Optimization of Compton source Performance through Electron Beam Shaping

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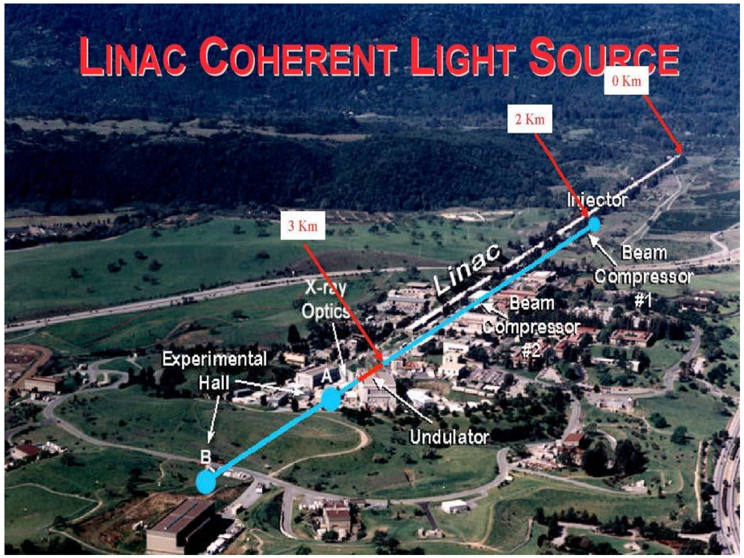
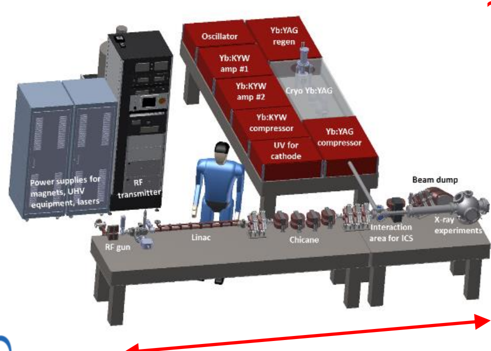
Advanced X-Ray Light Sources: FEL vs. ICS



~km

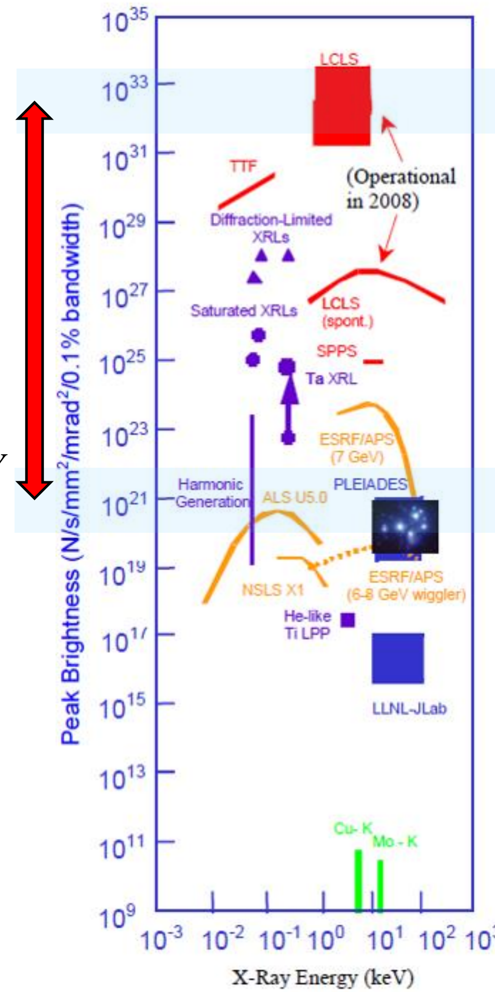
FEL typical Brightness
10^{30}-10^{33} ph/mm^2/mrad^2/s/0.1% BW

ICS typical Brightness
10^{20}-10^{22} ph/mm^2/mrad^2/s/0.1% BW



ICS source concept [2]

More than 10 orders of magnitude difference in Brightness



From [3]

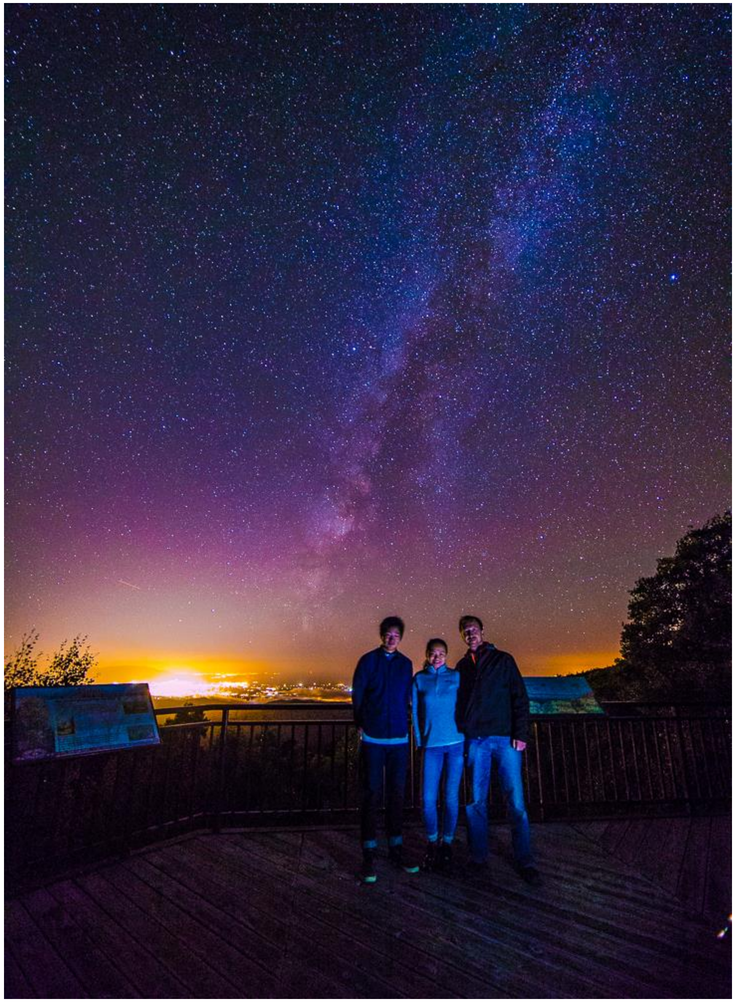
What is ~1 order difference of Brightness (number of photons)?

Exposure: 1 sec

What is ~1 order difference of Brightness (number of photons)?

Exposure: 1 sec

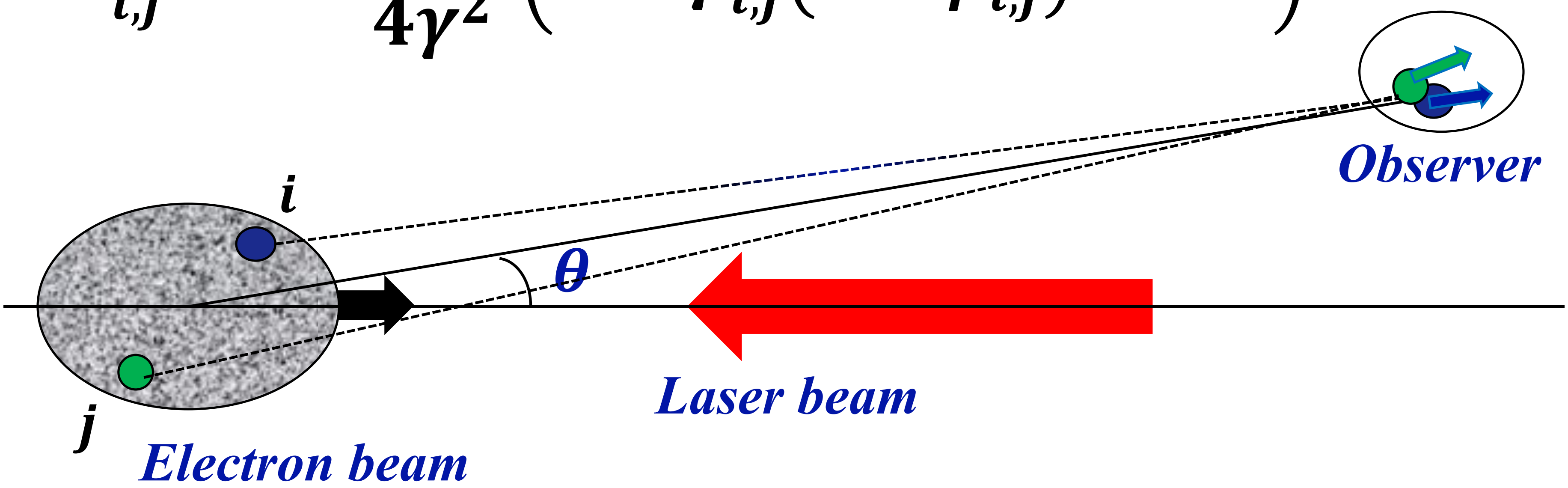
Exposure: 10 sec



ICS source is incoherent

Different electrons emit different photons

$$\lambda_{i,j}^{x-ray} = \frac{\lambda_L}{4\gamma^2}\left(1 + \gamma_{i,j}^2(\theta - \varphi_{i,j})^2 + \cdots\right)$$



Idea: squeeze the Bandwidth increase the peak Brightness

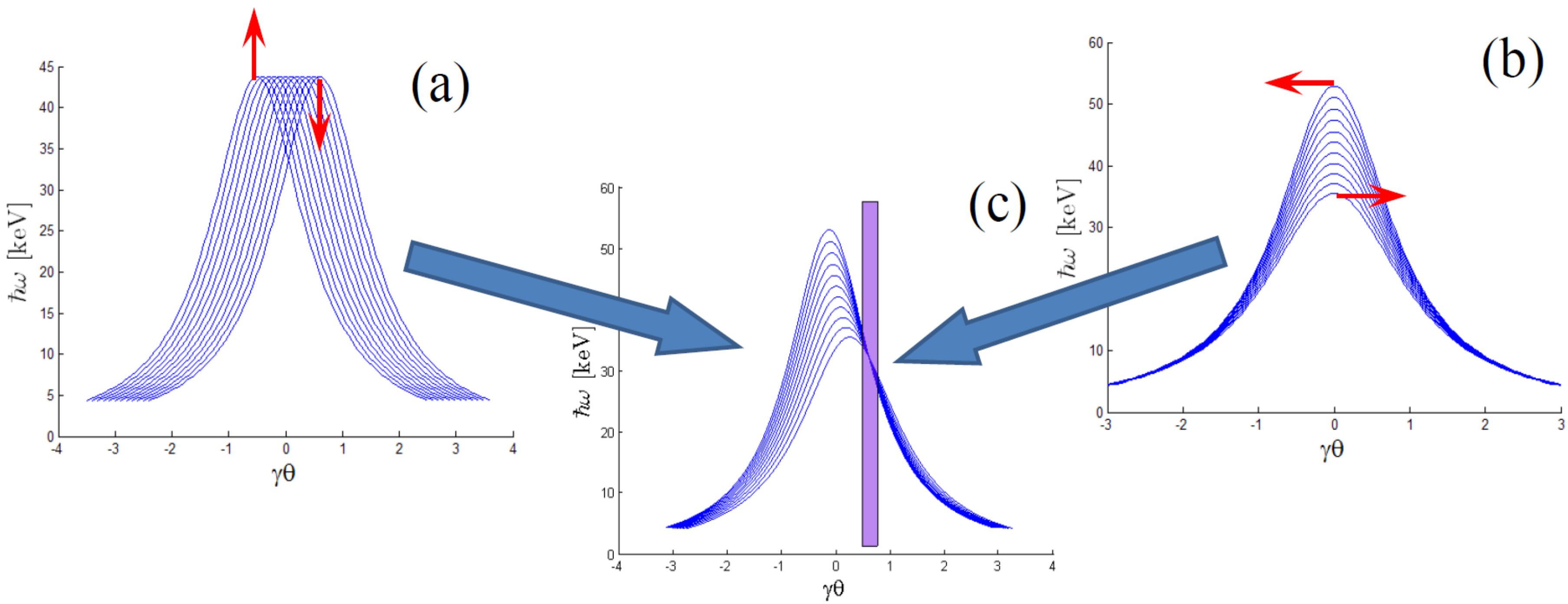
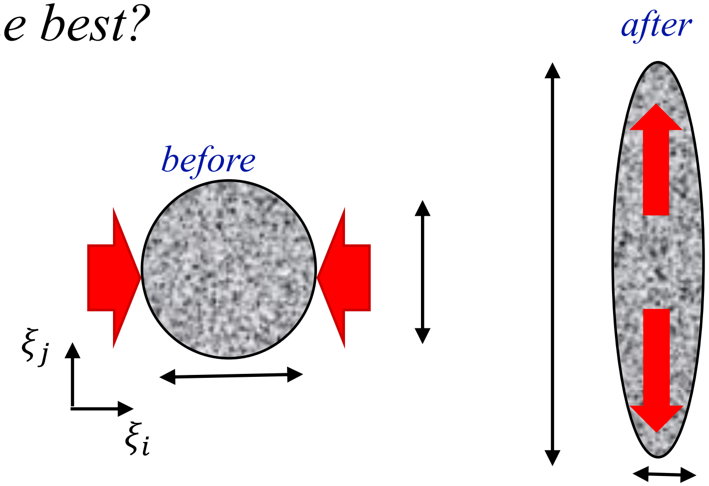


Fig. 2 ICS spectra emitted by different electrons due to (a) finite angular divergence, (b) finite energy spread, and (c) imposed x'-γ correlation in the beam.

Why do we need 6D model

Imposing an x'-γ correlation on the beam (via symplectic transformation) will create other correlations. Some of them may even increase the bandwidth. Which one is the best?



Brightness depends not only from the bandwidth, but from all other intervals as well.

Hope: squeezing the bandwidth interval via beam conditioning would mostly increase the Peak Brightness rather than spread the phase space in other directions

We need: Brightness as a 6-dimensional photon distribution function in the phase, dependent from 6d electron beam distribution :

$$B_{6d}(\Sigma^e{}_{6d}) \text{ - ?}$$

6+6=12 dimensions
Ideally we would want to have an answer in a Matrix Form
Don't worry ..Just 6d Matrix

Wigner function in 6d to characterize Brightness

Spectral Brightness Definition (Number of photons per 4d phase space): $B(x, y, \phi_x, \phi_y; z) = \dfrac{d^4F}{\mathrm{dx\ dy\ d}\phi_x \mathrm{d}\phi_y}$

4d Wigner function to characterize Brightness introduced for radiation sources by K.J.Kim in 1986 [4] as an analog of QM Wigner function:

$$W_{4d}(x, \xi_x, y, \xi_y) = \mathrm{W0} \int_{-\infty}^{\infty} \mathrm{d}\xi_x \int_{-\infty}^{\infty} \mathrm{d}\xi_y E\left(x + \frac{\xi_x}{2}, y + \frac{\xi_y}{2}\right) E^*\left(x - \frac{\xi_x}{2}, y - \frac{\xi_y}{2}\right) \mathrm{e}^{-\mathrm{i}k_x\xi_x - \mathrm{i}k_y\xi_y},$$

We would like to describe bandwidth changes, so we need to add two more dimensions:

Wigner function extension to 6d phase space (2 approaches)*

Radiation propagation can be described by evolution variable: t or z

$$W_{6d} = \mathrm{W0} \int_{-\infty}^{\infty} \mathrm{d}\,\vec{\xi}_{3d}\, E\left(\vec{r} + \frac{\vec{\xi}}{2}; t\right) E^*\left(\vec{r} - \frac{\vec{\xi}}{2}; t\right) \mathrm{e}^{-\mathrm{i}\vec{k}_w\vec{\xi}}, \ \vec{\xi}_{3d} = \{\xi_x, \xi_y, \xi_z\},$$

$$W_{6d} = \mathrm{W0} \int_{-\infty}^{\infty} \mathrm{d}\xi_t \int_{-\infty}^{\infty} \mathrm{d}\,\vec{\xi}_{2d}\, E\left(\vec{r} + \frac{\vec{\xi}}{2}, t + \xi_t; z\right) E^*\left(\vec{r} - \frac{\vec{\xi}}{2}, t - \xi_t; z\right) \mathrm{e}^{-\mathrm{i}\vec{k}_w\vec{\xi} - \mathrm{i}w_w\xi_t}; \ \vec{\xi}_{2d} = \{\xi_x, \xi_y\}$$

Autocorrelation function of the field

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Brightness convolution theorem

Approach: different electrons radiate incoherently

$$W_{\mathrm{beam}}(\vec{\xi}_{\mathrm{ph}}) = \int W_{1e}\left(\vec{\xi}_e, \vec{\xi}_{\mathrm{ph}}\right) f(\vec{\xi}_e)\, \mathrm{d}^4 \vec{\xi}_e\,,$$

Simplification in 4d:

$$W_{\mathrm{beam}}(\vec{\xi}_{\mathrm{ph}}) = \int W_{1e}\left(\vec{\xi}_e - \vec{\xi}_{\mathrm{ph}}\right) f(\vec{\xi}_e)\, \mathrm{d}^4 \vec{\xi}_e\,,$$

Would this simplification work in 6D?

$$W_{1e}\left(\vec{\xi}_e, \vec{\xi}_{\mathrm{ph}}\right) \longrightarrow W_{1e}\left(\vec{\xi}_e - \vec{\xi}_{\mathrm{ph}}\right)$$

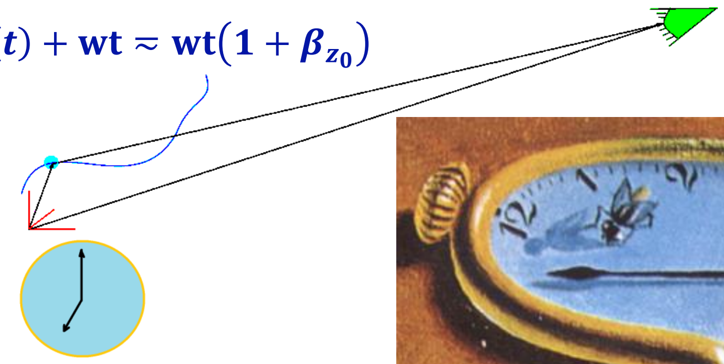
Hint: $\frac{\Delta \mathrm{w}}{w} \sim 2 \frac{\Delta \gamma}{\gamma}$

Single electron interacting with plane wave

Equations of motion of an electron: $(\vec{E} = E_0\,\vec{x}_0,\ \vec{k} = -kz\,\vec{z}_0)$:

Time transformation (squeezing)

$$\boldsymbol{\phi = kz(t) + wt \approx wt(1+\beta_{z_0})}$$



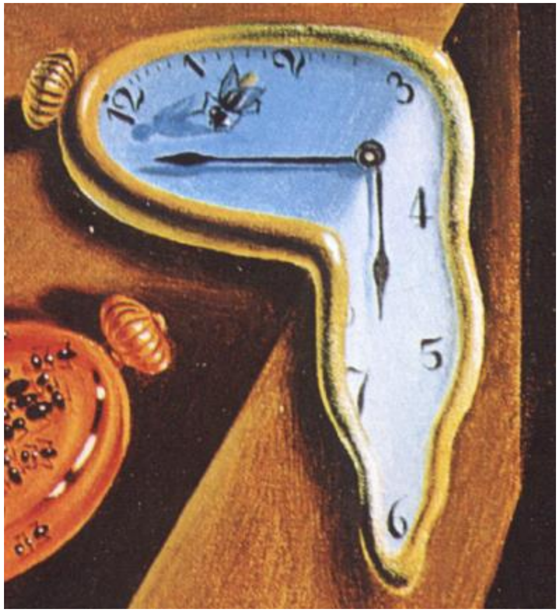
$$\frac{w}{c}(x-x_0) = -a_0\frac{\mathrm{mc}}{\mathrm{c1}}\cos\phi + \phi\frac{\mathrm{c2}}{\mathrm{c1}}$$

Linear approximation:

$$\frac{w}{c}(y-y_0) = \phi\frac{\mathrm{c4}}{\mathrm{c1}}$$

$$\boldsymbol{a_0 = \frac{eE_0}{mcw},\ a_0{}^2 \ll 1}$$

$$\frac{w}{c}(z-z_0) = \frac{a_0{}^2}{8}\frac{(\mathrm{mc})^2}{\mathrm{c1}^2}\sin2\phi + a_0\frac{\mathrm{mc}}{\mathrm{c1}}\frac{\mathrm{c2}}{\mathrm{c1}}\cos\phi + \phi\frac{\mathrm{c3}}{\mathrm{c1}}$$



Radiation field (far field):

$$\vec{E} = \frac{e}{c^2R}\left[\vec{n}\times\left[\vec{n}\times\frac{\mathrm{d}^2\vec{r}\,[t'(t)]}{\mathrm{d}t^2}\right]\right],$$

$$\boxed{\boldsymbol{E_{rad} = E_{rad_0}\cos\left(\frac{(1+\beta_{z_0})}{1-\vec{\beta}_0\,\vec{n}}\,w\left(t-\frac{\vec{R}\,\vec{n}}{c}\right)\right)}}$$

$$E_{\mathrm{rad}_0} \approx \frac{1}{2\gamma_0{}^5}\frac{e^2}{Rmc^2}E_0$$

$$\boldsymbol{t - \frac{1}{c}\sqrt{(\vec{R}-\vec{r}_0)^2} \approx Lt'}$$

$$\mathrm{L} = 1 - (\vec{\beta}_0\,\vec{\mathrm{n}}) \approx \frac{1}{2\gamma_0{}^2} \ll 1$$

We assume only E_x is a non zero component for an observer close to the axis

Wigner of a single electron interacting with a laser field (1d Theory*)

Laser field as a homogeneous continuous plane wave

$$W_{1e} \sim \delta\left(\frac{x - x_e}{z} - \phi_x\right) \delta\left(\frac{y - y_e}{z} - \phi_y\right) \delta(w - w_{\mathrm{rad}})$$

**We were able to get a Wigner function for the beam with it however it was not recognized as an appropriate answer: combination of several matrixes 3*3 – blocks of the initial matrixes 6*6*

Laser field as a homogeneous time-limited plane wave (Gaussian pulse, N_{osc}>>1):

$$\boxed{W_{1e} \sim \delta\left(\frac{x - x_e}{z} - \phi_x\right) \delta\left(\frac{y - y_e}{z} - \phi_y\right) e^{-\frac{(w - w_{\mathrm{rad}})^2}{1/T_{\mathrm{rad}}{}^2}} e^{-\frac{\left(\frac{z}{c} + t\right)^2}{T_{\mathrm{rad}}{}^2}}}$$

*3d theory for the realistic laser beam was analytically derived – please ask questions if interested

Wigner of a single electron in 6d matrix-form

Wigner of a single electron in 6d matrix-form:

Canonical coordinates

$$W_{1e} = W_0 \frac{1}{\sqrt{\pi}\delta_x} \frac{1}{\sqrt{\pi}\delta_y} e^{-\left(\vec{\xi}_{\mathrm{ph}}{}^{i} - \vec{\xi}_e\right)^T M_0 \left(\vec{\xi}_{\mathrm{ph}}{}^{i} - \vec{\xi}_e\right)}$$

$$\xi_e{}^c = \begin{pmatrix} x_e \\ \Delta\beta_x \\ y_e \\ \Delta\beta_y \\ z_e \\ \Delta\beta_z \end{pmatrix} \qquad \xi_{\mathrm{ph}} = \begin{pmatrix} \Delta x \\ \frac{\Delta k_{w_x}}{k_{w_{z0}}} \\ \Delta y \\ \frac{\Delta k_{w_y}}{k_{w_{z0}}} \\ \Delta z \\ \frac{\Delta k_{w_z}}{k_{w_{z0}}} \end{pmatrix}$$

$$\left(\vec{\xi}_{\mathrm{ph}}, \vec{\xi}_e\right) \to \cancel{\left(\vec{\xi}_{\mathrm{ph}}{}^{c} - \vec{\xi}_e{}^{c}\right)}$$

$$\searrow \left(\vec{\xi}_{\mathrm{ph}}{}^{c} - \vec{\xi}_e\right)$$

$$\Omega = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2\gamma_0{}^2 \end{pmatrix}$$

$$\vec{\xi}_e = \Omega\, \vec{\xi}_e{}^{c},$$

Ω - non-symplectic matrix

**Convolution Theorem fails to be simplified in canonical coordinates*

Wigner of an electron beam interacting with laser beam

$$W_{\text{beam}} = W_0 \lim_{\delta_x,\delta_y \to 0} \left\{ \frac{\sqrt{\text{Det}[\Sigma^{-1}]}}{\delta_x \delta_y \sqrt{\text{Det}[M_0 + \Sigma^{-1}]}} \text{e}^{-\left\{ \vec{\xi}_{\text{ph}}^{\,i\,T} (M_0 - M_0 M^{-1} M_0) \vec{\xi}_{\text{ph}}^{\,i} \right\}} \right\}$$

$M_0 = M_x + M_y + M_w + M_t$

M_x and M_y *have singularities*

$M = M_0 + \Sigma^{-1}$

Peak Brightness ~

$$\lim_{\delta_x,\delta_y \to 0} \left\{ \frac{\sqrt{\text{Det}[\Sigma^{-1}]}}{\delta_x \delta_y \sqrt{\text{Det}[M_0 + \Sigma^{-1}]}} \right\}$$

exists for an arbitrary electron beam matrix Σ

Next step is find it in a 6d Matrix form and find an optimum Σ_e maximizing the peak Brightness

Peak Brightness (Observer off-axis)

$$\mathrm{B}_{\mathrm{peak}} \sim \frac{1}{\sqrt{\mathrm{Det}[QM\,\Sigma + Q]}}$$

$QM = Q_0 \cdot$ M – matrix product of two matrixes with singularities → $\boldsymbol{\Sigma, QM, Q}$ $\boldsymbol{- matrixes\ with\ no\ singularities}$

$$Q_0 = \begin{pmatrix} \alpha & 0 & \beta - \frac{{\delta_y}^2 \alpha\beta}{\alpha+\beta} & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ {\delta_x}^2 + \alpha & 0 & \beta & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{\alpha+\beta}{\beta} & 0 & -\frac{\alpha+\beta}{\alpha} & 0 & \frac{{\delta_x}^2{\delta_y}^2 + \alpha + \beta}{\alpha\beta} & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}; \quad Q = \begin{pmatrix} \alpha & 0 & \beta & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \alpha & 0 & \beta & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\frac{\alpha+\beta}{\beta} & 0 & -\frac{\alpha+\beta}{\alpha} & 0 & \frac{1}{\alpha} + \frac{1}{\beta} & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$\alpha = \varphi_x,\ \beta = \varphi_y$ - angles to the observer

Peak Brightness (Undulator case + uncorrelated beam)

"Undulator case ": observer, electron beam and laser beam are on z-axis"

Uncorrelated beam: no correlation between xx'-yy'-zz'

$$B_{\text{peak}} \sim \frac{1}{\sqrt{\sigma_x^2 \sigma_y^2 \left(\frac{\sigma_z^2}{\sigma_{rad}^2} + k_{rad}^2 \epsilon_z^2\right)}} \Rightarrow$$

To maximize the brightness one need to compress the beam in the transverse plane;

Longitudinal size or emittance dominated regimes are possible depending on the beam parameters

$$k_{\text{rad}} = \frac{1+\beta_{z_0}}{1-\vec{\beta}_{z_0}\,\vec{n}} k_{las} \approx 4\gamma_0^2 k_{las}$$

$$\sigma_{\text{rad}} = \frac{1-\vec{\beta}_{z_0}\,\vec{n}}{1+\beta_{z_0}} \sigma_{las} \approx \frac{\sigma_{las}}{4\gamma_0^2}$$

Peak Brightness (Initial proposed case: x’-γ)

Observer, and laser beam are on z-axis

Electron beam upcoming with x- an angle to z-axis: $(\beta_x, \beta_y, \beta_z)$

$$\Sigma_{\max} = \begin{pmatrix} \frac{A^2}{k_{\mathrm{rad}}{}^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_x{}^2 & 0 & 0 & 0 & -\frac{\alpha_x{}^2\beta_{\mathrm{x0}}}{-1+\beta_{\mathrm{z0}}} \\ 0 & 0 & \frac{B^2}{k_{\mathrm{rad}}{}^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_y{}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\mathrm{rad}}{}^2 & 0 \\ 0 & -\frac{\alpha_x{}^2\beta_{\mathrm{x0}}}{-1+\beta_{\mathrm{z0}}} & 0 & 0 & 0 & \frac{\alpha_x{}^2\beta_{\mathrm{x0}}{}^2}{(-1+\beta_{\mathrm{z0}})^2}+\frac{1}{k_{\mathrm{rad}}{}^2\sigma_{\mathrm{rad}}{}^2} \end{pmatrix};$$

Where A and B as small as possible, α_x and α_y as big a as possible

Correlation required: x’-γ

From uncorrelated beam via R_{62}:
Via drift + transverse deflecting cavity

Benefits of 6d Theory

- Can derive actual ICS radiation for any correlated in the phase space beam, even for the beam from advanced accelerators (DLA, Plasma/Dielectric Wake Field, etc.) – one step closer to the compact, high brightness X-ray source
- Increasing Brightness of the ICS source via electron beam shaping
- Extension to the FEL case
- Electron beam 6d phase space diagnostic via ICS:

$$\mathbf{e}^{-\left\{\vec{\xi}_{\mathrm{ph}}{}^{i^{T}}\left(M_0 - M_0 M^{-1} M_0\right)\vec{\xi}_{\mathrm{ph}}{}^{i}\right\}}$$

References

1. J. Galayda, LCLS presentation to 20-Year BES Facilities subcommittee, Feb. 2003
2. Compact x-ray source based on burst-mode inverse Compton scattering at 100 kHz - Graves, W.S. et al.Phys.Rev.ST Accel.Beams 17 (2014) no.12, 120701 arXiv:1409.6954 [physics.acc-ph]
3. J. Kuba, et al., "PLEIADES: High Peak Brightness, Subpicosecond Thomson Hard-X-ray Source", Proceedings of the Society of Photo-Optical Instrumentation Engineers (SPIE) 4978, 101 (2003).
4. Characteristics of Synchrotron Radiation, K.J.Kim, 1989, Berkley, CA

Additional information

- **Why do we need X-Ray light Sources?**
- **Laser 3D Theory**

Why do we need X-Ray light Sources?

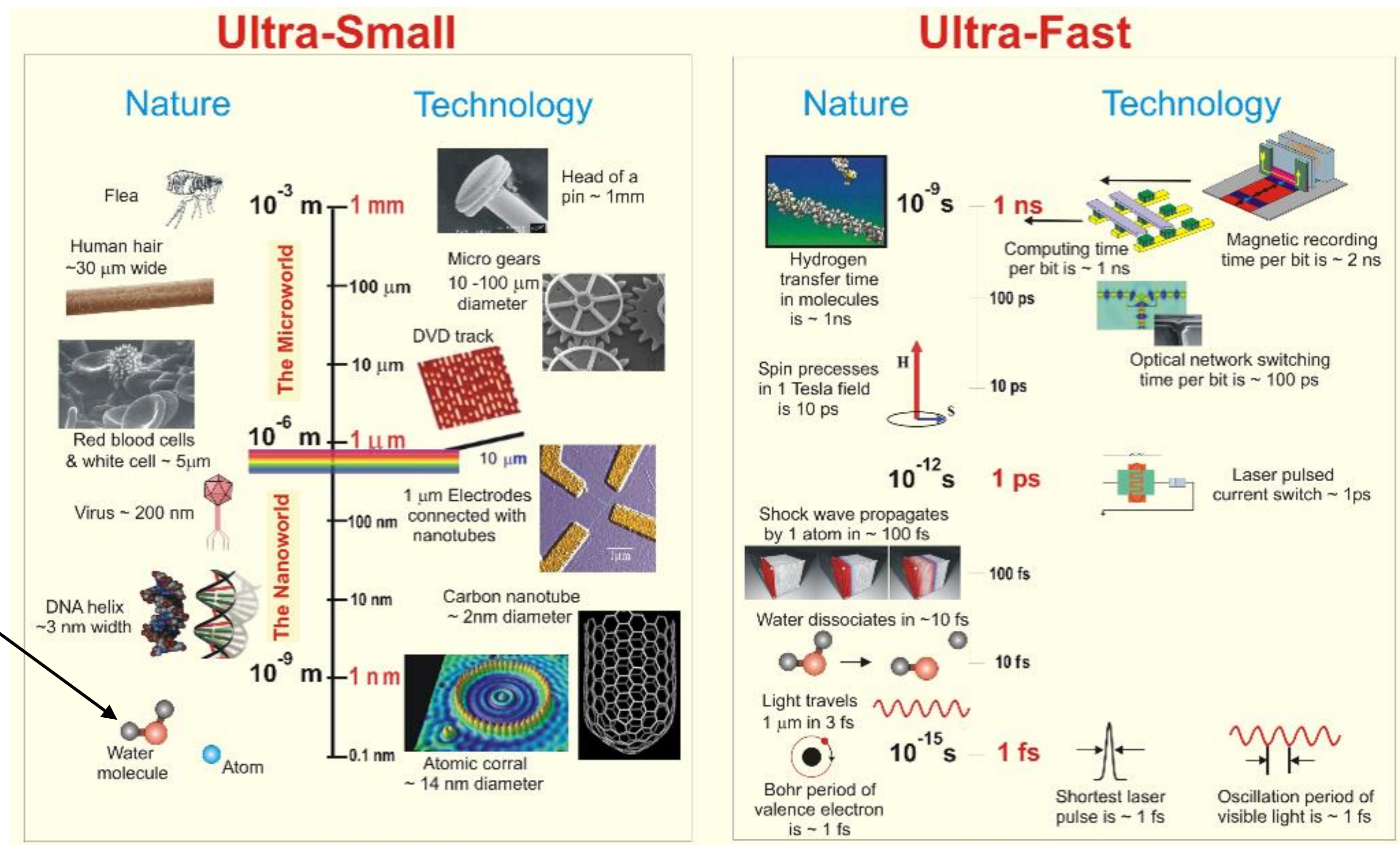
To discover and explore: material science, biology, medicine, etc.

Imagine to watch a video how two hydrogen atoms and one atom of oxygen become water in a real time… with high resolution … and even in 3D…

$$H_2 + O = H_2O$$

~ 10^9 $

"Science is priceless, for everything else there is …"



From [1]

Wigner of a single electron interacting with a laser field (3d Theory)

A problem: for strongly focused beam, homogeneous approximation will fail…

Typical approach: $E_{\mathrm{L}}(x, y, \frac{z}{c} + t)\mathrm{e}^{-i(\mathrm{kz+wt})}$, *but off axis the plane wave would travel with an angle to z axis*

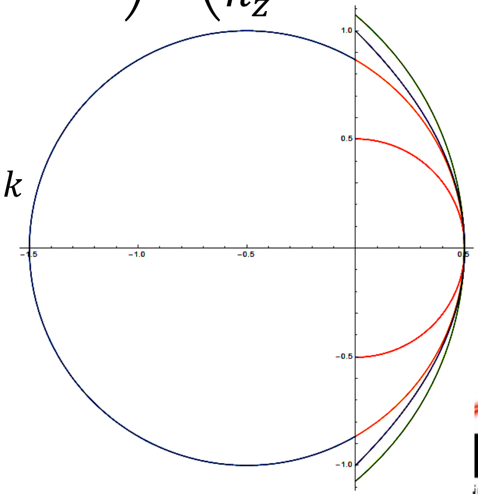
Laser field as a continuous superposition of plane waves with different direction and amplitude

We already know the answer for each mode: $E_{\mathrm{rad1mode}} = \alpha_f E_{\mathrm{k0}}(\vec{k})\mathrm{e}^{-\mathrm{i}\vec{q}(\vec{k})\vec{r}+\mathrm{i}q(\vec{k})ct}$

Approach: *single electron with different laser modes radiates coherently*

$$W \sim \delta\left(\frac{n_x}{n_z} k_{w_z} - k_{w_x}\right)\delta\left(\frac{n_y}{n_z} k_{w_z} - k_{w_y}\right)\int_{-\infty}^{\infty} \ldots \delta\left(\frac{q_z(\vec{k}) + q_z(\vec{k'})}{2} - k_{w_z}\right) \mathbf{d}^3\,\vec{k}\;\; \mathbf{d}^3\,\vec{k'}$$

$$\vec{q}\,(\vec{k}) = \frac{1 - \vec{\beta}_0\,\vec{k}}{1 - \vec{\beta}_0\,\vec{n}}\,\vec{n}\,k$$

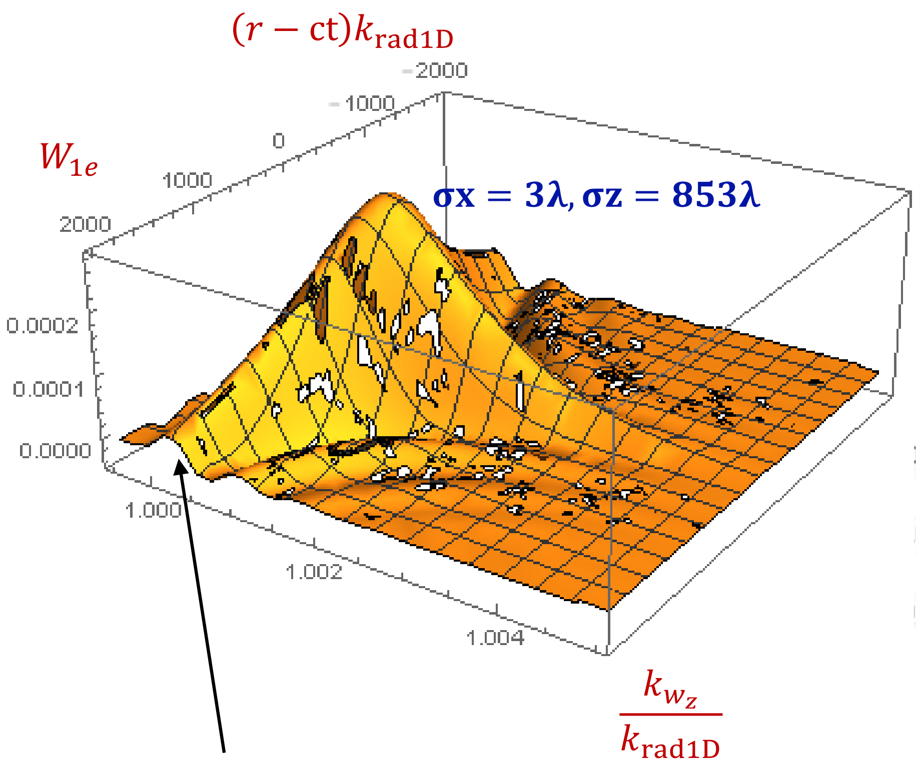


From 6d Integral we can get 1d integral by changing the integration over an ellipsoid to paraboloid :

$$if\,\boldsymbol{\sigma x} = \boldsymbol{\sigma y}\;\; and$$

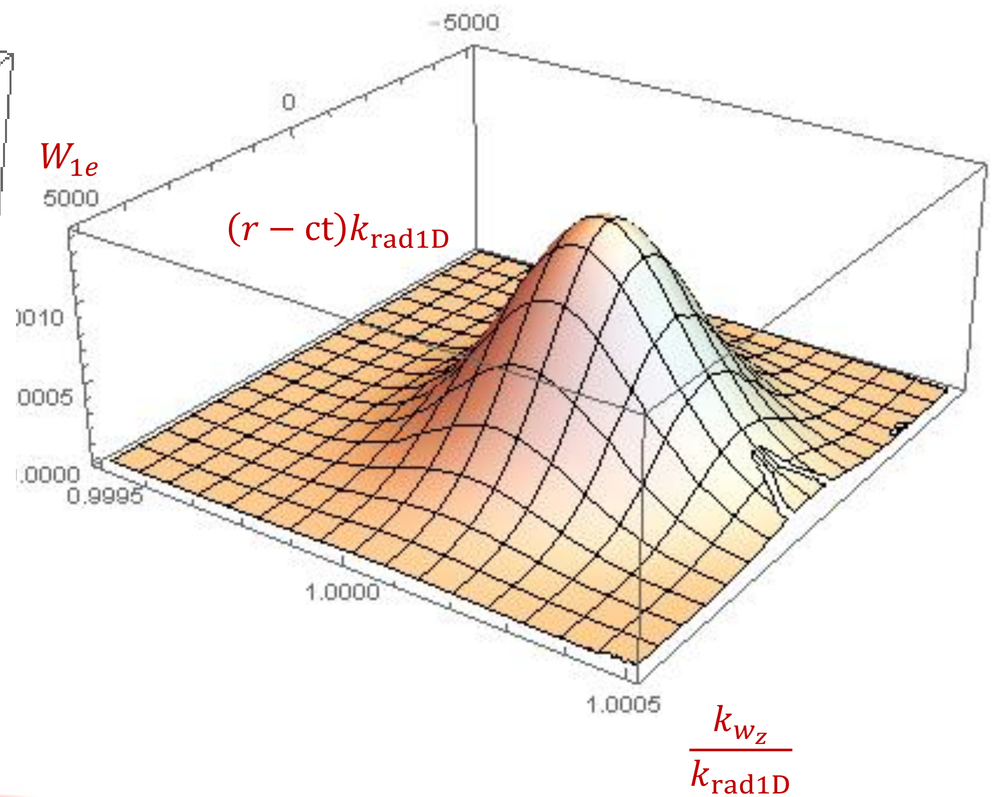
$$\boldsymbol{k_{L_z}}^{3}\,\boldsymbol{\sigma x^2 \sigma z} >> \mathbf{1}$$

Wigner of a single electron…
(3d Theory, time-frequency dependence)

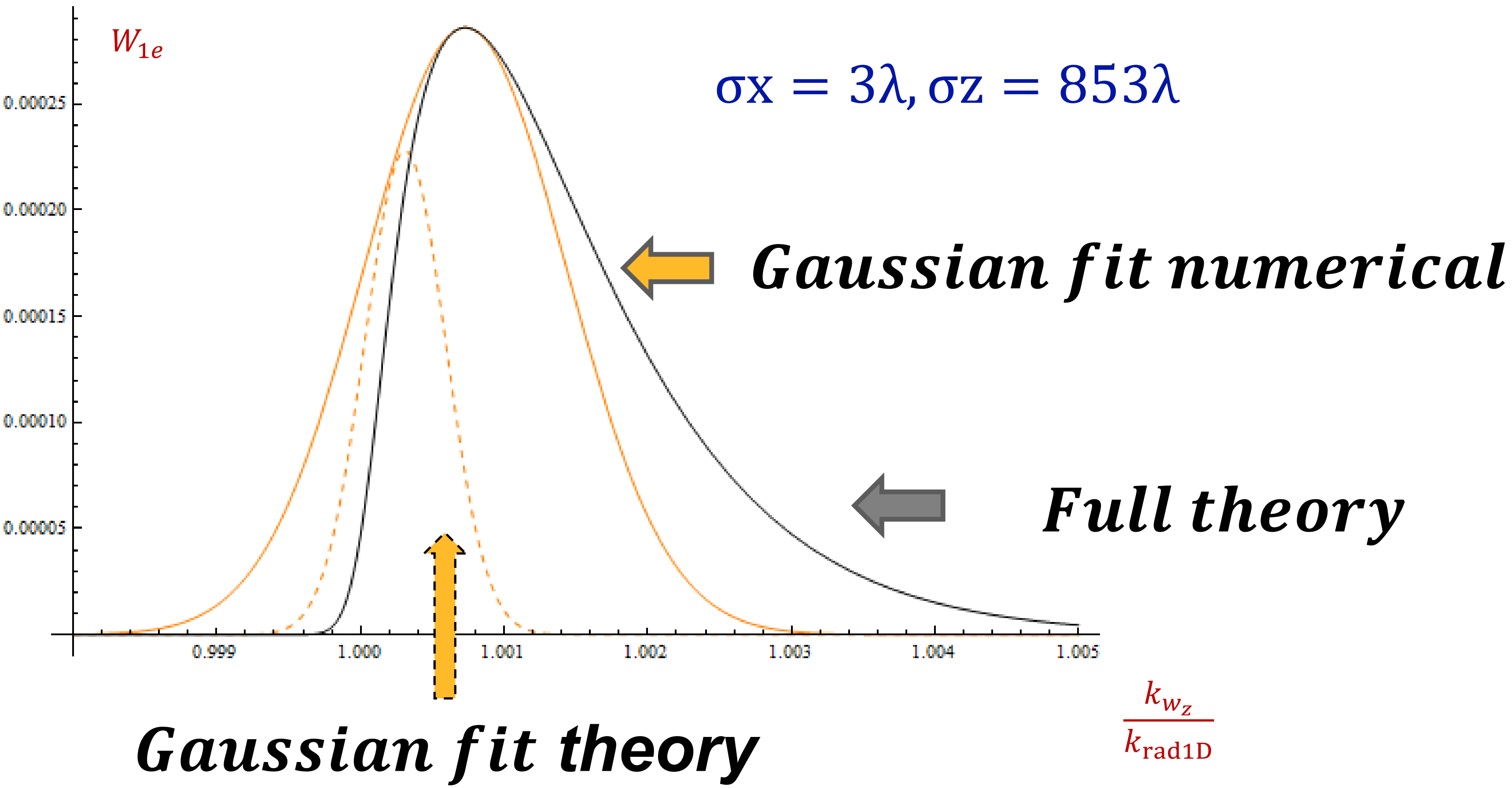


Oscillations to W<0

$\sigma x = 10\lambda, \sigma z = 853\lambda,$
FAST at FNAL



Wigner of a single electron…(3d Theory, spectrum)



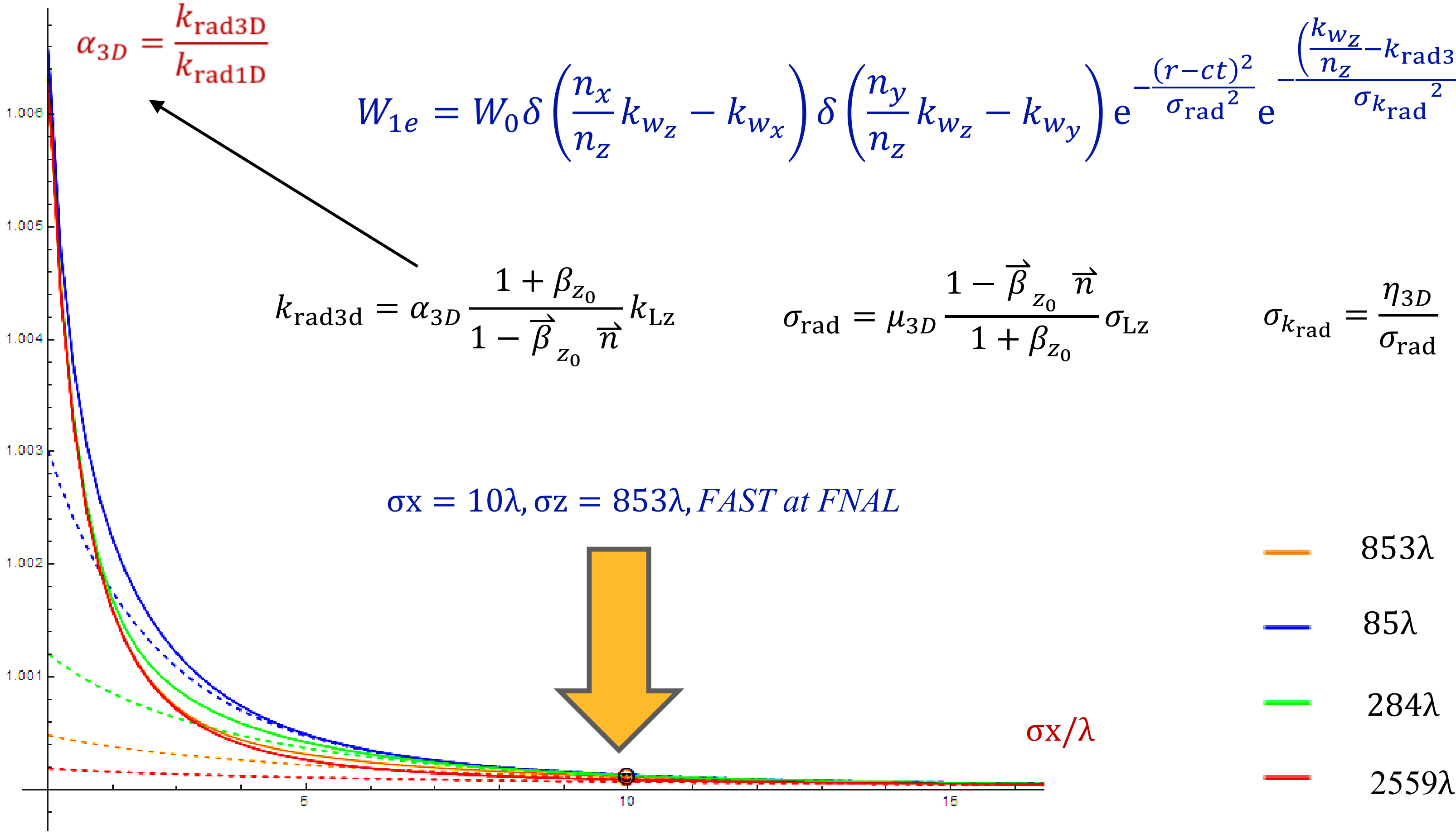
Wigner of a single electron…
(3d Theory, peak frequency shift)

$$\alpha_{3D} = \frac{k_{\text{rad3D}}}{k_{\text{rad1D}}}$$

$$W_{1e} = W_0 \delta\left(\frac{n_x}{n_z} k_{w_z} - k_{w_x}\right) \delta\left(\frac{n_y}{n_z} k_{w_z} - k_{w_y}\right) \mathrm{e}^{-\frac{(r-ct)^2}{\sigma_{\text{rad}}{}^2}} \mathrm{e}^{-\frac{\left(\frac{k_{w_z}}{n_z} - k_{\text{rad3d}}\right)^2}{\sigma_{k_{\text{rad}}}{}^2}}$$

$$k_{\text{rad3d}} = \alpha_{3D} \frac{1+\beta_{z_0}}{1-\vec{\beta}_{z_0}\,\vec{n}} k_{\text{Lz}} \qquad \sigma_{\text{rad}} = \mu_{3D} \frac{1-\vec{\beta}_{z_0}\,\vec{n}}{1+\beta_{z_0}} \sigma_{\text{Lz}} \qquad \sigma_{k_{\text{rad}}} = \frac{\eta_{3D}}{\sigma_{\text{rad}}}$$

σx = 10λ, σz = 853λ, *FAST at FNAL*



Wigner of a single electron…(3d Theory, spectrum fit error)