunches***, Proc.ICAP1998, Monterey, USA.

Motivation

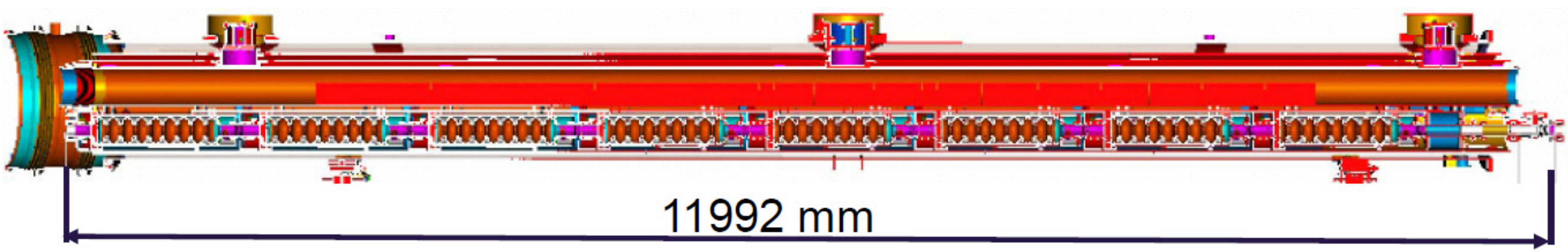
New projects with

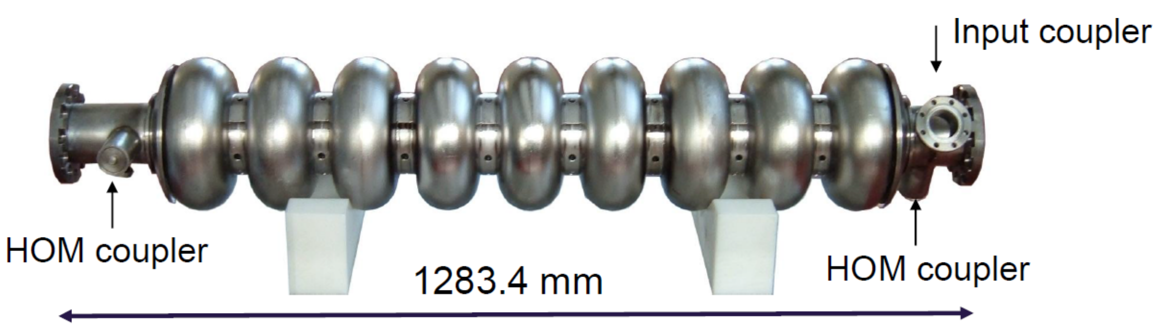
- short bunches;
- long structures;
- tapered collimators

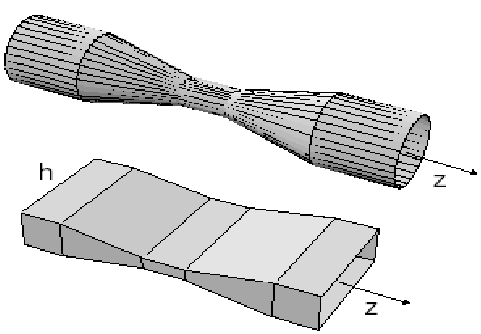
Solutions

- zero dispersion in longitudinal direction;
- “conformal” meshing;
- moving mesh and “explicit” or “split” methods

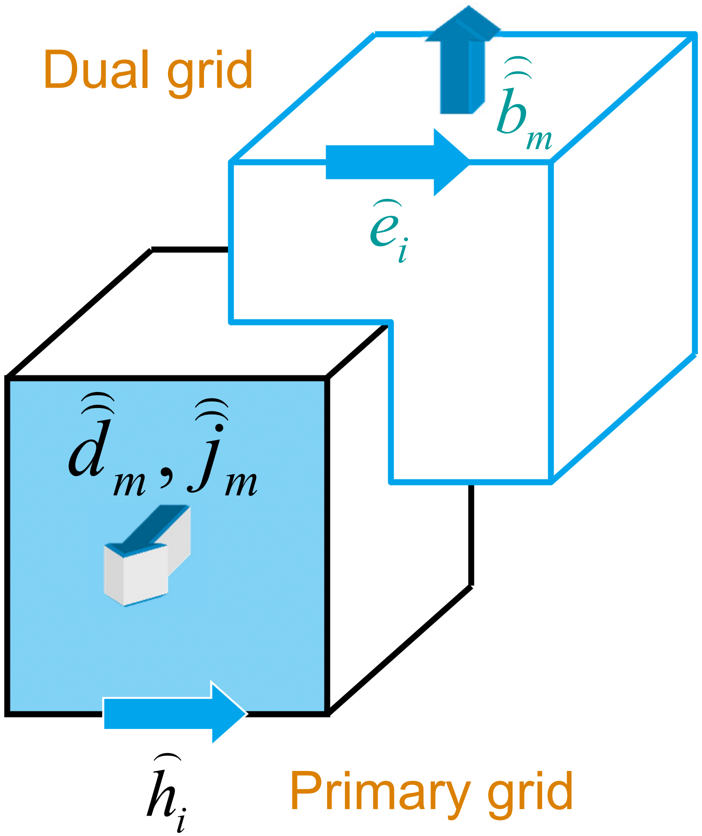
Cryomodule housing: 8 cavities, quadrupole and BPM







Low-dispersive schemes



Maxwell Grid Equations*

$$\mathbf{C}\widehat{\mathbf{e}} = -\frac{d}{dt}\widehat{\widehat{\mathbf{b}}}\,, \qquad \tilde{\mathbf{C}}\widehat{\mathbf{h}} = \frac{d}{dt}\widehat{\widehat{\mathbf{d}}} + \widehat{\widehat{\mathbf{j}}},$$

$$\mathbf{S}\widehat{\widehat{\mathbf{b}}} = \mathbf{0}, \qquad \tilde{\mathbf{S}}\widehat{\widehat{\mathbf{d}}} = \mathbf{q},$$

$$\widehat{\mathbf{e}} = \mathbf{M}_{\varepsilon^{-1}}\widehat{\widehat{\mathbf{d}}}\,, \qquad \widehat{\mathbf{h}} = \mathbf{M}_{\mu^{-1}}\widehat{\widehat{\mathbf{b}}}$$

$$curl \;\Box\; \mathbf{C} = \begin{pmatrix} \mathbf{0} & -\mathbf{P}_z & \mathbf{P}_y \\ \mathbf{P}_z & \mathbf{0} & -\mathbf{P}_x \\ -\mathbf{P}_y & \mathbf{P}_x & \mathbf{0} \end{pmatrix}$$

$$div \;\Box\; \mathbf{S} = \begin{pmatrix} \mathbf{P}_x^* & \mathbf{P}_y^* & \mathbf{P}_z^* \end{pmatrix}$$

* T. Weiland, ***A discretization method for the solution of Maxwell's equations for six-component fields***, Electronics and Communication (AEÜ) **31**, p. 116 (1977)

Low-dispersive schemes

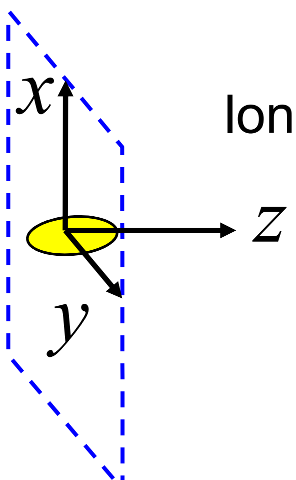
Curl operator splitting (2002)

$$\mathbf{C} = \begin{pmatrix} 0 & -\mathbf{P}_z & \mathbf{P}_y \\ \mathbf{P}_z & 0 & -\mathbf{P}_x \\ -\mathbf{P}_y & \mathbf{P}_x & 0 \end{pmatrix}$$

$$\mathbf{T} = \begin{pmatrix} 0 & 0 & \mathbf{P}_y \\ 0 & 0 & -\mathbf{P}_x \\ -\mathbf{P}_y & \mathbf{P}_x & 0 \end{pmatrix} \qquad \mathbf{L} = \begin{pmatrix} 0 & -\mathbf{P}_z & 0 \\ \mathbf{P}_z & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

transverse plane

longitudinal direction



I. Zagorodnov, T. Weiland, ***A Conformal Scheme for Wake Field Calculation***, Proc. *EPAC 2002, Paris*

Low-dispersive schemes

FDTD (1966)

$$\frac{\mathbf{e}^{n+1/2}-\mathbf{e}^{n-1/2}}{\Delta t}=\mathbf{M}_{\varepsilon^{-1}}(\mathbf{C}^{*}\mathbf{h}^{n}-\mathbf{j}^{n})$$

$$\frac{\mathbf{h}^{n+1}-\mathbf{h}^{n}}{\Delta t}\mathbf{h}^{n}=\mathbf{M}_{\mu^{-1}}\mathbf{C}\mathbf{e}^{n+1/2}$$

$$\mathbf{h}^{n}=\begin{pmatrix}\mathbf{h}_x^n\\ \mathbf{h}_y^n\\ \mathbf{h}_z^n\end{pmatrix} \qquad \mathbf{e}^{n+0.5}=\begin{pmatrix}\mathbf{e}_x^{n+0.5}\\ \mathbf{e}_y^{n+0.5}\\ \mathbf{e}_z^{n+0.5}\end{pmatrix}$$

Implicit Scheme* (2002)

$$\mathbf{C}=\mathbf{T}+\mathbf{L}$$

$$\frac{\mathbf{e}^{n+1/2}-\mathbf{e}^{n-1/2}}{\Delta t}=\mathbf{M}_{\varepsilon^{-1}}(\mathbf{T}^{*}\overline{\mathbf{h}}^{n}+\mathbf{L}^{*}\mathbf{h}^{n}-\mathbf{j}^{n})$$

$$\frac{\mathbf{h}^{n+1}-\mathbf{h}^{n}}{\Delta t}\mathbf{h}^{n}=\mathbf{M}_{\mu^{-1}}\mathbf{C}\mathbf{e}^{n+1/2}$$

$$\overline{\mathbf{h}}^{n}\equiv\theta\mathbf{h}^{n+1}+(1-2\theta)\mathbf{h}^{n}+\theta\mathbf{h}^{n-1}$$

* I. Zagorodnov, R. Schuhmann, T. Weiland, ***Long-Time Numerical Computation of Electromagnetic Fields in the Vicinity of a Relativistic Source***, Journal of Computational Physics **191**, No.2 , pp. 525-541 (2003)

Low-dispersive schemes

Implicit Scheme (2002)

$$\mathbf{a}^n = \int_{-\infty}^{t_n} \widehat{\mathbf{h}}_s d\tau$$

$$(\mathbf{I} + \theta \mathbf{T}_1)\mathbf{a}^{n+1} = 2\mathbf{a}^n - \mathbf{a}^{n-1} - \mathbf{T}_1\left((1-2\theta)\mathbf{a}^n + \theta \mathbf{a}^{n-1}\right) - \mathbf{L}_1\mathbf{a}^n + \mathbf{F}^n$$

$$\mathbf{T}_1 = \Delta t^2 \mathbf{M}_{\mu^{-1}} \mathbf{C} \mathbf{M}_{\varepsilon^{-1}} \mathbf{T}^*, \quad \mathbf{L}_1 = \Delta t^2 \mathbf{M}_{\mu^{-1}} \mathbf{C} \mathbf{M}_{\varepsilon^{-1}} \mathbf{L}^*$$

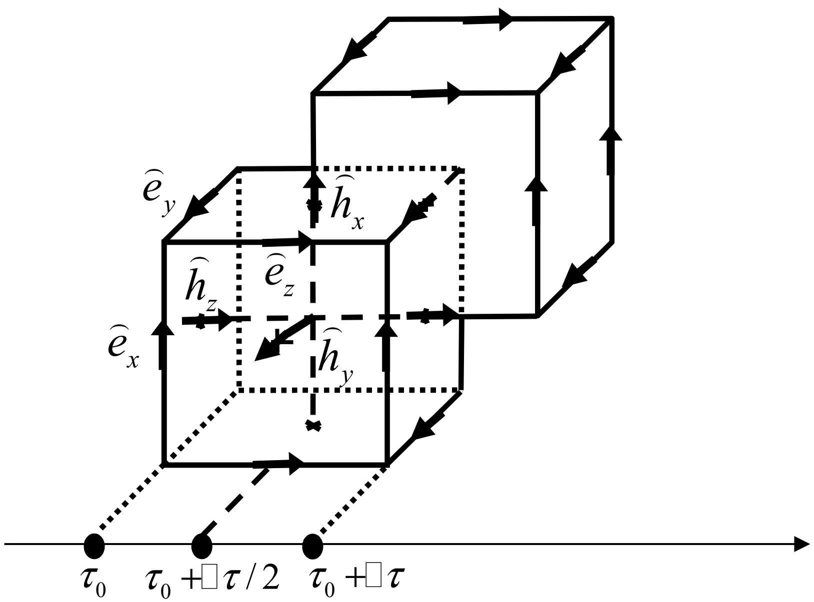
For fully rotationally symmetric problems (**monopole** mode m=0) our scheme in staircase approximation with θ=0.5 coincides with the equation realized in code NOVO**.

$$\mathbf{M}^{-1}_{\mu^{-1}_{\varphi}} \Delta t^{-2}\left(\mathbf{a}^{n+1}_{\varphi} - 2\mathbf{a}^{n}_{\varphi} + \mathbf{a}^{n-1}_{\varphi}\right) = \mathbf{P}_z \mathbf{M}_{\varepsilon^{-1}_r} \mathbf{P}^T_z \mathbf{a}^n_{\varphi} + \mathbf{P}_r \mathbf{M}_{\varepsilon^{-1}_z} \mathbf{P}^T_r \frac{\mathbf{a}^{n+1}_{\varphi} + \mathbf{a}^{n-1}_{\varphi}}{2} + \mathbf{F}^n_{\varphi},$$

** A. Novokhatski, M. Timm, T. Weiland, ***Transition Dynamics of the Wake Fields of Ultra Short Bunches,*** Proc. ICAP 1998, Monterey, California, USA.

Low-dispersive schemes

E/M and TE/TM* splitting (2004)



Subdue the updating procedure to the bunch motion

* Zagorodnov I.A, Weiland T., ***TE/TM Field Solver for Particle Beam Simulations without Numerical Cherenkov Radiation***, Phys. Rev. ST Accel. Beams **8**, 042001 (2005)

E/M splitting (Yee's Scheme)

$$\mathbf{h}^n = \begin{pmatrix} \mathbf{h}_x^n \\ \mathbf{h}_y^n \\ \mathbf{h}_z^n \end{pmatrix} \qquad \mathbf{e}^{n+0.5} = \begin{pmatrix} \mathbf{e}_x^{n+0.5} \\ \mathbf{e}_y^{n+0.5} \\ \mathbf{e}_z^{n+0.5} \end{pmatrix}$$

TE/TM splitting

$$\mathbf{v}^n = \begin{pmatrix} \mathbf{e}_x^n \\ \mathbf{e}_y^n \\ \mathbf{h}_z^n \end{pmatrix} \qquad \mathbf{u}^{n+0.5} = \begin{pmatrix} \mathbf{h}_x^{n+0.5} \\ \mathbf{h}_y^{n+0.5} \\ \mathbf{e}_z^{n+0.5} \end{pmatrix}$$

Low-dispersive schemes

Curl operator splitting

$$\mathbf{C}_0 = c^{-1}\mathbf{M}_{\mu^{-1}}^{1/2}\mathbf{C}\mathbf{M}_{\varepsilon^{-1}}^{1/2} = \begin{pmatrix} \mathbf{0} & -\mathbf{P}_z^0 & \mathbf{P}_y^0 \\ \mathbf{P}_z^1 & \mathbf{0} & -\mathbf{P}_x^0 \\ -\mathbf{P}_y^1 & \mathbf{P}_x^1 & \mathbf{0} \end{pmatrix}$$

transversal plane

longitudinal direction

$$\mathbf{T}_\mathrm{i} = \begin{pmatrix} \mathbf{0} & \mathbf{0} & -\mathbf{P}_y^i \\ \mathbf{0} & \mathbf{0} & \mathbf{P}_x^i \\ \left(\mathbf{P}_y^i\right)^* & -\left(\mathbf{P}_x^i\right)^* & \mathbf{0} \end{pmatrix}, i = 0,1$$

$$\mathbf{L} = \begin{pmatrix} \mathbf{0} & \mathbf{P}_z^0 & \mathbf{0} \\ -\mathbf{P}_z^1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$$

Low-dispersive schemes

E/M splitting (1966)

$$\frac{\mathbf{e}^{n+0.5}-\mathbf{e}^{n-0.5}}{\Delta\tau}=\mathbf{C}_0{}^*\mathbf{h}^n+\mathbf{j}^n,$$

$$\frac{\mathbf{h}^{n+1}-\mathbf{h}^n}{\Delta\tau}=-\mathbf{C}_0\mathbf{e}^{n+0.5}$$

$$\mathbf{h}^n=\begin{pmatrix}\mathbf{h}_x^n\\ \mathbf{h}_y^n\\ \mathbf{h}_z^n\end{pmatrix}\qquad \mathbf{e}^{n+0.5}=\begin{pmatrix}\mathbf{e}_x^{n+0.5}\\ \mathbf{e}_y^{n+0.5}\\ \mathbf{e}_z^{n+0.5}\end{pmatrix}$$

dispersion error

$$\Delta\tau<\Delta z$$

TE/TM splitting (2004)

$$\frac{\mathbf{u}^{n+0.5}-\mathbf{u}^{n-0.5}}{\Delta\tau}=\mathbf{T}_0\frac{\mathbf{u}^{n+0.5}+\mathbf{u}^{n-0.5}}{2}+\mathbf{L}\mathbf{v}^n+\mathbf{j}_u^n,$$

$$\frac{\mathbf{v}^{n+1}-\mathbf{v}^n}{\Delta\tau}=\mathbf{T}_1\frac{\mathbf{v}^{n+1}+\mathbf{v}^n}{2}-\mathbf{L}^*\mathbf{u}^{n+0.5}+\mathbf{j}_v^{n+0.5}$$

$$\mathbf{v}^n=\begin{pmatrix}\mathbf{e}_x^n\\ \mathbf{e}_y^n\\ \mathbf{h}_z^n\end{pmatrix}\qquad \mathbf{u}^{n+0.5}=\begin{pmatrix}\mathbf{h}_x^{n+0.5}\\ \mathbf{h}_y^{n+0.5}\\ \mathbf{e}_z^{n+0.5}\end{pmatrix}$$

dispersion error suppressed for

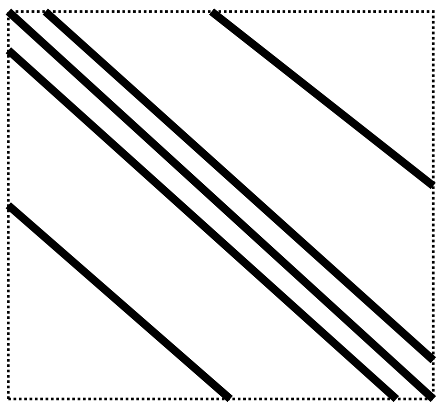
$$\Delta\tau=\Delta z$$

Low-dispersive schemes

Implicit TE/TM formulation

$$\mathbf{W}^e \frac{\widehat{\mathbf{e}}_z^{\,n+0.5} - \widehat{\mathbf{e}}_z^{\,n-0.5}}{\Delta\tau} = \mathbf{M}_{\varepsilon_z^{-1}} \left[\mathbf{P}_y^* \widehat{\mathbf{h}}_x^{\#} - \mathbf{P}_x^* \widehat{\mathbf{h}}_y^{\#} + \widehat{\mathbf{j}}_z^{\,n} \right]$$

$$\mathbf{W}^h \frac{\mathbf{h}_z^{n+1} - \mathbf{h}_z^{n}}{\Delta\tau} = \mathbf{M}_{\mu_z^{-1}} \left[\mathbf{P}_y \widehat{\mathbf{e}}_x^{\#} - \mathbf{P}_x \widehat{\mathbf{e}}_y^{\#} \right]$$

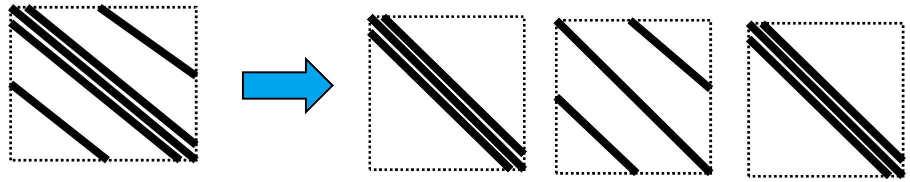


$$\mathbf{W}^e = \mathbf{W}^e_{CN} = \mathbf{I} + \frac{\Delta\tau^2}{4} \mathbf{M}_{\varepsilon_z^{-1}} \mathbf{P}_y^* \mathbf{M}_{\mu_x^{-1}} \mathbf{P}_y + \frac{\Delta\tau^2}{4} \mathbf{M}_{\varepsilon_z^{-1}} \mathbf{P}_x^* \mathbf{M}_{\mu_y^{-1}} \mathbf{P}_x$$

Low-dispersive schemes

ADI and Explicit TE/TM formulations (2009)

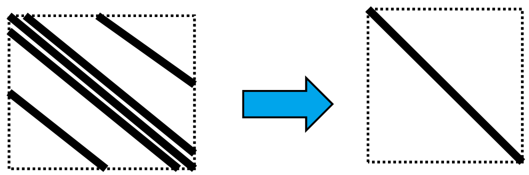
TE/TM-ADI scheme



$$\mathbf{W}_{CN}^{e} = \mathbf{W}_{ADI2}^{e} + O(\Delta\tau^{4})$$ - splitting error

Explicit TE/TM scheme*

$$\mathbf{W}^{e} = \mathbf{W}^{h} = \mathbf{I}$$



$$\mathbf{W}_{CN}^{e} = \mathbf{I} + O(\Delta\tau^{2})$$ - splitting error

* Dohlus M., Zagorodnov I., ***Explicit TE/TM Scheme for Particle Beam Simulations***, Journal of Computational Physics **225**, No. 8, pp. 2822-2833 (2009)

Low-dispersive schemes

Stability, energy and charge conservation

$$\mathbf{B}\frac{\mathbf{y}^{n+1} - \mathbf{y}^{n}}{\Delta\tau} + \mathbf{A}\mathbf{y}^{n} = \mathbf{f}^{n}$$

The stability condition $\mathbf{Q} \equiv \mathbf{B} - 0.5\Delta\tau\mathbf{A} \geq 0$

E/M splitting

(FDTD scheme)

$$\Delta\tau \leq \frac{1}{\sqrt{\Delta x^{-2} + \Delta y^{-2} + \Delta z^{-2}}}$$

TE/TM implicit

$$\Delta\tau \leq \Delta z$$

TE/TM explicit

$$\Delta\tau \leq \min\left(\frac{1}{\sqrt{\Delta x^{-2} + \Delta y^{-2}}}, \Delta z\right)$$

dispersion error suppressed for $\Delta\tau = \Delta z$

Low-dispersive schemes

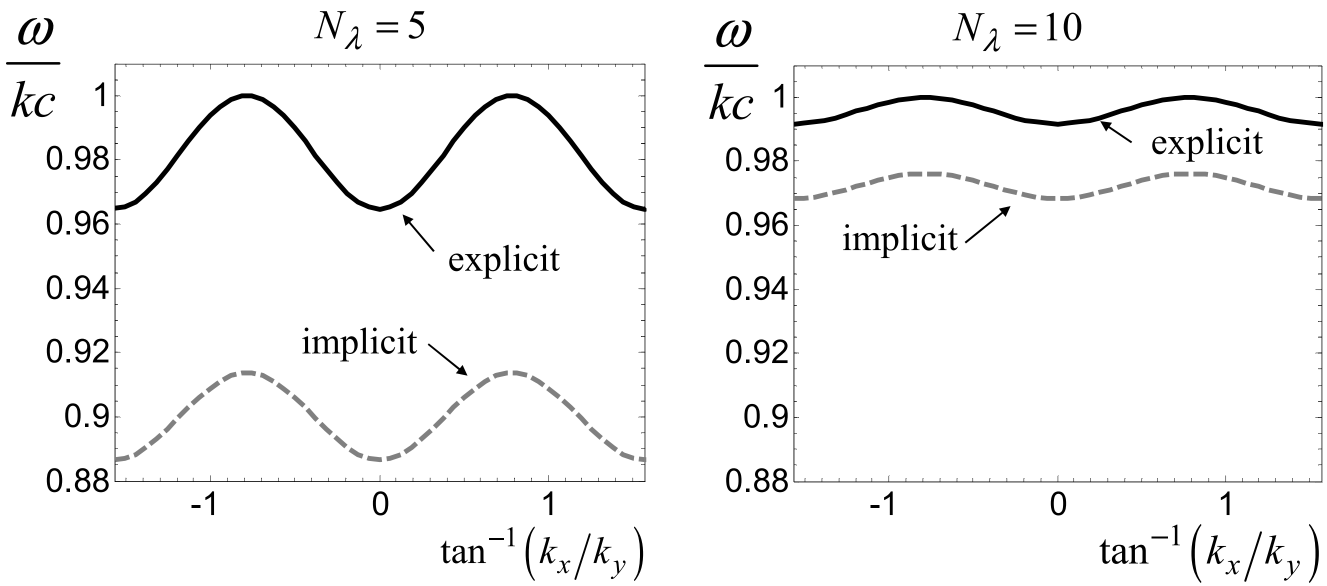
Dispersion relation in the transverse plane

$$\frac{\sin^2\Omega}{\Delta\tau^2}=\frac{\sin^2 K_z}{\Delta z^2}+\left(\frac{\sin^2 K_x}{\Delta x^2}+\frac{\sin^2 K_y}{\Delta y^2}\right)\cos^2\Omega$$

Implicit TE/TM scheme

$$\frac{\sin^2\Omega}{\Delta\tau^2}=\frac{\sin^2 K_z}{\Delta z^2}+\left(\frac{\sin^2 K_x}{\Delta x^2}+\frac{\sin^2 K_y}{\Delta y^2}\right)\left(1-\frac{\Delta\tau^2}{\Delta z^2}\sin^2 K_z\right)$$

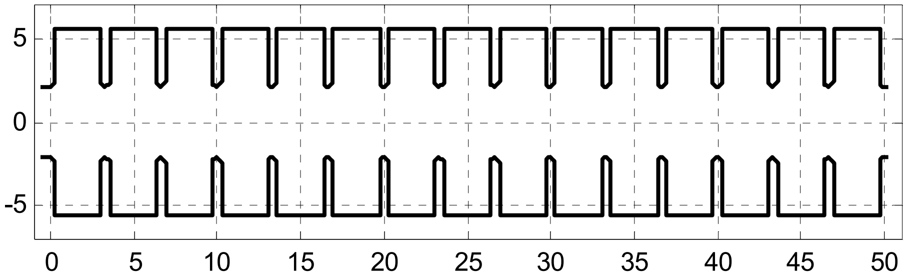
Explicit TE/TM scheme



No dispersion in z-direction + no dispersion along XY diagonals

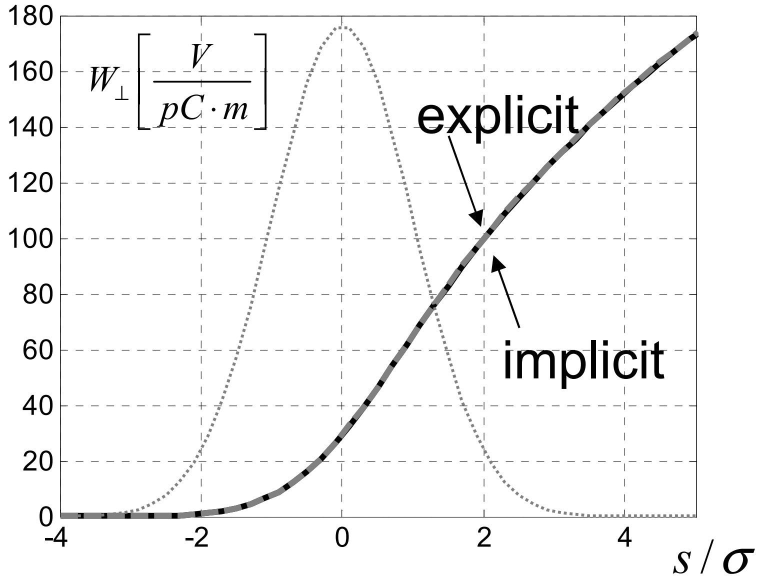
Low-dispersive schemes

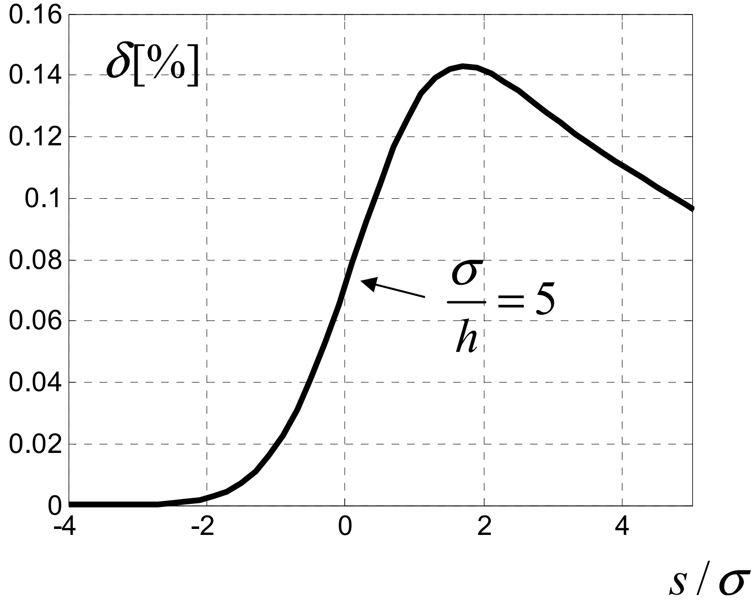
Transverse Deflecting Structure



$$\delta = \frac{\left| W_{\perp}^{\exp} - W_{\perp}^{imp} \right|}{\max W_{\perp}^{\exp} - \min W_{\perp}^{\exp}} * 100\%$$

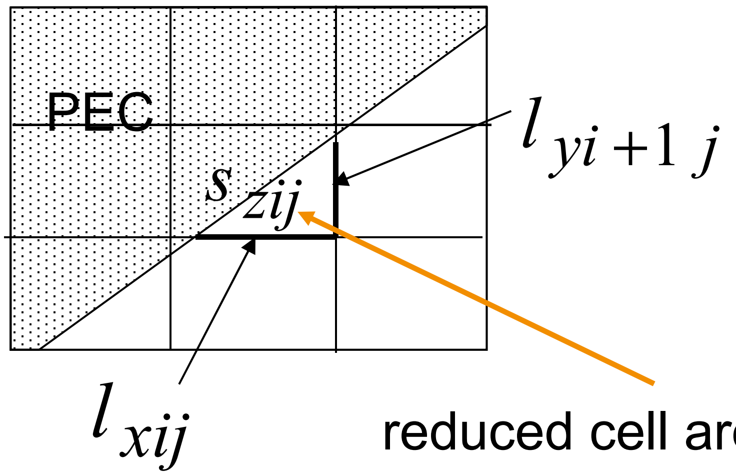
Gaussian bunch with sigma=300μm





Boundary approximation

Standard Conformal Scheme (1997)



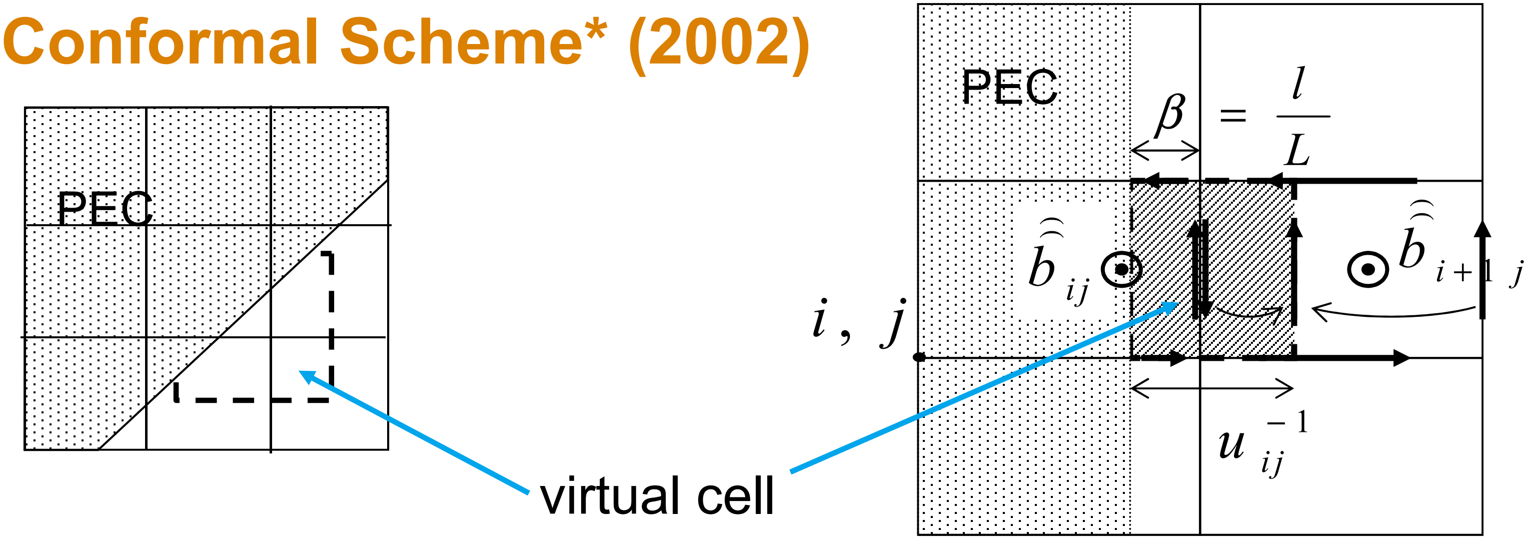
time step must be reduced

Dey S, Mittra R. ***A locally conformal finite-difference time-domain (FDTD) algorithm for modeling threedimensional perfectly conducting objects***, IEEE Microwave and Guided Wave Letters 7(9):273–275 (1997)

Thoma P. ***Zur numerischen Lösung der Maxwellschen Gleichungen im Zeitbereich***, Dissertation DI7: TH Darmstadt,1997.

Boundary approximation

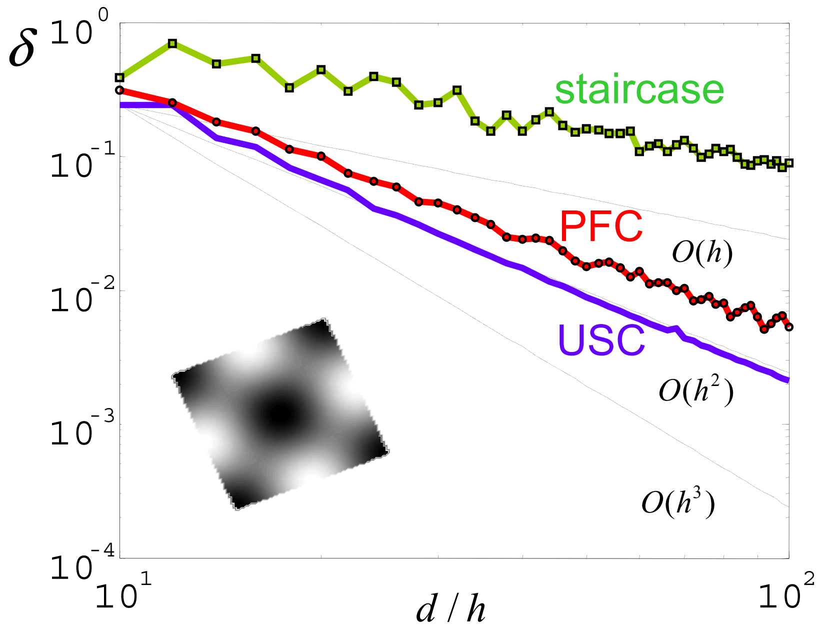
New Conformal Scheme* (2002)

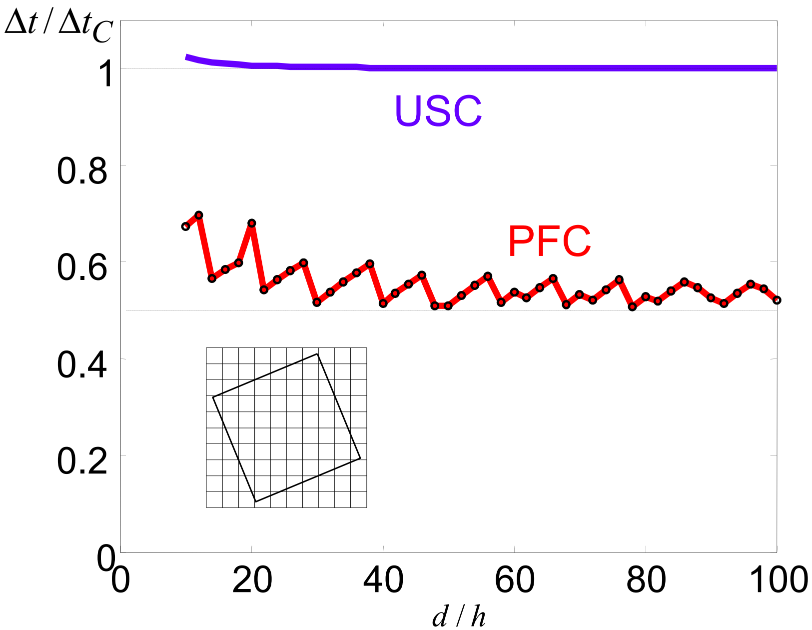


Time step is not reduced!

* Zagorodnov I., Schuhmann R.,Weiland T., ***A Uniformly Stable Conformal FDTD-Method on Cartesian Grids***, International Journal on Numerical Modeling, vol. 16, No.2, pp. 127-141 (2003)

Boundary approximation



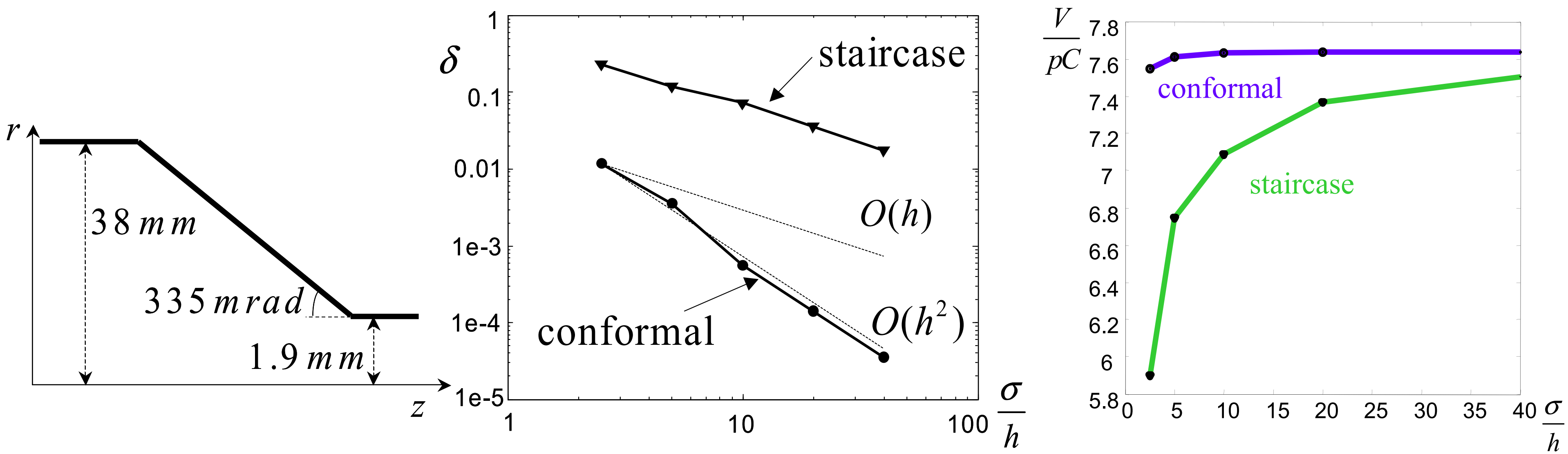


Square rotate by angle $\pi/8$.

$$\delta = \frac{\left\| H_z - \tilde{H}_z \right\|_{L_2^h}}{\left\| H_z \right\|_{L_2^h}}$$

PFC (Partially Filled Cells) ~ Dey-Mittra
USC (Uniformly Stable Conformal)

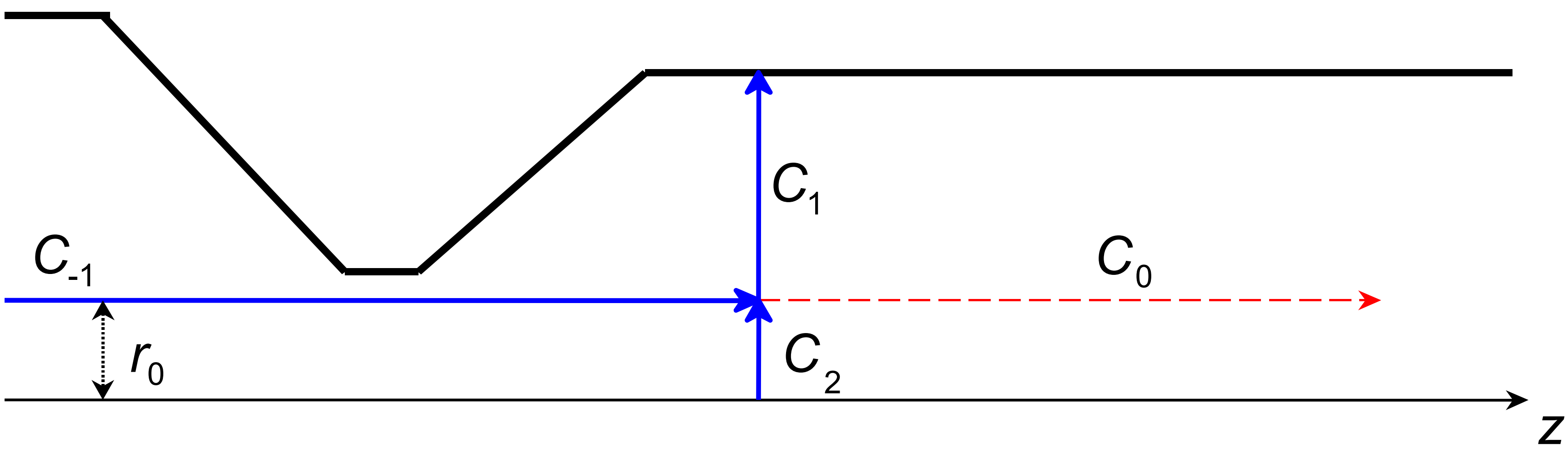
Boundary approximation



Error in loss factor for a taper $\delta = \left| L_{calc} - L \right| L^{-1}$

The error δ relative to the extrapolated loss factor L=-7.63777 V/pC for bunch with σ=1 mm is shown.

Indirect Integration Algorithm



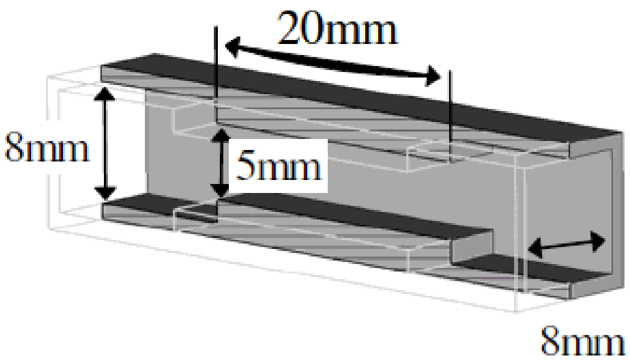
$$QW_{\parallel}^{m} = \int_{C_{-1}} e_z^s dz + \int_{C_0} e_z^s dz$$

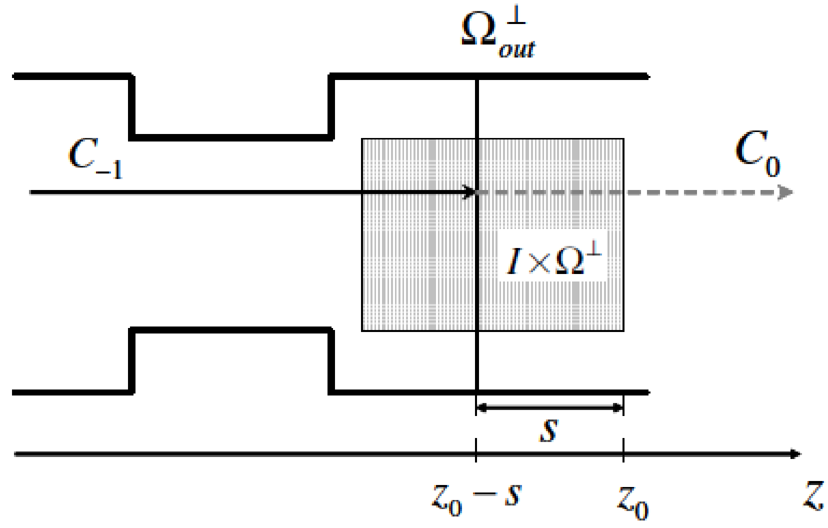
$$\int_{C_0} e_z^s dz = -\frac{1}{2}\left(\int_{C_1} \left(r_0^m \omega_D + r_0^{-m} \omega_S - \frac{\beta}{a^m} \omega_S \right) - \frac{\beta}{a^m} \int_{C_2} \omega_S \right)$$

I. Zagorodnov, R. Schuhmann, T. Weiland, Journal of Computational Physics **191**, No.2 , pp. 525-541 (2003)

O. Napoly, Y. Chin, and B. Zotter, Nucl. Instrum. Methods Phys. Res. Sect. A **334**, 255 (1993)

Indirect Integration Algorithm





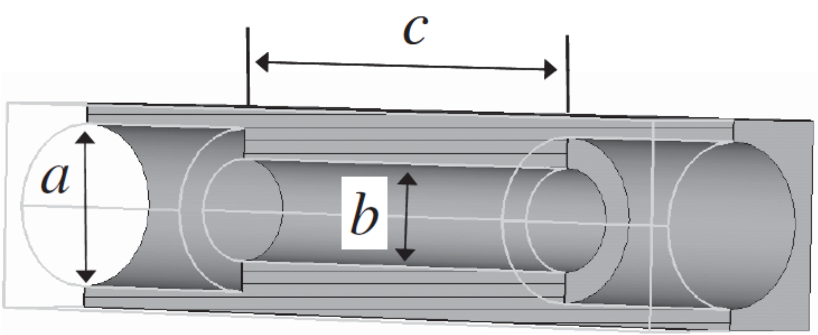
$$QW_{\|}(\vec{r}_0, s) = -\int_{C_{-1}(\vec{r}_0, z_0 - s)} E_z^{sc}[\vec{r}_0, z, t(z, s)]dz - u(\vec{r}_0, s), \qquad t(z, s) = \frac{z + s}{c}$$

$$\Delta u(\vec{r}, s) = -\left[\frac{\partial}{\partial s} + \frac{\partial}{c\,\partial t}\right] E_z^{sc}(\vec{r}, z_0 - s, t_0), \qquad \vec{r} \in \Omega^{\perp}_{\text{out}}, \qquad u(\vec{r}, s) = 0, \qquad \vec{r} \in \partial\Omega^{\perp}_{\text{out}}.$$

Zagorodnov I., ***Indirect Methods for Wake Potential Integration***, Phys. Rev. STAB **9**, 102002 (2006)

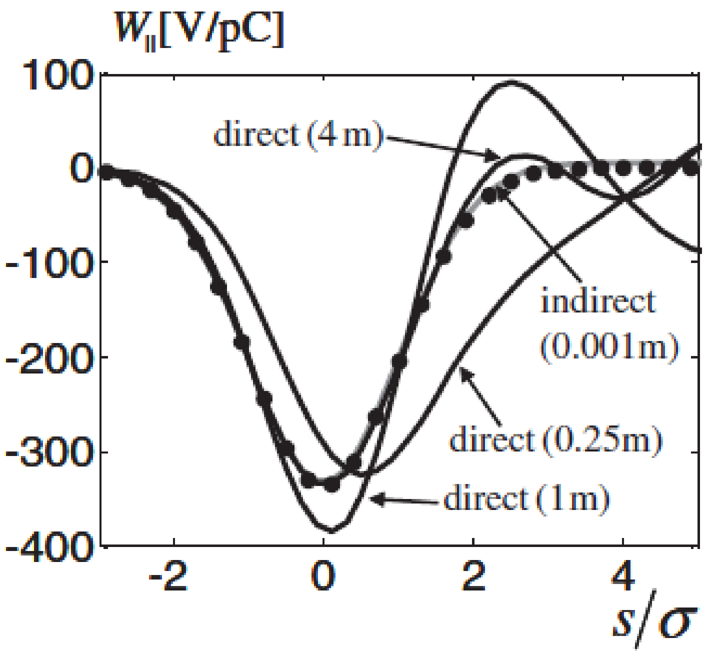
H. Henke and W. Bruns, in Proceedings of EPAC 2006, Edinburgh, Scotland (WEPCH110, 2006)

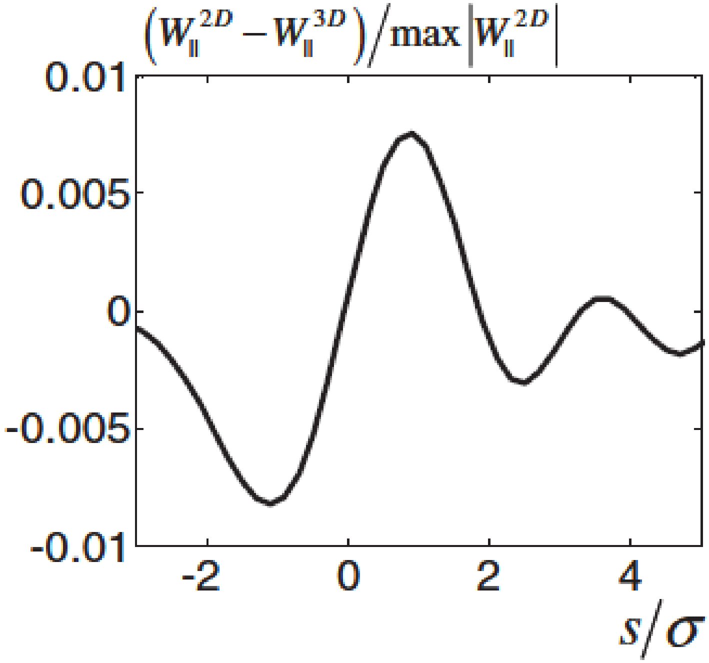
Indirect Integration Algorithm



$a = 8$ mm, $b = 5$ mm, and $c = 20$ mm

Gaussian bunch with rms length 25 µm





Modelling of Conductive Walls

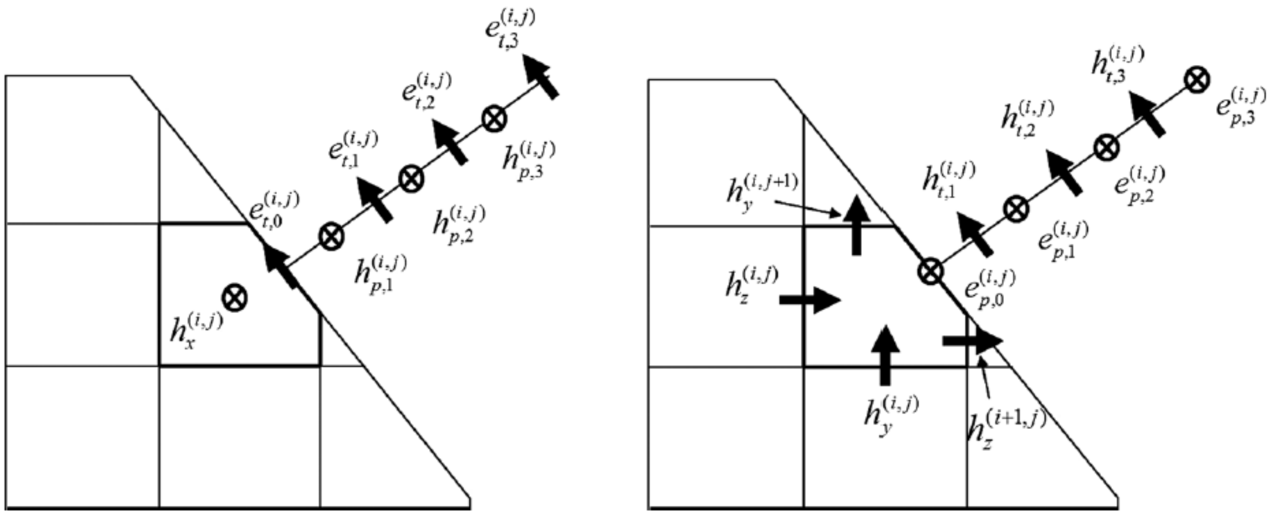
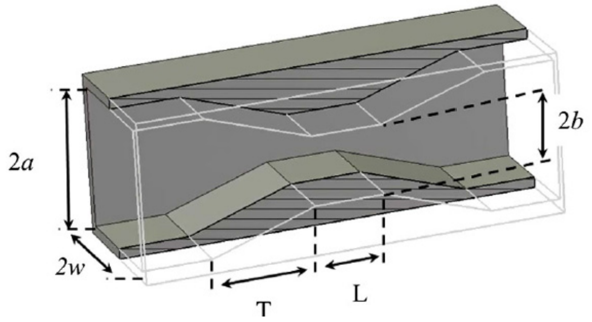


FIG. 6. A boundary cell in the vacuum and 1D conductive line in the metal.

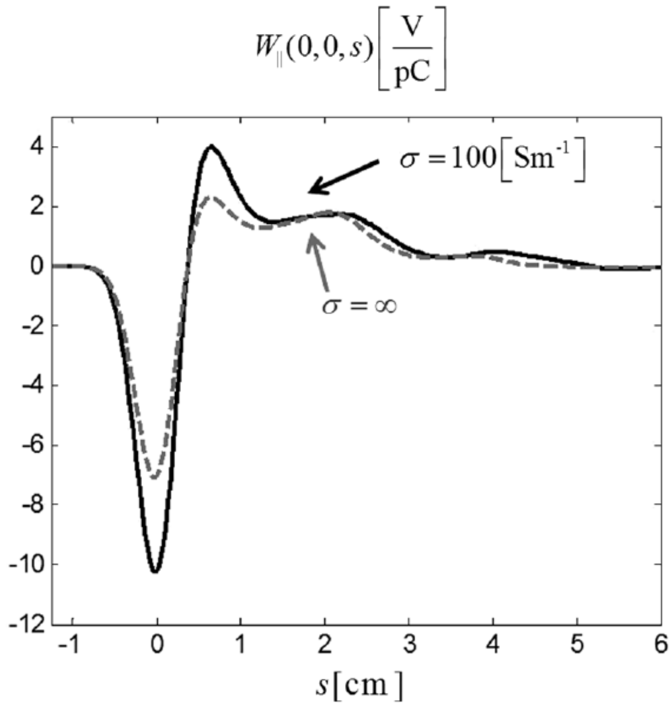
Tsakanian A., Dohlus M., Zagorodnov I., ***Hybrid TE-TM scheme for time domain numerical calculations of wakefields in structures with walls of finite conductivity***, Phys. Rev. STAB **15**, 054401 (2012)

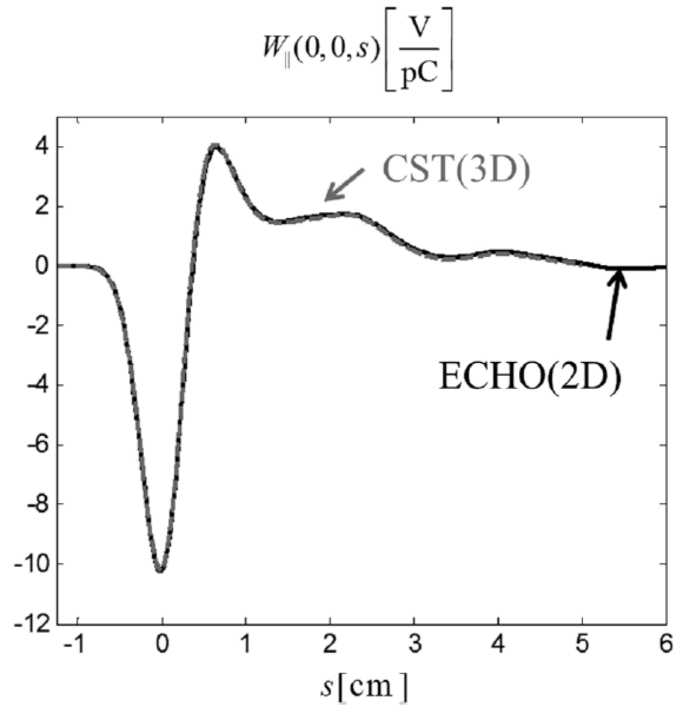
Zagorodnov I., Bane K., Stupakov G., **Calculations of wakefields in 2D rectangular structures**, Phys. Rev. STAB **18**, 104401 (2015)

Modelling of Conductive Walls



2*w*= 10 cm, *T*=5 cm, 2*b*=2cm, *L*=12 cm.
Gaussian bunch with rms length 25mm





Longitudinal wake potential of tapered collimator

Code Status

- ECHOz1, ECHOz2 (rotationally symmetric)
- ECHO2D (rectangular and rotationally symmetric)
- ECHO3D (fully 3D)
- ECHO-PIC (Particle-In-Cell for rotationally symmetric and rectangular)

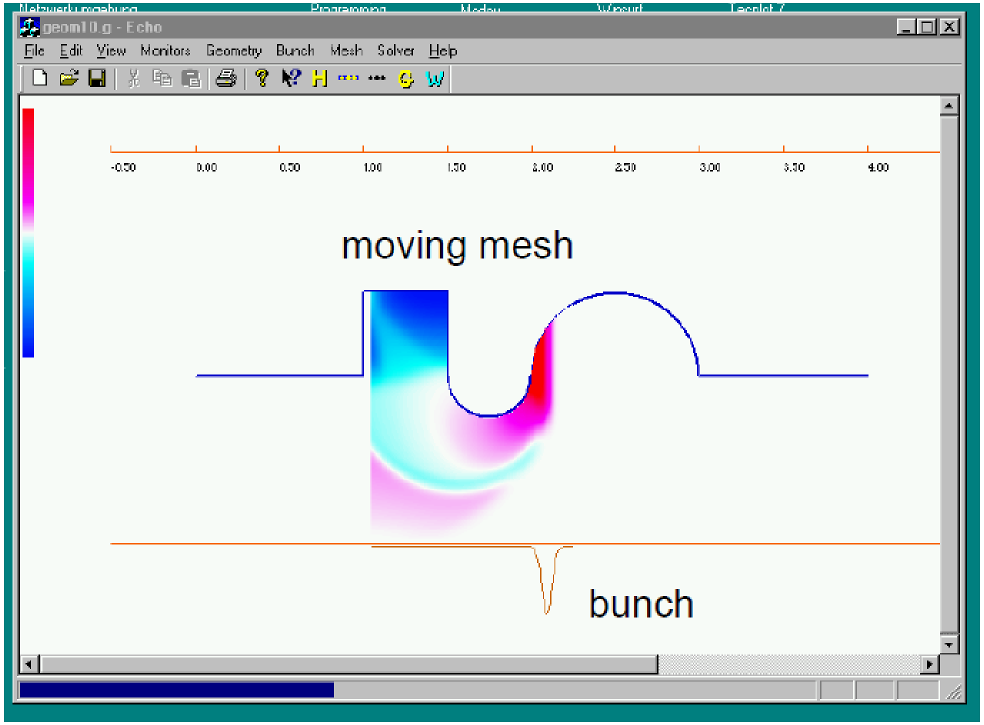
https://www.desy.de/~zagor/WakefieldCode_ECHOz/

Rotationally Symmetric Geometries

$$\begin{pmatrix} E_r \\ H_\theta \\ E_z \end{pmatrix} = \sum_{m=0}^{\infty} \begin{pmatrix} E_{r,m} \\ H_{\theta,m} \\ E_{z,m} \end{pmatrix} \cos(m\theta) \qquad \begin{pmatrix} H_r \\ E_\theta \\ H_z \end{pmatrix} = \sum_{m=0}^{\infty} \begin{pmatrix} H_{r,m} \\ E_{\theta,m} \\ H_{z,m} \end{pmatrix} \sin(m\theta)$$

$$W_{\parallel}(r_0, \theta_0, r, \theta, s) = \sum_{m=0}^{\infty} W_m(s) r_0^m r^m \cos\left(m(\theta - \theta_0)\right)$$

ECHOz1



ECHO

Electromagnetic
Code for
Handling
Of
Harmful
Collective
Effects

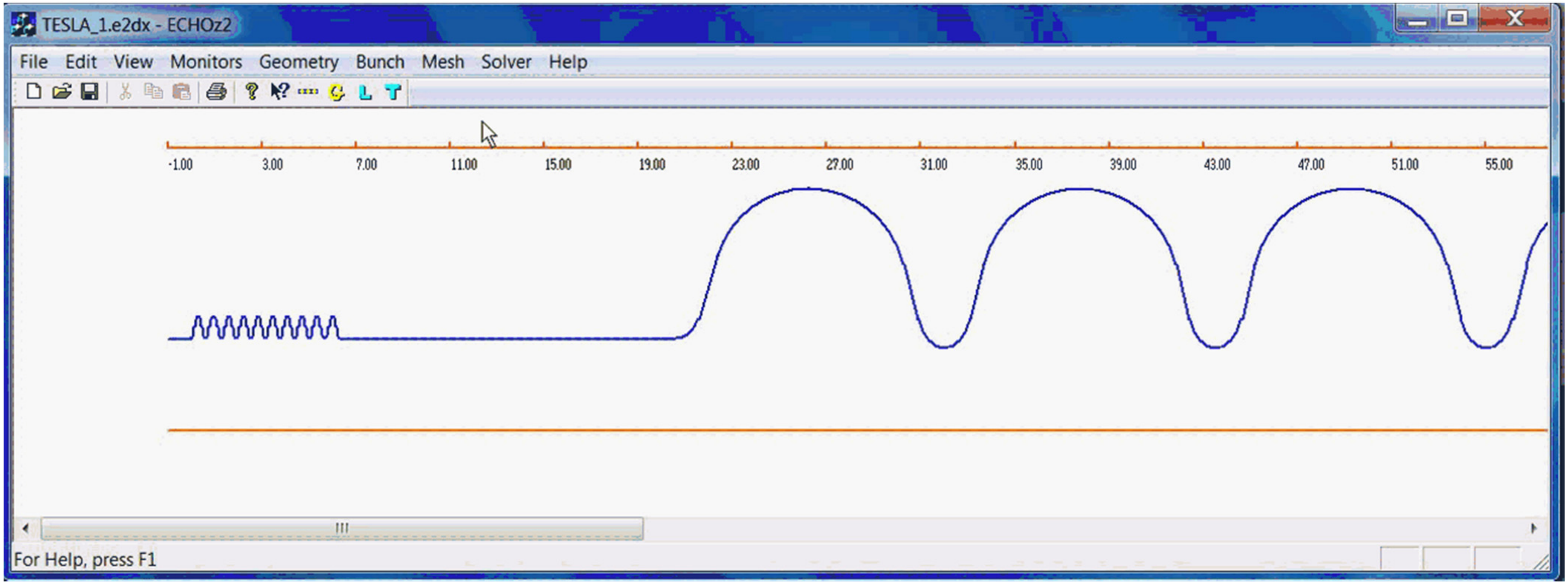
Only fully rotationally symmetric problems (m=0 mode)

- vector potential wave equation (slide 9)
- only PEC
- stand-alone Windows GUI application

ECHOz2

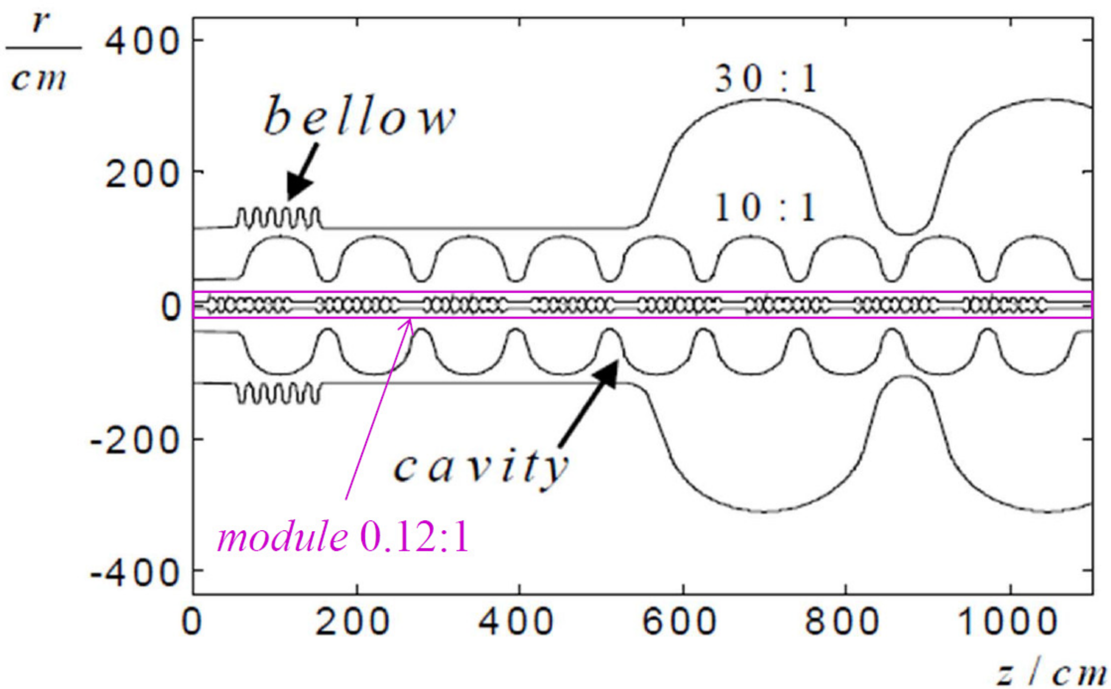
Rotationally symmetric geometries (all modes)

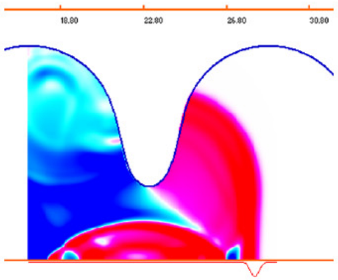
- TE/TM implicit (slide 10)
- surface conductivity
- stand-alone Windows GUI application



ECHOz2

Short-Range Wake Functions for TESLA Cryomodule





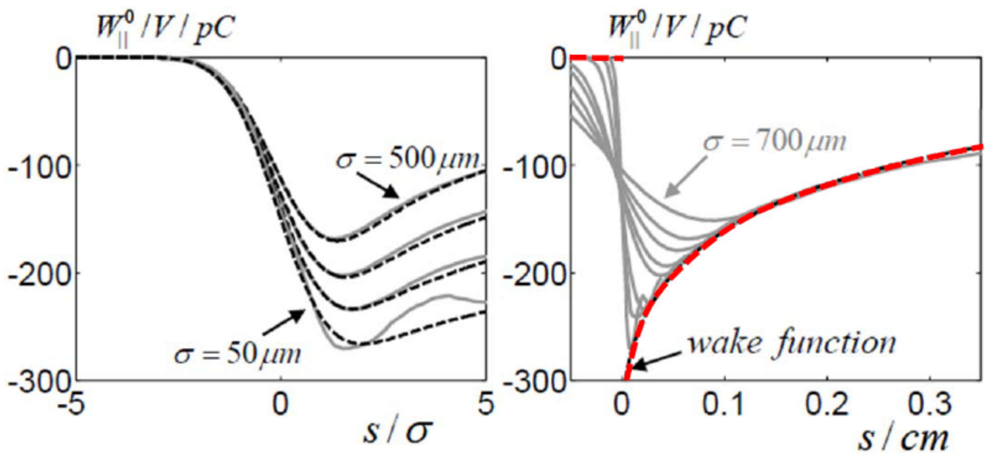
σ = 700 ... **50 µm**

Longitudinal wake functions was found earlier with code NOVO
Novokhatski A., Timm M., Weiland T., ***Single Bunch Enetgy Spread in the TESLA Cryomodule***, DESY TESLA-99-16, 1999

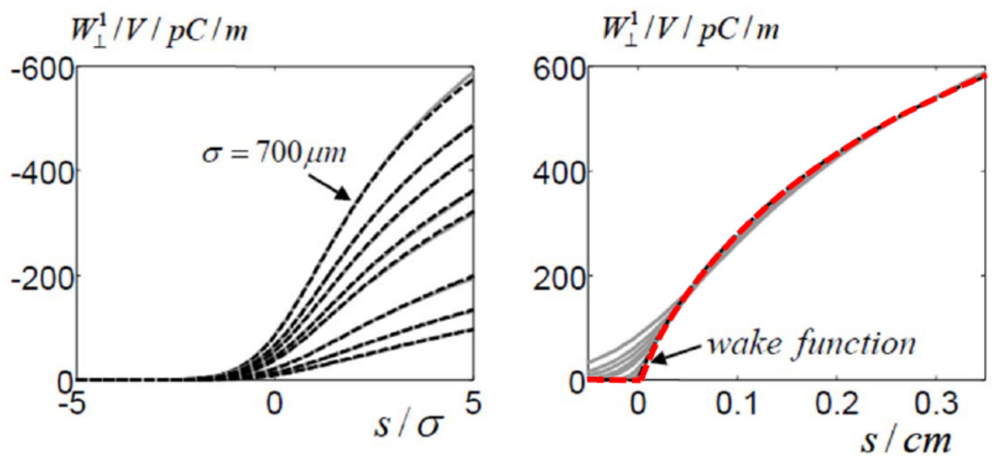
Transverse wake functions is found with code ECHO
T. Weiland, I. Zagorodnov, ***The short-range transverse wake function for TESLA accelerating structure***, TESLA Report 2003-19, 2003

ECHOz2

longitudinal wake (monopole)



transverse wake (dipole)



Novokhatski A., Timm M., Weiland T. (1999, NOVO)

$$W_{\parallel}(s) = A\left[(1+\beta_0)e^{-\sqrt{\frac{s}{s_0}}} - \beta_0\right]$$

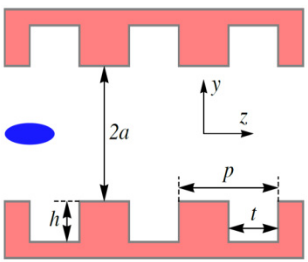
Bane K.L.F., SLAC-PUB-9663, LCC-0116, 2003

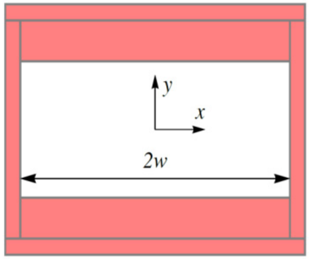
Weiland T., Zagorodnov I. (2003, ECHO)*

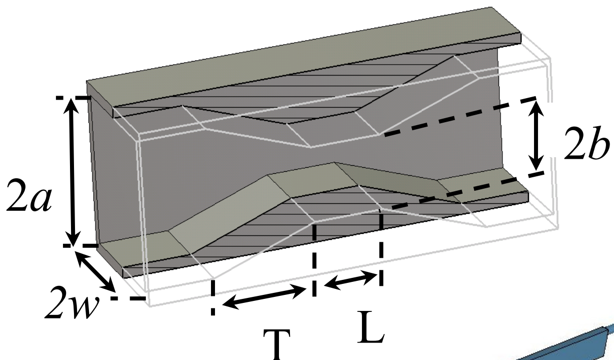
$$W_{\parallel}(s) = Ae^{-\sqrt{\frac{s}{s_0}}}$$

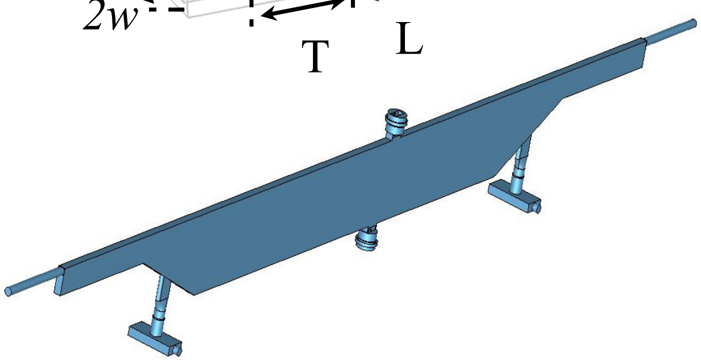
$$W_{\perp}(s) = A_1\left[1-\left(1+\sqrt{\frac{s}{s_1}}\right)e^{-\sqrt{\frac{s}{s_1}}}\right]$$

Rectangular Geometries with Constant Width









$$\begin{pmatrix} E_x \\ H_y \\ H_z \end{pmatrix} = \frac{1}{w} \sum_{m=1}^{\infty} \begin{pmatrix} E_{x,m} \\ H_{y,m} \\ E_{z,m} \end{pmatrix} \cos\left(\frac{\pi}{2w} m x \right)$$

$$\begin{pmatrix} H_x \\ E_y \\ E_z \end{pmatrix} = \frac{1}{w} \sum_{m=1}^{\infty} \begin{pmatrix} H_{x,m} \\ E_{y,m} \\ E_{z,m} \end{pmatrix} \sin\left(\frac{\pi}{2w} m x \right)$$

Zagorodnov I., Bane K., Stupakov G. ***Calculations of wakefields in 2D rectangular structures***, Phys. Rev. STAB **18**, 104401 (2015)

Novokhatski A., ***Wakefield potentials of corrugated structures***, Phys. Rev. STAB **18**, 104402 (2015)

ECHO2D

Wake field expansion is new (to our knowledge)

$$W_{\parallel}(x_0, y_0, x, y, s) = \frac{1}{w}\sum_{m=0}^{\infty} W_m(y_0, y, s)\sin\left(k_{x,m}x_0\right)\sin\left(k_{x,m}x\right)$$

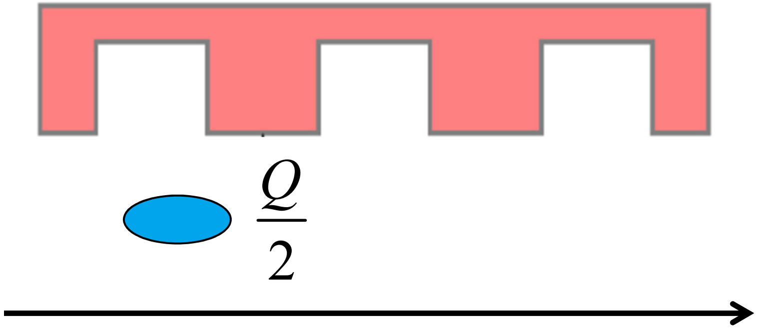
General case

$$W_m(y_0, y, s) = \begin{pmatrix}\cosh(k_{x,m}y_0) & \sinh(k_{x,m}y_0)\end{pmatrix} \begin{pmatrix} W_m^{cc}(s) & W_m^{cs}(s) \\ W_m^{sc}(s) & W_m^{ss}(s)\end{pmatrix} \begin{pmatrix}\cosh(k_{x,m}y) \\ \sinh(k_{x,m}y)\end{pmatrix}$$

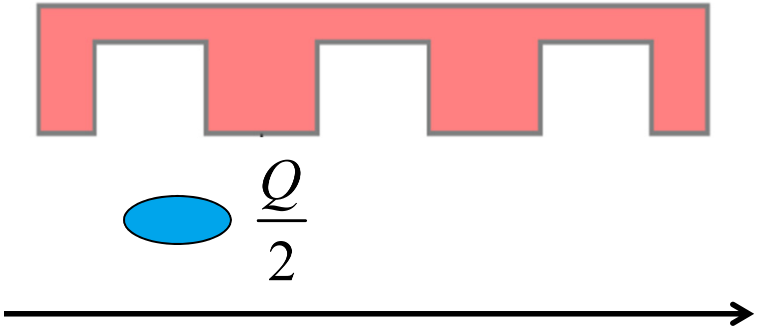
With symmetry in y

$$W_m(y_0, y, s) = W_m^{cc}(s)\cosh(k_{x,m}y_0)\cosh(k_{x,m}y) + \\ + W_m^{ss}(s)\sinh(k_{x,m}y_0)\sinh(k_{x,m}y)$$

ECHO2D



$$\frac{Q}{2}, \quad H_z = 0$$



$$\frac{Q}{2}, \quad E_z = 0$$

$$W_m(y_0, y, s) = W_m^H(y_0, y, s) + W_m^E(y_0, y, s)$$

$$W_m^{cc}(s) = \frac{W_m^H(y_0, y_0, s)}{\cosh(k_{x,m} y_0)^2}$$

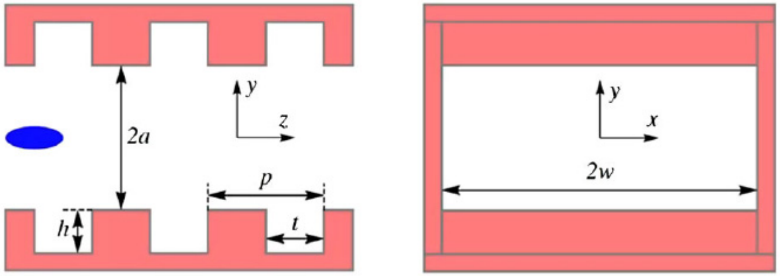
$$W_m^{ss}(s) = \frac{W_m^E(y_0, y_0, s)}{\sinh(k_{x,m} y_0)^2}$$

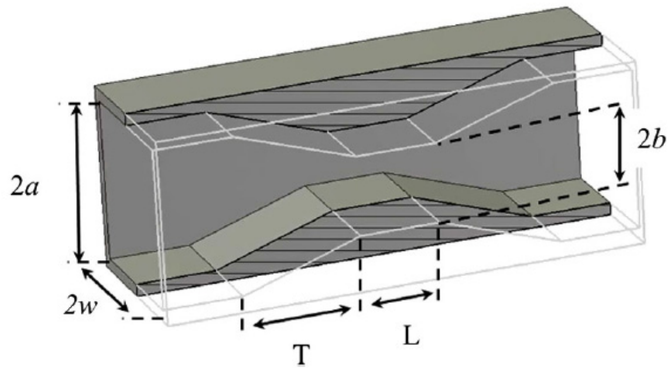
$$W_m(y_0, y, s) = W_m^{cc}(s)\cosh(k_{x,m} y_0)\cosh(k_{x,m} y) + W_m^{ss}(s)\sinh(k_{x,m} y_0)\sinh(k_{x,m} y)$$

ECHO2D

Rectangular and rotationally symmetric problems (all modes)

- TE/TM implicit (slide 10)
- surface conductivity
- stand-alone console application (Windows, Mac OS, Linux)
- Parallelized (MPI and threads)

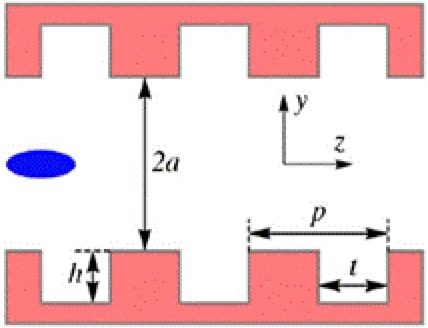


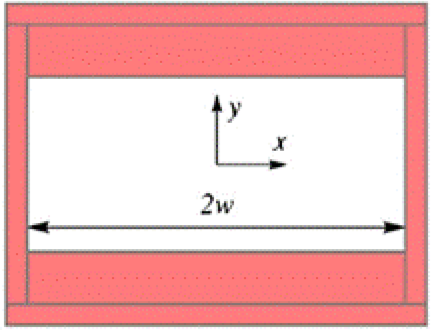


Zagorodnov I., Bane K., Stupakov G., **Calculations of wakefields in 2D rectangular structures**, Phys. Rev. STAB **18**, 104401 (2015)

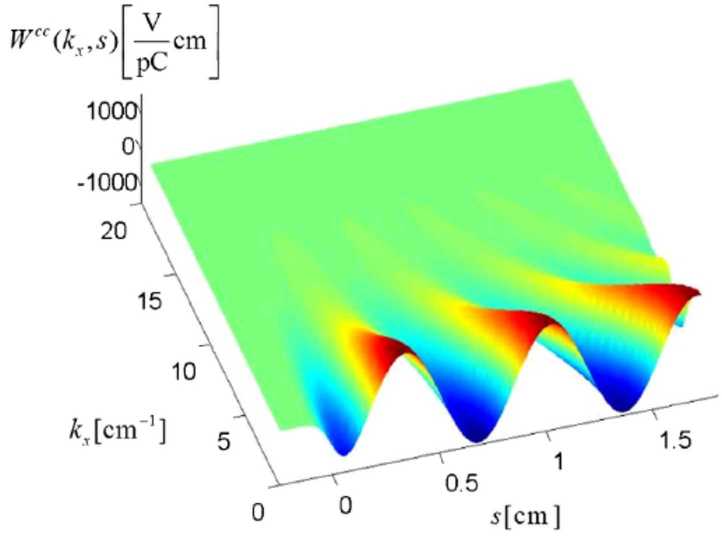
ECHO2D

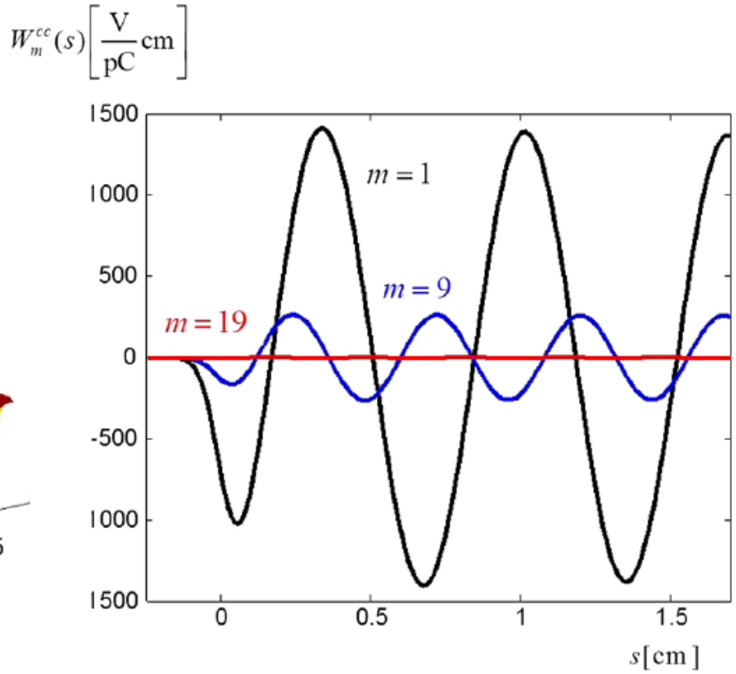
Pohang Dechirper Experiment





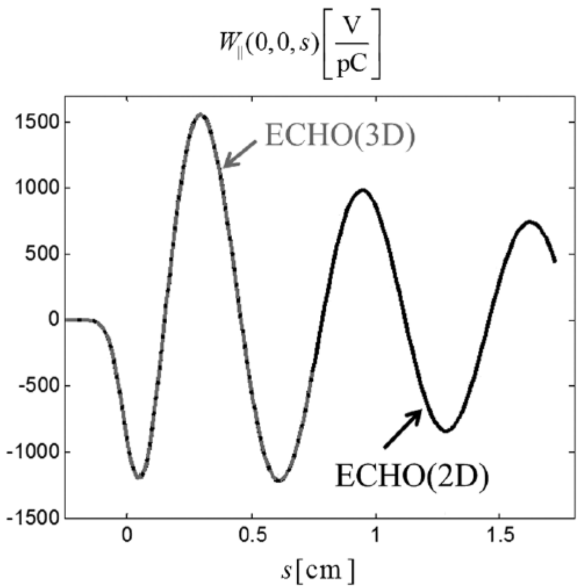
2*w*= 5 cm, *p*=0.5 mm,
h=0.6mm,
t=p/2, *L*=1 m, 2a= 6mm.
Gaussian bunch with rms length 0.5mm

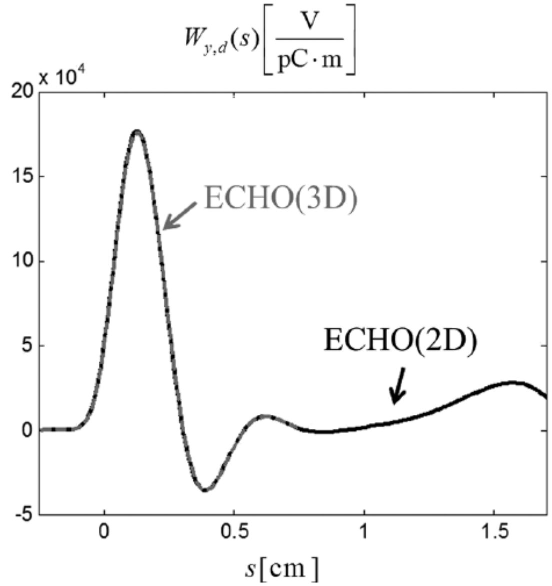


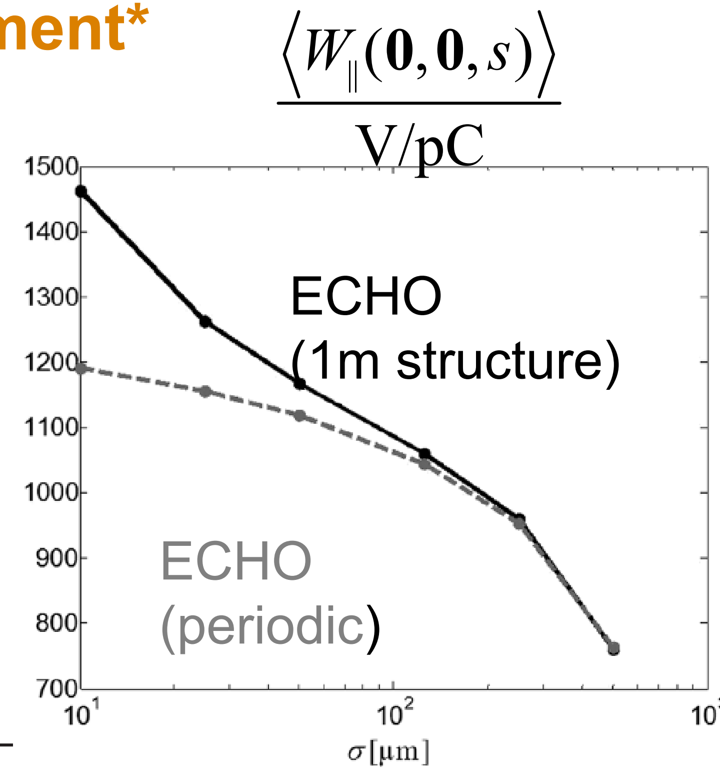


ECHO2D

Pohang Dechirper Experiment*





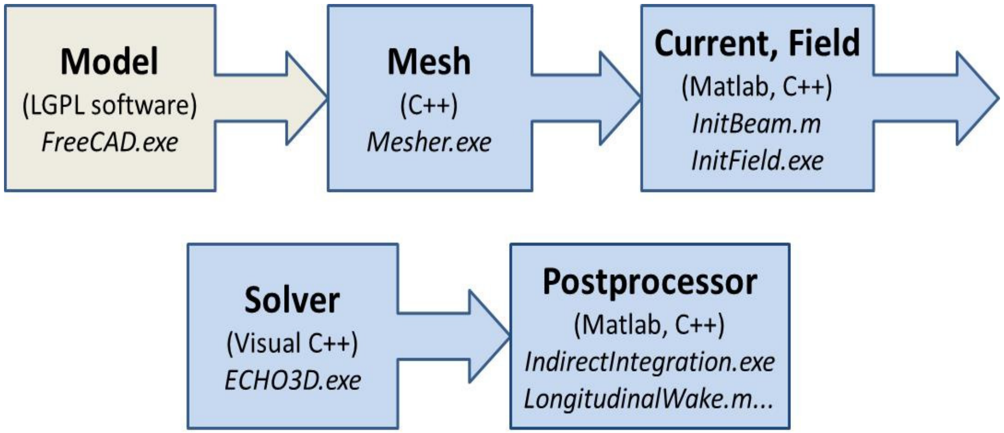


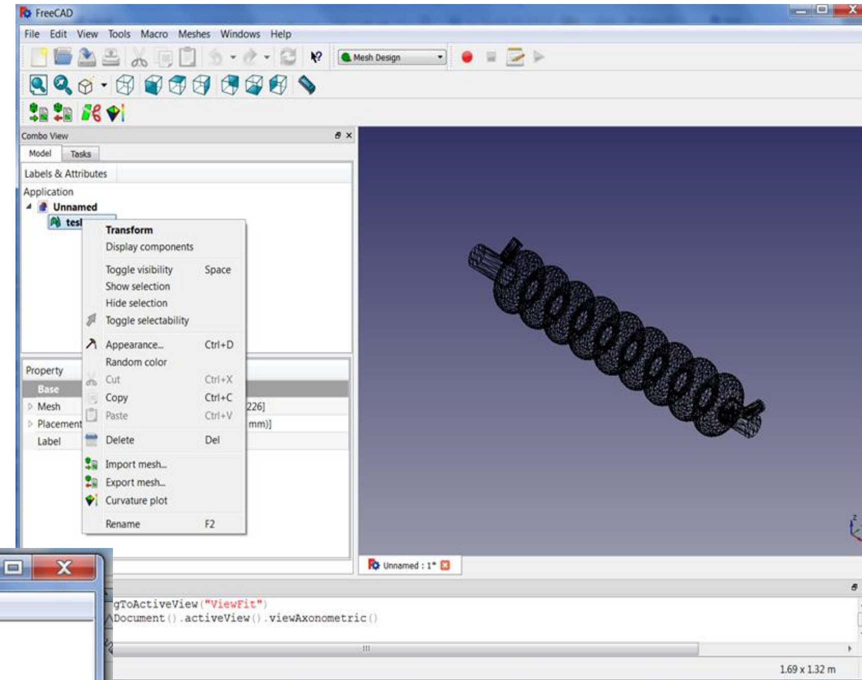
Wake	r[T3P]	r[ECHO]
Longitudinal, loss factor	0.84	0.83
Dipole, kick factor	1.08	0.79
Quad, kick factor	0.73	0.73

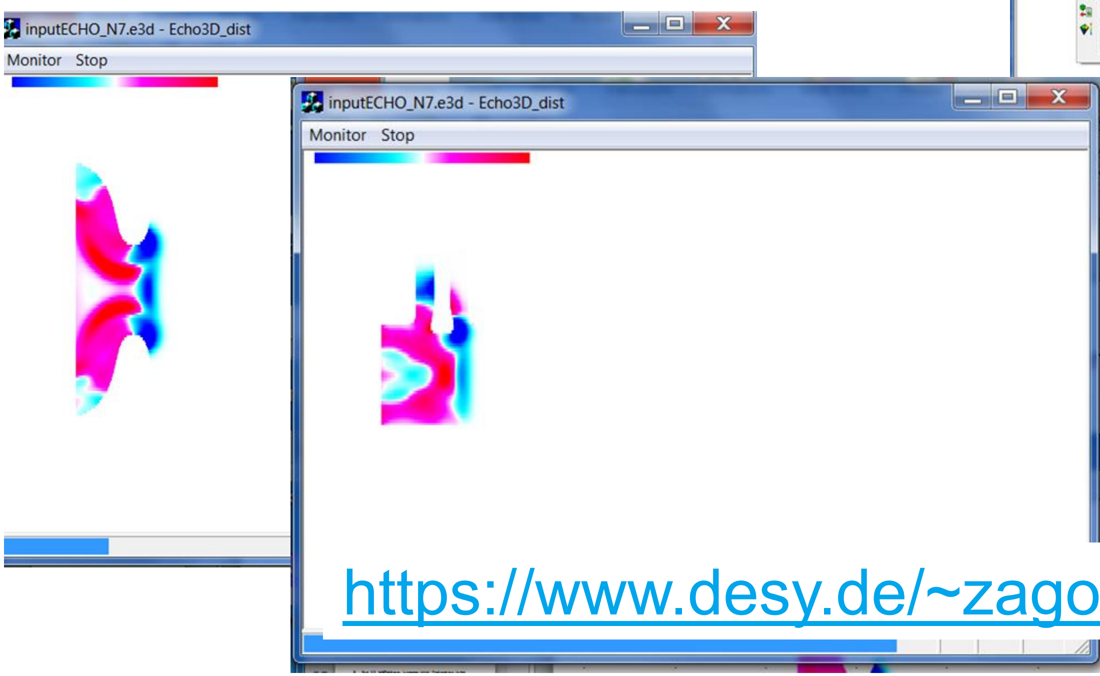
$$r = \frac{\text{measured}}{\text{analytical}} = 0.75$$

*P. Emma et al., Phys. Rev. Lett. **112**, 034801 (2014)

ECHO3D





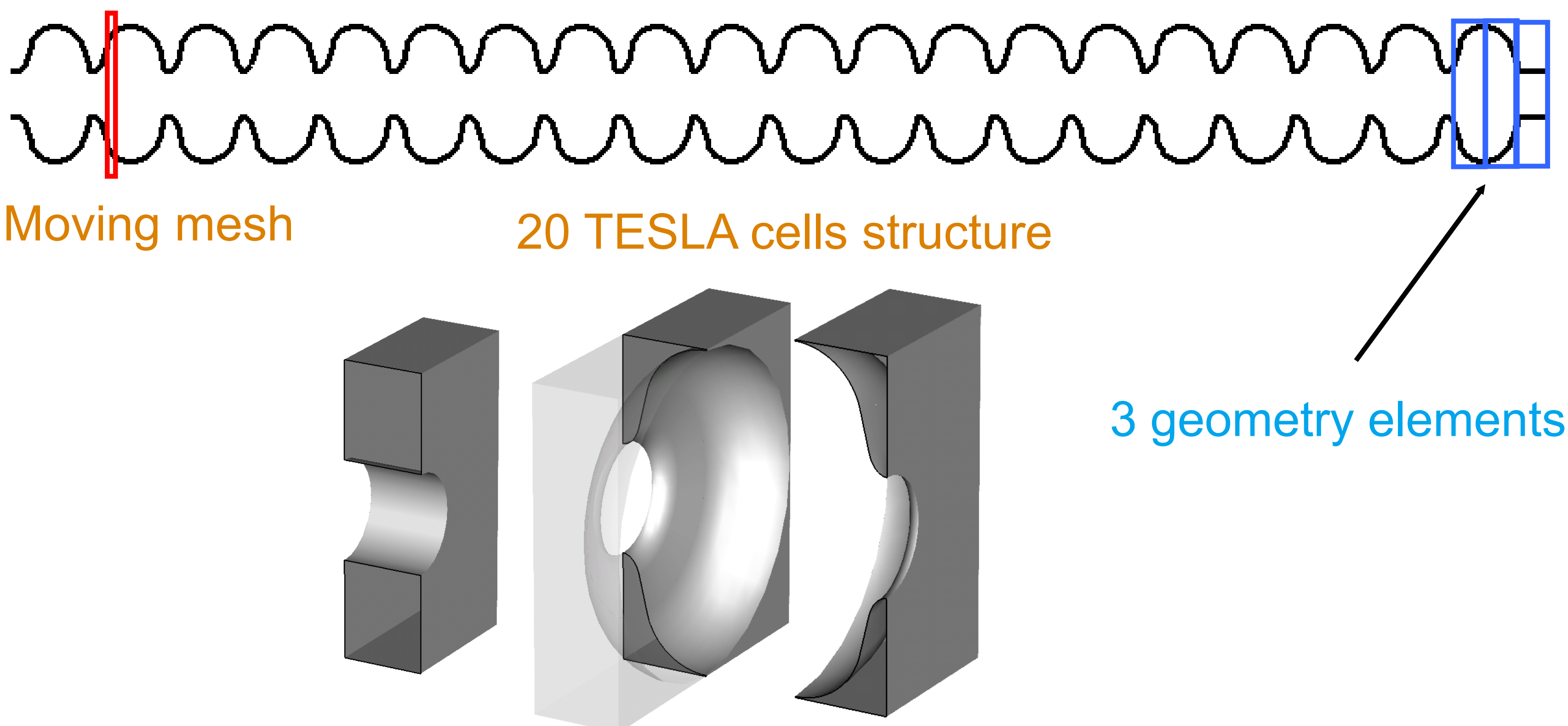


Arbitrary 3D geometry

- TE/TM ADI scheme
- Only PEC
- Serial version only

https://www.desy.de/~zagor/WakefieldCode_ECHOz/

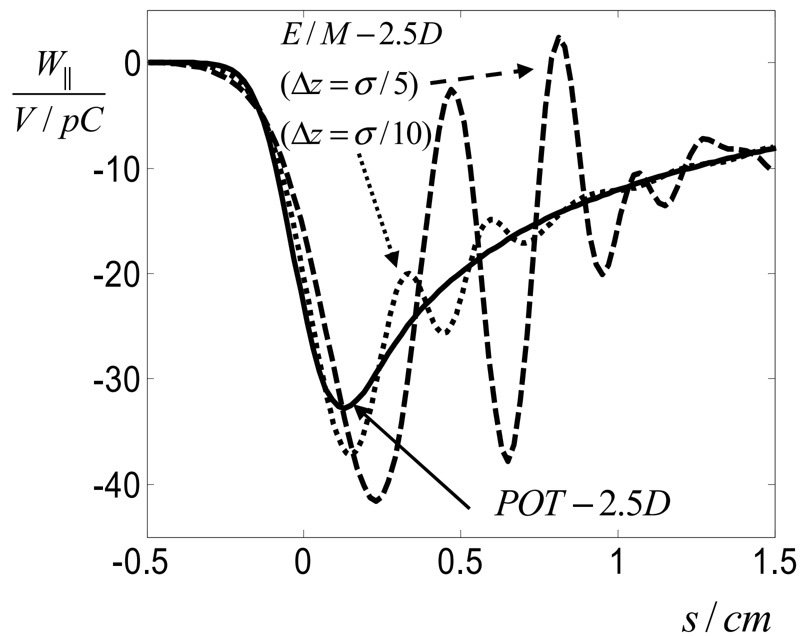
3D simulation. Cavity



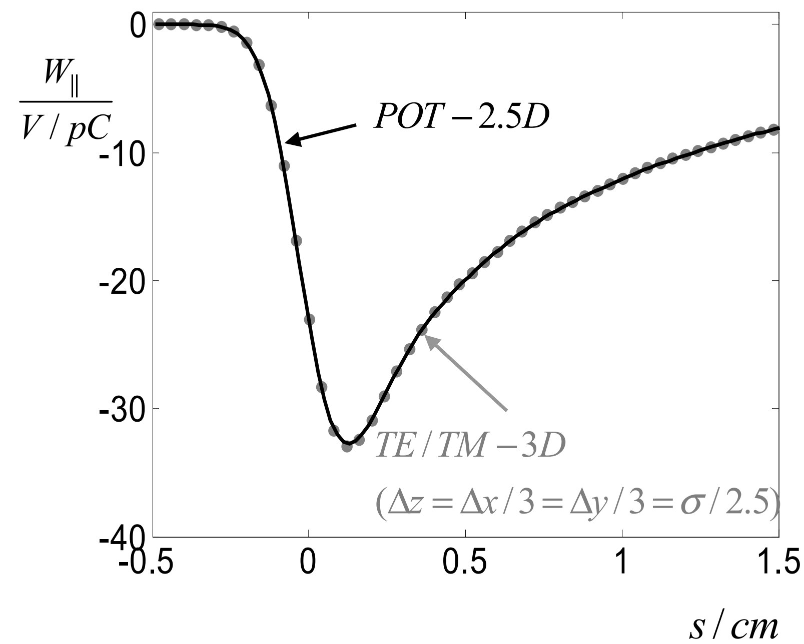
The geometric elements are loaded when the moving mesh reaches them. During the calculation only 2 geometric elements are in memory.

ECHO3D

E/M



ECHO-3D (TE/TM)



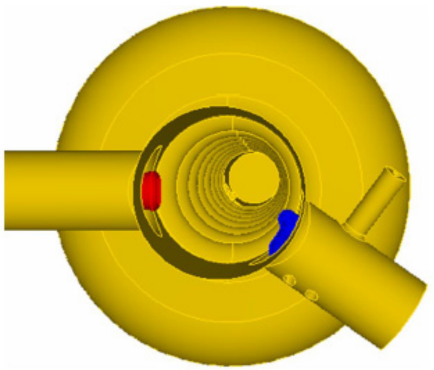
Comparison of the wake potentials obtained by different methods for structure consisting of 20 TESLA cells excited by Gaussian bunch $\sigma = 1mm$

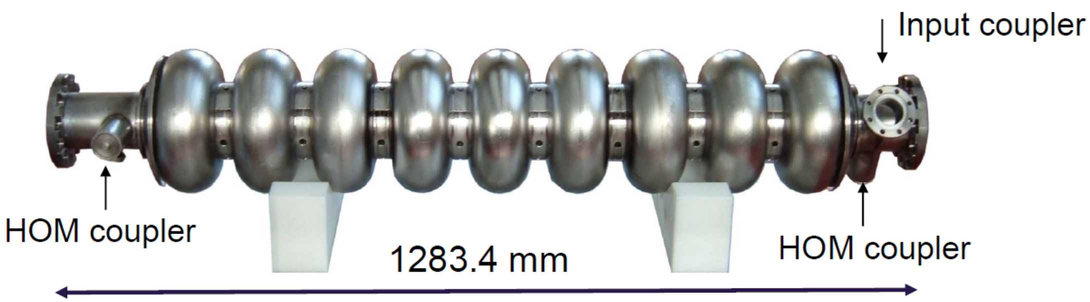
E/M splitting $\Delta z \sim \sqrt{\frac{\sigma^3}{L}}$

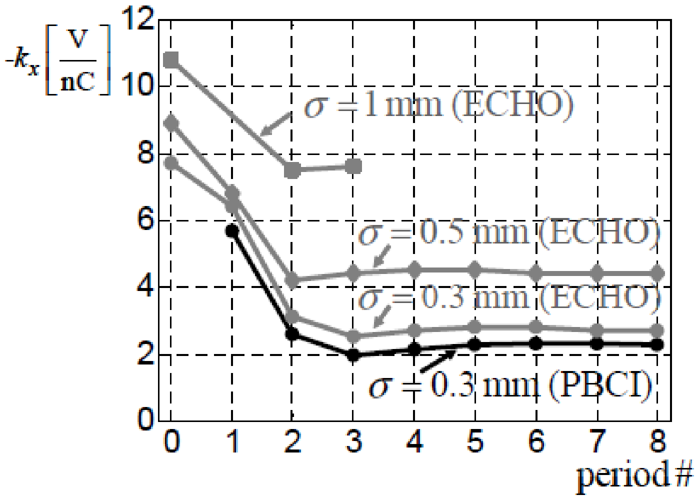
TE/TM splitting $\Delta z \sim \sigma$

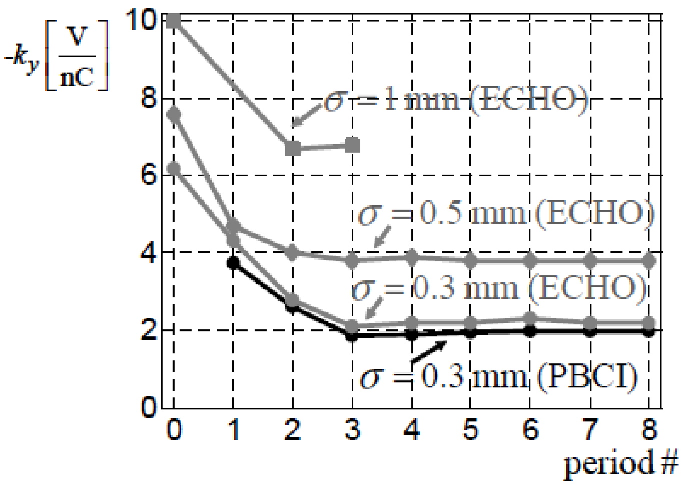
ECHO3D

Coupler Kick (One cryomodule ~ 12 meters)









M. Dohlus, I. Zagorodnov, E. Gjonaj, T. Weiland, ***Coupler Kick for Very Short Bunches and its Compensation***, Proc.EPAC 2008

ECHO-PIC

Stupakov G., ***Using pipe with corrugated walls for a subterahertz free electron laser***, PR-STAB **18**, 030709 (2015)

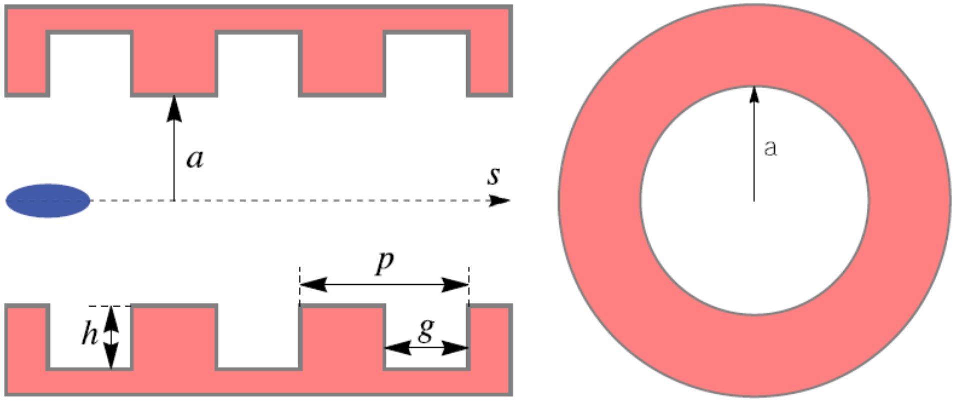
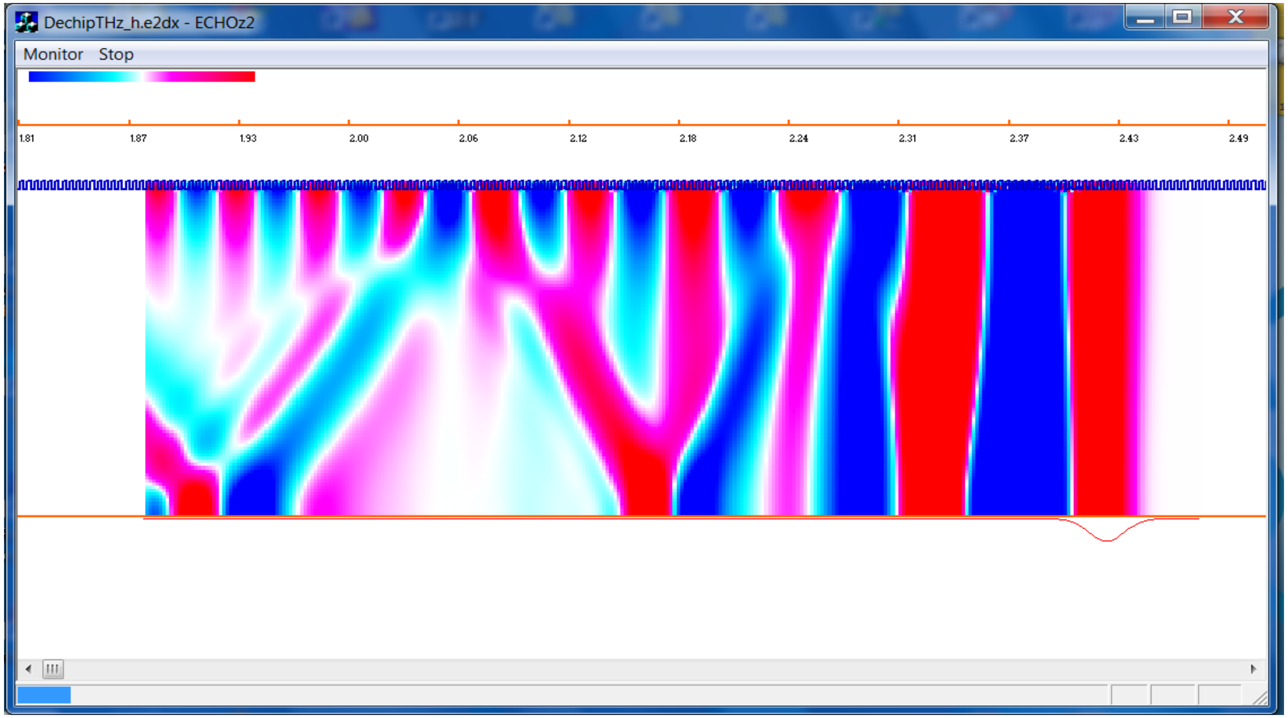


TABLE I. Corrugation and beam parameters.

Pipe radius (mm)	2
Depth h (μm)	50
Period p (μm)	40
Gap g (μm)	10
Bunch charge (nC)	1
Energy (MeV)	5
Bunch length (ps)	10

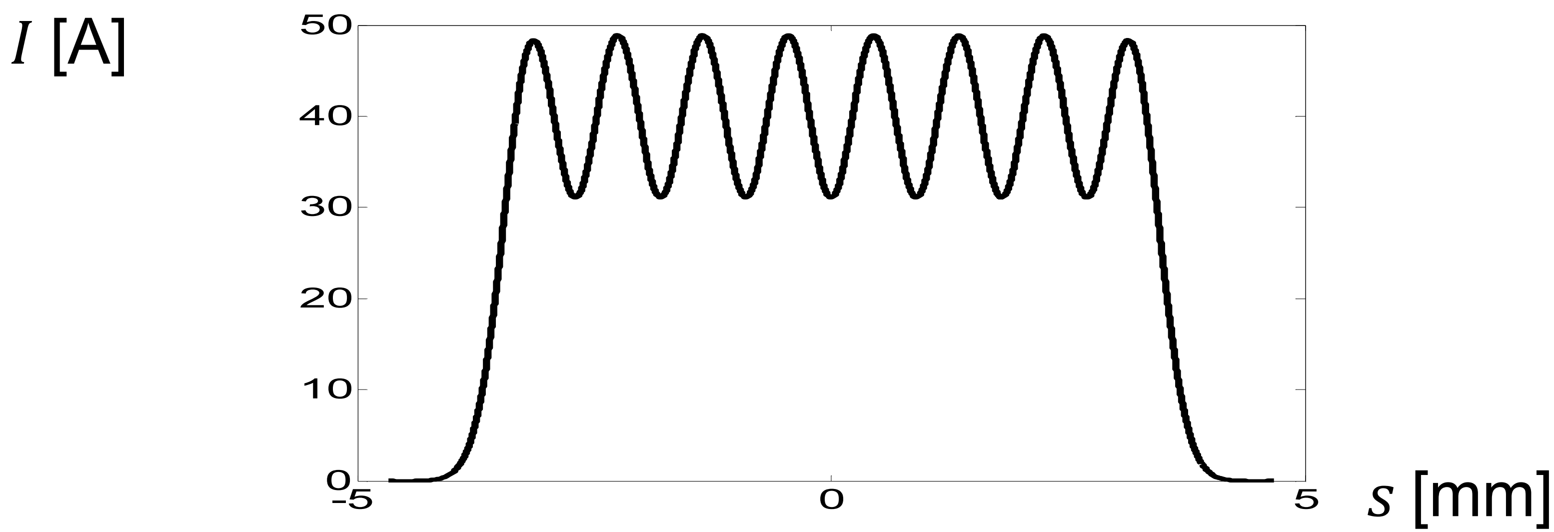
ECHO-PIC



Wakefield code ECHO (with resistivity) → Particle-in-Cell code ECHO-PIC (only longitudinal dynamics)

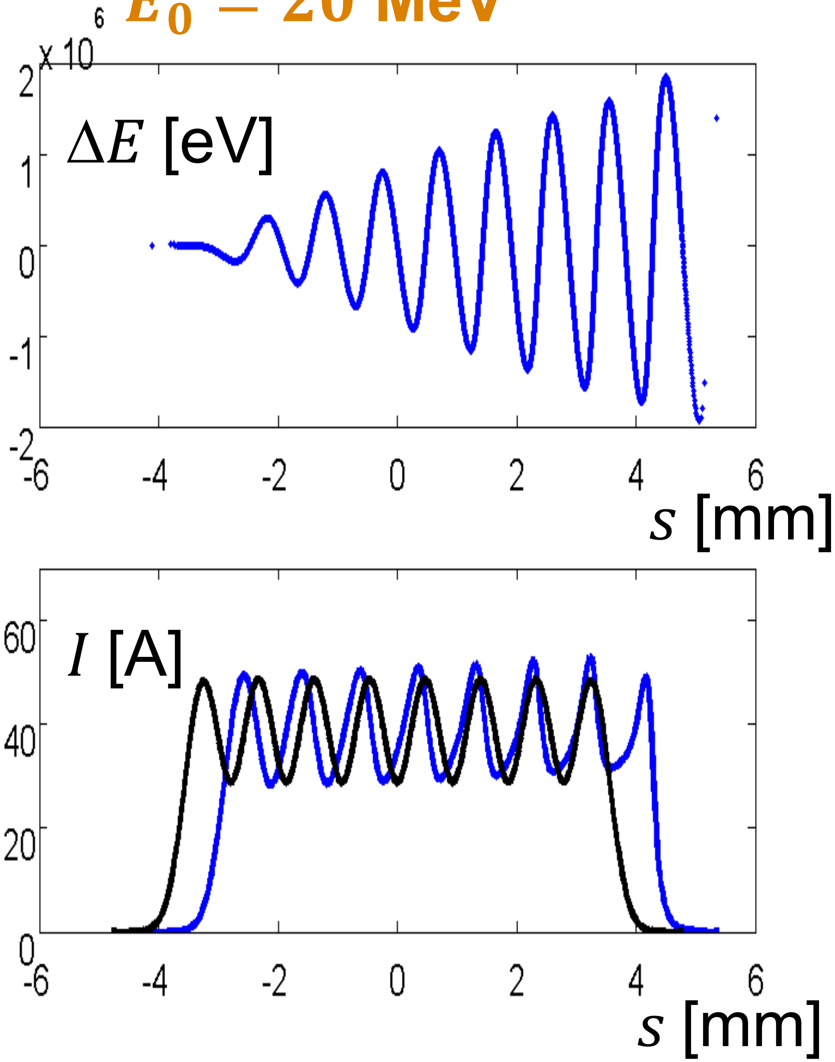
ECHO-PIC

Parameter	Value
Pipe length L, m	1-2
Bunch enegry E_0, MeV	5-20
Gausian bunch rms σ, mm	0.3*8
Charge Q, nC	0.96

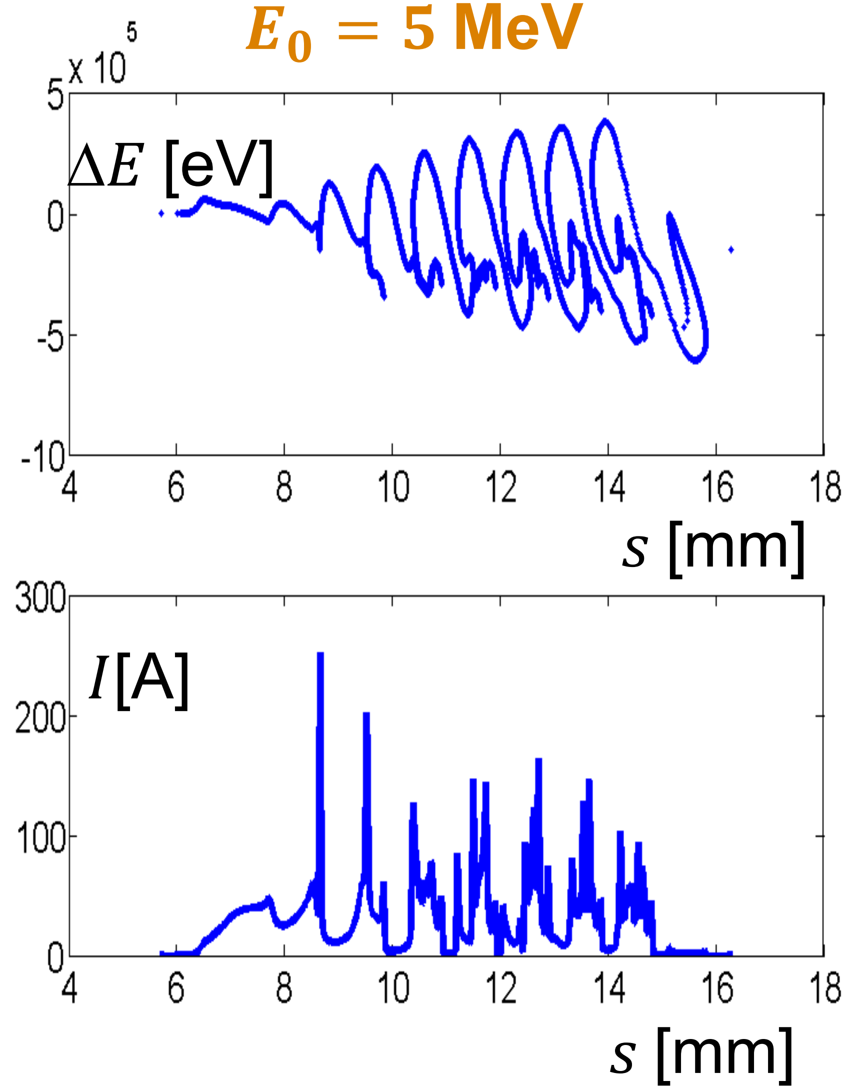


ECHO-PIC

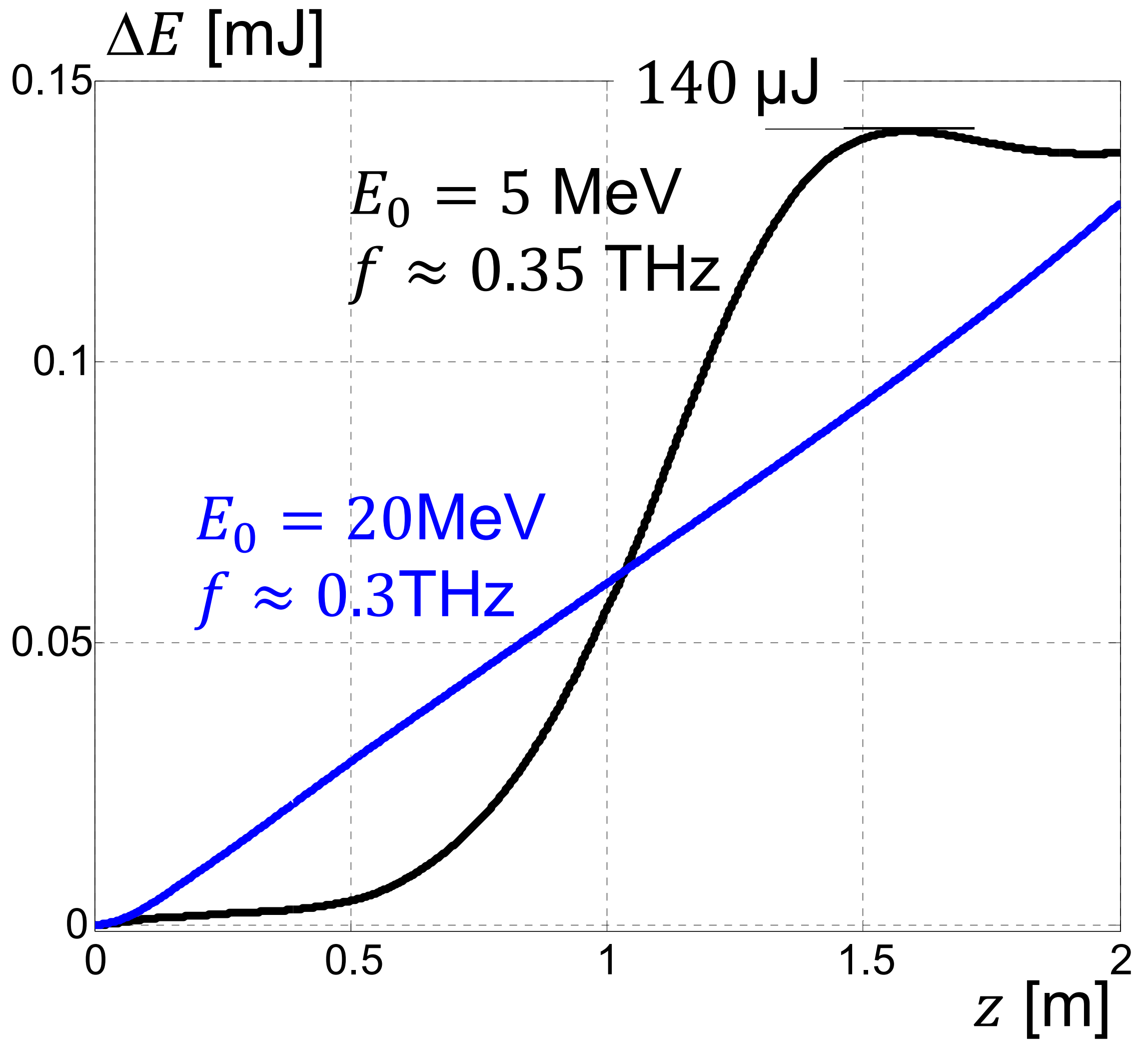
$E_0 = 20$ MeV



$E_0 = 5$ MeV

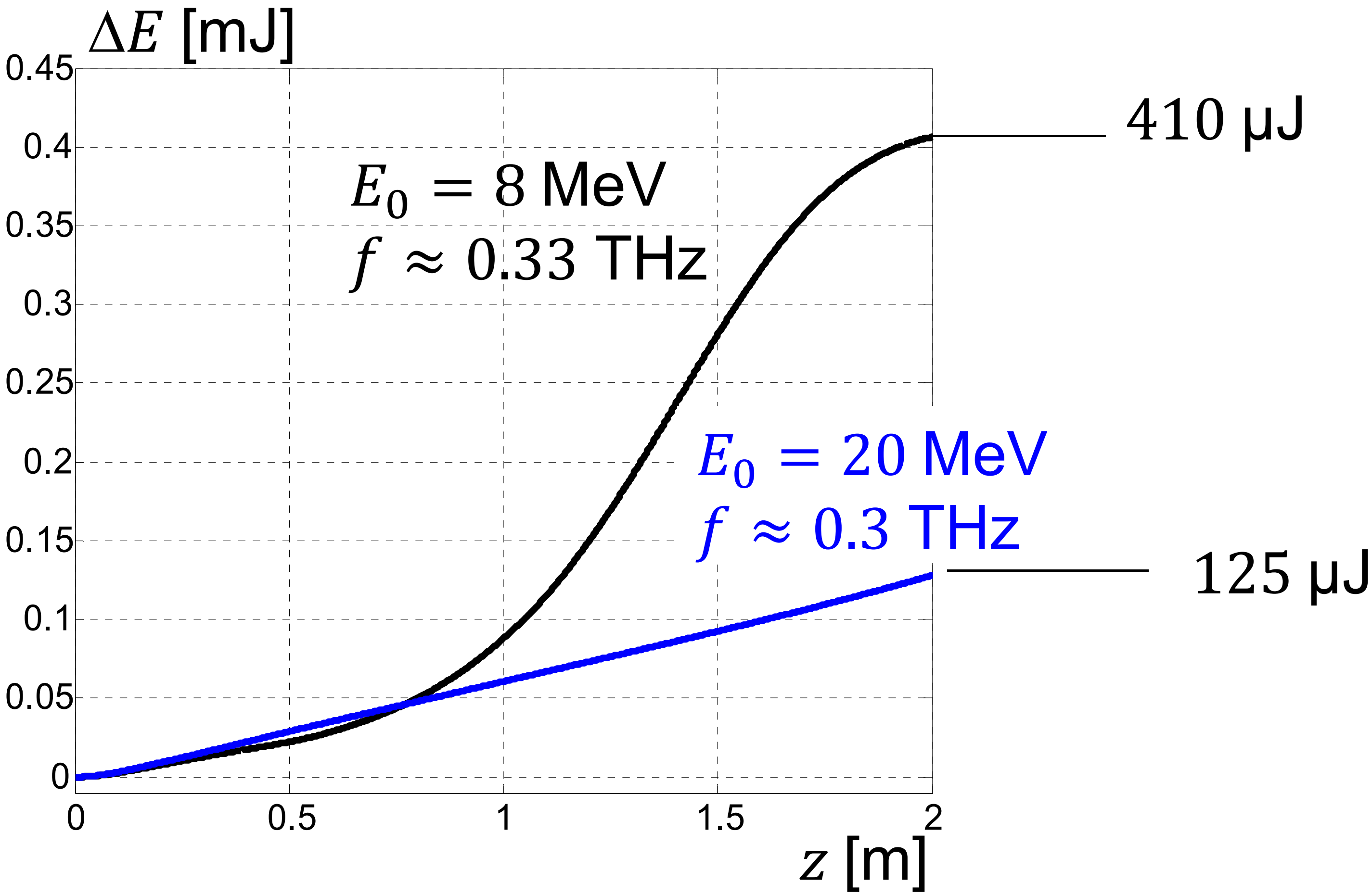


ECHO-PIC



ΔE [mJ]
0.15
0.1
0.05
0
140 μJ
E_0 = 5 MeV
$f \approx$ 0.35 THz
E_0 = 20MeV
$f \approx$ 0.3THz
0
0.5
1
1.5
2
z [m]

ECHO-PIC



ΔE [mJ]
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.5
1
1.5
2
z [m]
$E_0 = 8$ MeV
$f \approx 0.33$ THz
$E_0 = 20$ MeV
$f \approx 0.3$ THz
410 μJ
125 μJ