Theoretical study of novel concepts for compact, high-gain free electron lasers

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Overview

- ❑ Introduction-basics of free electron laser (FEL) physics.

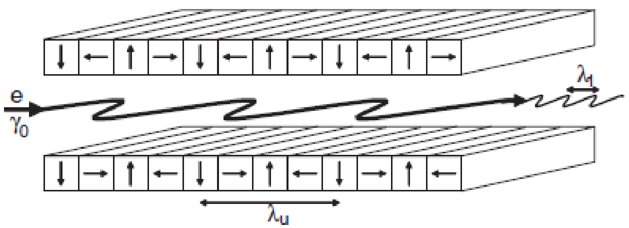
- ❑ FEL with variable electron beam/undulator parameters.
  - ✓ Solution of the three-dimensional, linearized initial value problem (IVP).
  - ✓ Comparison with simulation-applications.

- ❑ FEL based on a transverse gradient undulator (TGU).
  - ✓ attractive option for FEL driven by laser-plasma accelerator.
  - ✓ theoretical analysis in terms of the FEL eigenmodes.
  - ✓ gain/transverse coherence optimization studies.

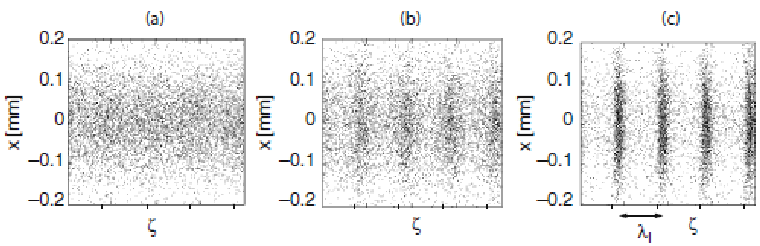
- ❑ Conclusions-discussion.

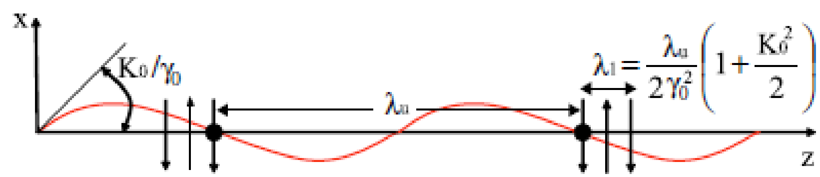
Introduction

- FEL operation is based on the resonant interaction between an electron beam and the radiation emitted during its passage through an undulator.

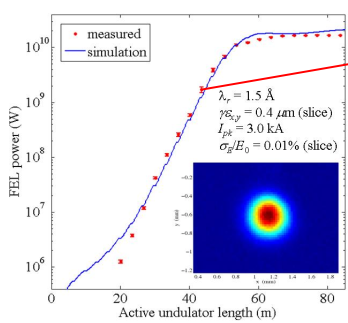


- ✓ A planar undulator induces a transverse wiggle motion which couples to the radiation field.
- ✓ When the resonance condition $\lambda_r = \lambda_u(1 + K^2/2)/(2\gamma^2)$ is satisfied, the interaction with the radiation field leads to an electron energy modulation.





energy modulation → density modulation (micro-bunching) → coherent radiation



linear regime

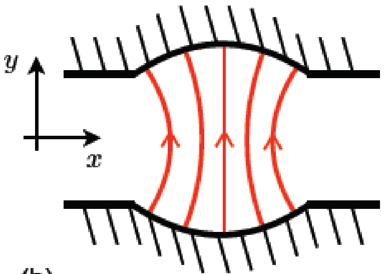
The interaction region for a high-gain FEL spans two regimes:

a) The linear, low/exponential-gain regime (described analytically).

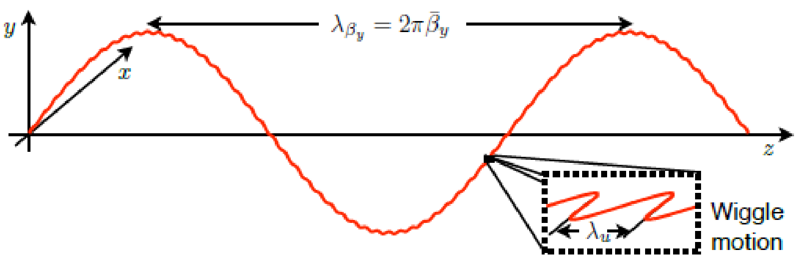
b) The nonlinear, saturation regime, in which power roughly stabilizes (described through simulation).

Introduction

➢ Understanding the physics of X-ray FELs requires a fully 3D theory-including the effects of betatron oscillations (due to undulator focusing-external and natural), transverse emittance and energy spread as well as the diffraction of the radiation.



(b)



➢ Moreover, we focus on cases where the beam and undulator parameters vary longitudinally. Such a "z-dependence" can be introduced by:

- Betatron mismatch effects (size and/or centroid mismatch).
- Undulator tapering, wakefields and electron beam chirp.
- Absence of focusing, which may be a feature of some special FEL schemes (such as a laser undulator-based FEL).

➢ We select the case of size mismatch as a representative example. We follow a perturbative, self-consistent, Maxwell-Vlasov analysis that is applicable in the linear regime of the interaction.

FEL with a mismatched e-beam

- The radiation complex amplitude satisfies a driven, paraxial wave equation.

$$\frac{\partial E_\nu}{\partial z} + \frac{\nabla_\perp^2 E_\nu}{2ik_r} = \int_{-\infty}^{\infty} d^2\bar{\mathbf{x}} \int_0^z d\zeta \overbrace{\Lambda(\mathbf{x}, \bar{\mathbf{x}}, z, \zeta)}^{\text{integral kernel known analytically}} E_\nu(\bar{\mathbf{x}}, \zeta)$$

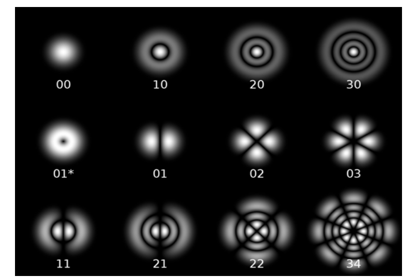
- Expand the rad. amplitude in terms of a set of generalized, Gauss-Laguerre modes (axial symmetry assumed):

$$E_\nu(\mathbf{x}, z) = \varepsilon_\nu \sum_{p=0}^{\infty} \sum_{q=-\infty}^{\infty} C_{pq}(z)\psi_{pq}(\mathbf{x}, z)$$

$$\psi_{pq}(\mathbf{x}, z) = \left(\frac{p!}{(p+|q|)!}\right)^{1/2} \left(\frac{\sqrt{2}r}{w}\right)^{|q|}$$

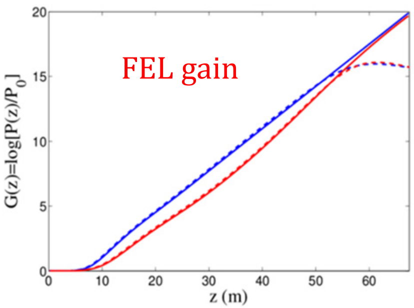
$$\times L_p^{|q|}\left(\frac{2r^2}{w^2}\right) \psi_{00}(\mathbf{x}, z) e^{iq\phi} e^{-i(2p+|q|)u}, \quad \psi_{00}(\mathbf{x}, z) = \left(\frac{k_r \beta_1}{\pi}\right)^{1/2} \frac{1}{z - i\beta} \exp\left(\frac{ik_r r^2}{2(z - i\beta)}\right)$$

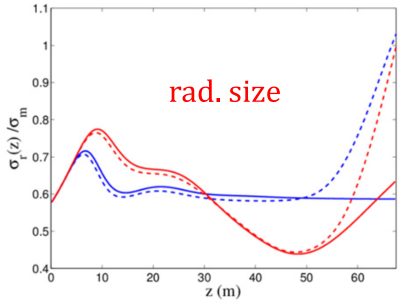
P. Baxevanis et.al, Phys. Rev. ST-AB, 16, 010705 (2013)

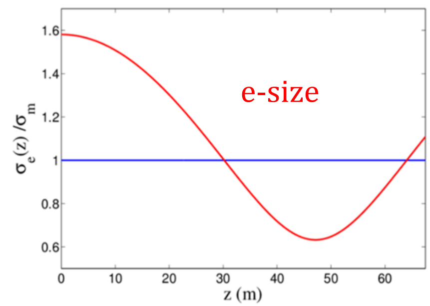


- This result in an infinite set of evolution equations for the expansion coefficients and the mode parameters. A truncated solution can be used to approximately determine the key radiation and e-beam parameters in the linear regime.

FEL with a mismatched e-beam



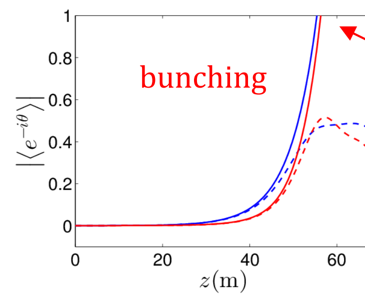




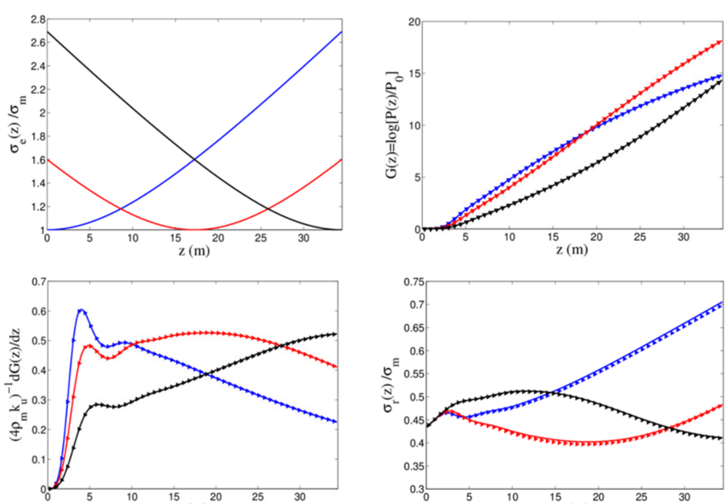
Parameter	Set 1	Set 2
Undulator parameter K	3.7	0.5
Undulator period λ_u	3 cm	0.5 cm
beam energy $\gamma_0 mc^2$	14.31 GeV	2.21 GeV
Resonant wavelength λ_r	1.5 A°	1.5 A°
Peak current I	3 kA	3 kA
Energy spread σ_η	10^{-4}	10^{-4}
Normalized emittance $\gamma_0\epsilon$	0.5 μm	0.5 μm
Matched beta β_m	30 m	13.78 m
Matched beam size σ_m	23.14 μm	39.89 μm
ρ_m (for $\sigma = \sigma_m$)	5.4×10^{-4}	2.3×10^{-4}
External focusing	Yes	No

Table 1: Undulator and electron beam parameters

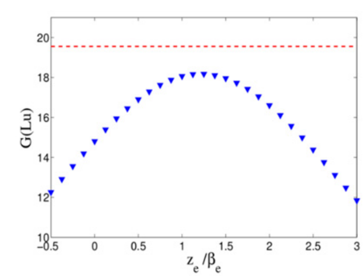
- Good agreement observed between linearized results (solid lines) and GENESIS 3D simulation (dashed lines - LCLS parameters).



estimate saturation length from linearized solution

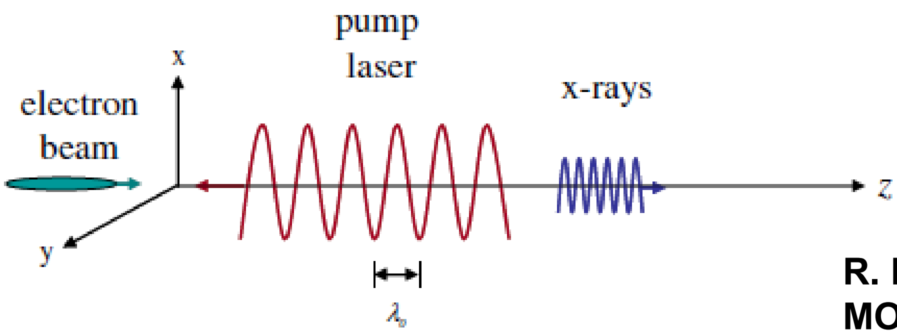


- Consider a set of **unfocused e-beams (2 GeV energy).** Compare 1-mode (solid lines) to 5-mode results (symbols) → **rapid convergence**
- Thus, the linearized results can be used for fast optimization studies.



Laser undulator-based FEL

- Essentially the same formalism can be used to analyze a high-gain FEL based on a **"laser undulator"** (a counter-propagating laser pulse acting as a wiggler).



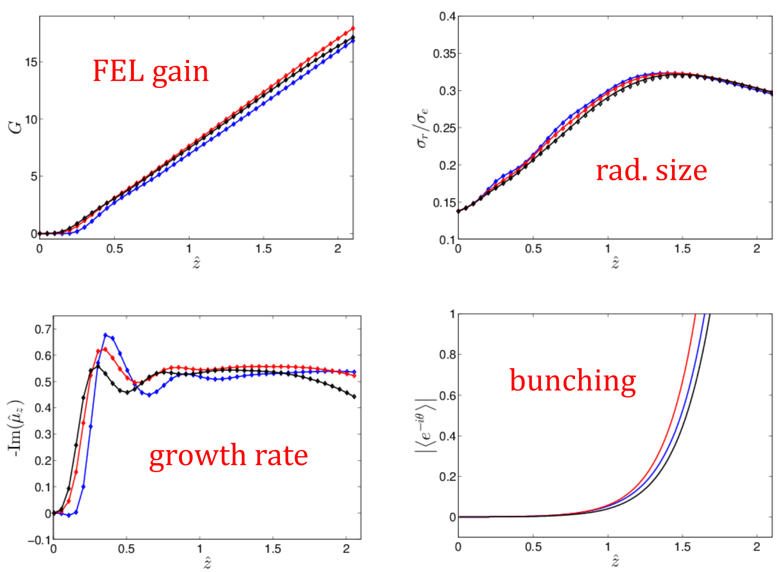
P. Sprangle et.al, Phys. Rev. ST-AB, 12-050702 (2009)

R. Ruth, P. Baxevanis, MOPSO04, FEL-2013

Table 3.1: Laser undulator-based FEL parameters

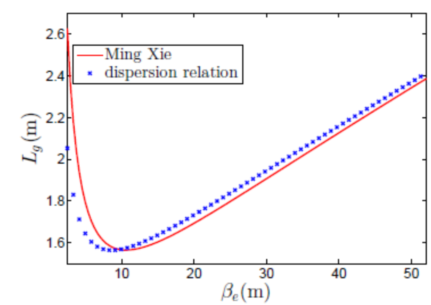
Laser wavelength λ_L	10 μm
Laser power P_L	1 TW
Pulse energy U_L	70 J
Minimum spot size w_0	330 μm
Rayleigh length z_R	3 cm
Beam energy $\gamma_0 m_e c^2$	12.9 MeV
Undulator parameter K_0	0.2
Resonant wavelength λ_r	4 nm
Peak current I_p	1 kA
Energy spread σ_δ	10^{-4}
Normalized emittance $\gamma_0\epsilon = \gamma_0\sigma\sigma'$	0.1 μm
Minimum beta (e-beam) $\beta_e = \sigma/\sigma'$	5 mm
Interaction length L_I	1 cm
FEL parameter ρ	6.7×10^{-4}
Saturation power P_{sat}	5 MW

soft X-ray parameters



- The linearized results can be used to validate the basic FEL concept.

- One can also optimize parameters such as the e-beam waist size (beta value).



TGU-based FEL: theory

- FEL performance depends on parameters of the electron beam, such as transverse emittance and energy spread.
  - efficient lasing requires that $\boldsymbol{\sigma_\delta/\rho < 1}$

$$\rho = \left(\frac{K_0^2 JJ^2}{16\gamma_0^3\, k_u^2 \sigma_x \sigma_y} \frac{I}{I_A} \right)^{1/3}$$

- For beams from storage rings or plasma-based accelerators, it is extremely hard to satisfy this condition (due to large energy spread).

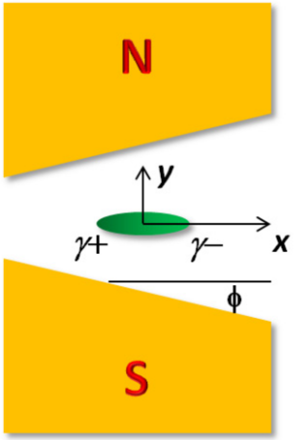
- One possible solution is the transverse gradient undulator (TGU).

T. Smith et. al., J. Appl. Phys. 50, 4580 (1979).
N. Kroll et. al., IEEE Journ. Quant. Electronics, QE-17, 1496 (1981).

✓ Use a dispersive element to spread out the beam in the horizontal direction:
$\boldsymbol{\delta = (\gamma - \gamma_0)/\gamma_0 = x/\eta \rightarrow \gamma = \gamma_0(1 + x/\eta)}$

✓ Introduce a linear dependence of the undulator strength K with x by sloping the undulator poles:

$$\boldsymbol{K = K_0(1 + \alpha x)}$$



TGU-based FEL: theory

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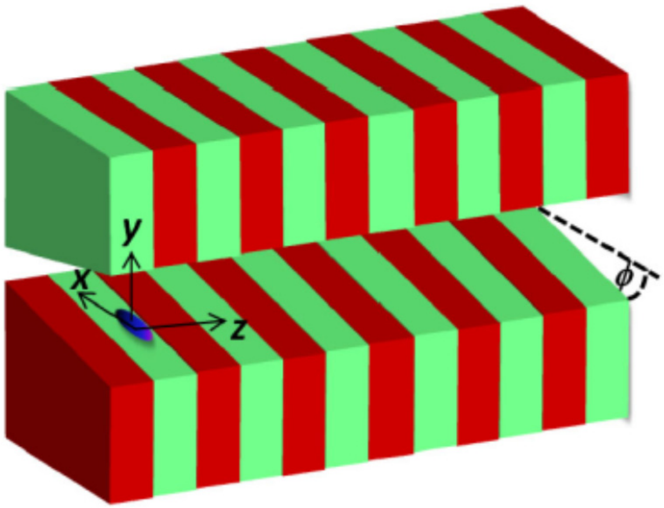
- The basic FEL relation $\lambda_r = \lambda_u(1+K^2/2)/(2\gamma^2)$ leads to the TGU resonance condition $\bar{\boldsymbol{a}} = \boldsymbol{\alpha K_0^2/(2+K_0^2) = 1/\eta}$
- We seek the self similar, guided eigenmodes of the FEL : $\boldsymbol{E_\nu(x,z) = A(x)e^{i\mu z}}$
- This leads to an eigenmode equation (neglecting emittance and focusing effects):



$$\left(\mu - \frac{\nabla_\perp^2}{2k_r}\right) A(\mathbf{x}) = U(\mathbf{x},\mu)A(\mathbf{x})$$

$\boldsymbol{\rho_T = \rho R^{-1/3}}$ (effective ρ-parameter)

$$U(\mathbf{x},\mu) = -8\rho_T^3 k_u^3 \exp\left(-\frac{x^2}{2\sigma_T^2} - \frac{y^2}{2\sigma_y^2}\right)$$
$$\times \int_{-\infty}^{0} d\xi\, \xi e^{i(\mu - \Delta\nu k_u)\xi} e^{-2(\sigma_\delta^{ef})^2\xi^2} \exp\left(-2ik_u C_p \frac{x}{\eta}\xi\right)$$

$\boldsymbol{R = \sigma_T/\sigma_x =}$
$\boldsymbol{= (1+\eta^2\sigma_\delta^2/\sigma_x^2)^{1/2}}$
(dispersed size increase)

$\boldsymbol{\sigma_\delta^{ef} = \sigma_\delta R^{-1}}$
(effective energy spread)

"new term" ($C_p = R^{-2} + \bar{a}\eta - 1$)

TGU-based FEL: theory

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- Approximate solutions to the eigenmode equation can obtained:

- using a simplified model that admits exact, analytical solutions:

$$A_{mn}(\hat{\mathbf{x}}) = H_m(\sqrt{2\hat{a}_x}(\hat{x} - \hat{b}/(2\hat{a}_x)))e^{-\hat{a}_x\hat{x}^2 + \hat{b}\hat{x}} \times H_n(\sqrt{2\hat{a}_y}\hat{y})e^{-\hat{a}_y\hat{y}^2}$$

m,n=0,1,2,3…

$$\hat{\mu} + p_{dx}[(4m+2)\hat{a}_x - \hat{b}^2] + (4n+2)p_{dy}\hat{a}_y = -\hat{I}_0$$

$$\hat{a}_x^2 = -\frac{\hat{I}_0 + 4p_0^2\hat{I}_2}{8p_{dx}} \quad , \quad \hat{a}_x\hat{b} = \frac{ip_0}{2p_{dx}}\hat{I}_1 \quad , \quad \hat{a}_y^2 = -\frac{\hat{I}_0}{8p_{dy}}$$

P. Baxevanis et.al, Phys. Rev. ST-AB, 17, 020701 (2014)

P. Baxevanis et.al, Phys. Rev. ST-AB, 18, 010701 (2015)

- through a variational technique (for fundamental + higher-order modes):

$$\int d^2\hat{\mathbf{x}} A(\hat{\mathbf{x}}) \left(\hat{\mu} - p_{dx}\frac{\partial^2}{\partial \hat{x}^2} - p_{dy}\frac{\partial^2}{\partial \hat{y}^2}\right) A(\hat{\mathbf{x}}) = \int d^2\hat{\mathbf{x}} A^2(\hat{\mathbf{x}})\hat{U}(\hat{\mathbf{x}}, \hat{\mu})$$

- Each mode is characterized by a **power gain length L_g** ($P_r \propto e^{z/L_g}$)

$$L_g = \underbrace{\frac{\lambda_u}{4\pi\sqrt{3}\rho_T}}_{\text{1D value (excluding diffraction and e-spread effects)}} \frac{\sqrt{3}/2}{|\mathrm{Im}(\hat{\mu})|}$$

1D value (excluding diffraction and e-spread effects)

Soft X-ray FEL with an LPA beam

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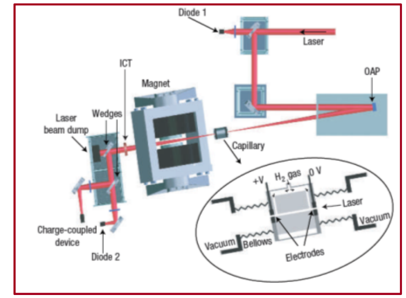
- Laser-plasma accelerators (LPAs) have demonstrated the capability to produce e-beams in the GeV range.

LETTERS

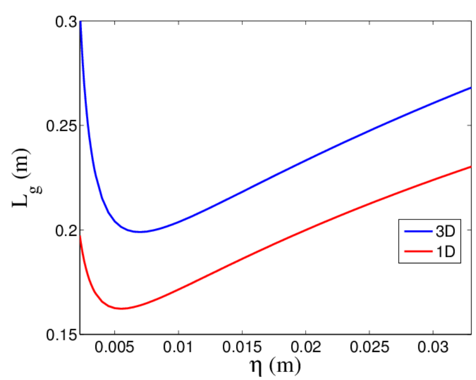
GeV electron beams from a centimetre-scale accelerator

W. P. LEEMANS[1*], B. NAGLER[1], A. J. GONSALVES[2], Cs. TÓTH[1], K. NAKAMURA[1,3], C. G. R. GEDDES[1], E. ESAREY[1*], C. B. SCHROEDER[1] AND S. M. HOOKER[2]

Nat. Phys. 2, 696 (2006)



- **1 GeV, λ_u = 1 cm, K_0 = 2 radiation wavelength λ_r = 3.9 nm**
- **10 kA peak current,**
- **0.1 mm-mrad norm. emittance**
- **1% energy spread ($\rho \approx 6 \times 10^{-3}$)**



✓ **Optimize the gain length** as a function of the dispersion for the fundamental (00) mode.

✓ Results also compared with a 1D fitting formula.

Z. Huang et.al, PRL 109, 204801 (2012)

$$L_g = \frac{\lambda_u}{4\pi\sqrt{3}\rho_T}\left[1 + \left(\frac{\sigma_\delta^{ef}}{\rho_T}\right)^2\right]$$

Optimum working point (1D)

$$L_g \approx 1.75(\lambda_u/4\pi\sqrt{3}\rho)(\sigma_\delta/\rho\ )^{1/2} \quad \eta \approx 2.28\sigma_x\sigma_\delta^{1/2}/\rho^{3/2}$$

(square root dependence on the energy spread)

Soft X-ray FEL with an LPA beam

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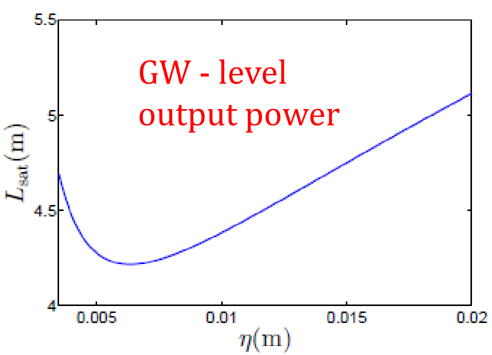
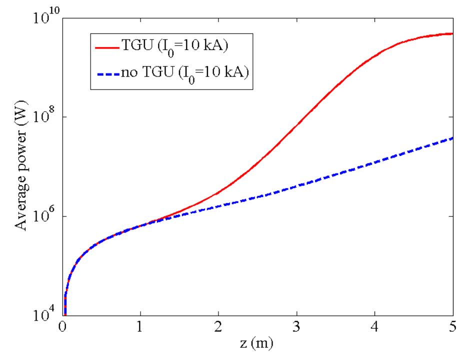


(b) Saturation length.



- ✓ The saturation length L_{sat} is estimated through the gain length:

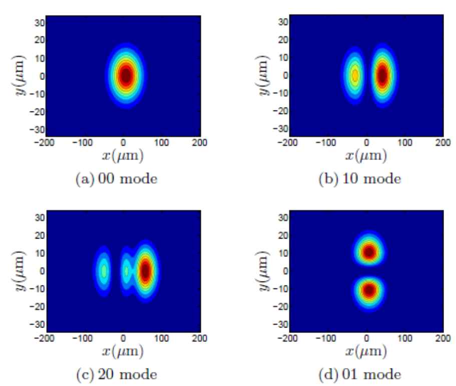
$$L_{sat} \sim N_g L_g^{00} \quad N_g \sim 18 - 20$$

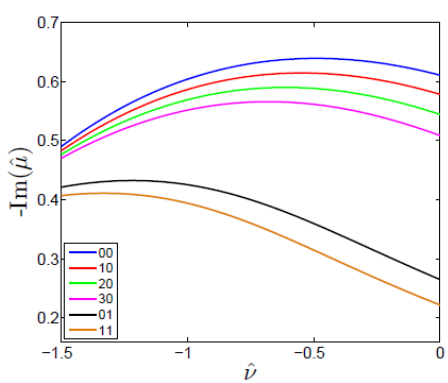
- ✓ L_{sat} depends rather weakly on η.

- ✓ For a **1 cm** dispersion, we compare with GENESIS simulation (courtesy of Y. Ding) ⇒ e-beam sizes: **100 μm x 10 μm**, opt. gain length ~ **20.4 cm**, saturation within **5 m**

- ✓ Use the parabolic model to determine the FEL mode spectrum (1 cm dispersion).

- **Define the gain ratios**
  $f_{mn} = exp(z/L_g^{mn})/exp(z/L_g^{00})$.
- **For** $z = L_{sat} \sim$ **4.5 m**





ratio	value
f_{10}	40%
f_{20}	15%
f_{30}	6%
f_{01}	<0.1%

expect multiple modes in output radiation ($\zeta_{coh} \sim 20\%$)

Soft X-ray FEL with an LPA beam

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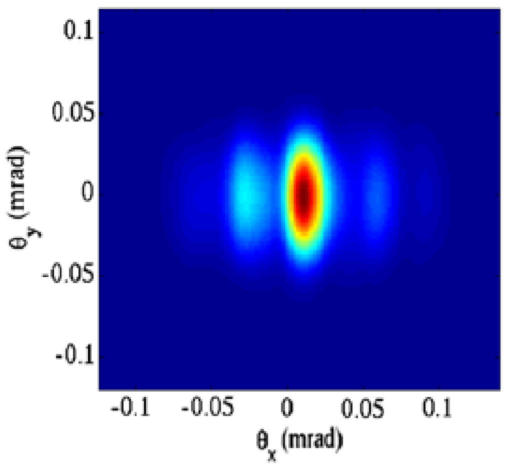
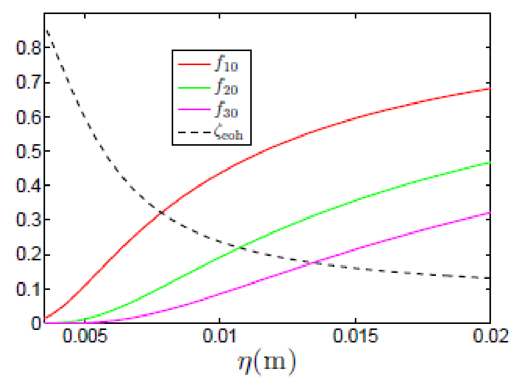


Z. Huang et.al, PRL109



(a) Gain factor ratios and degree of transverse coherence.

P. Baxevanis et.al, Phys. Rev. ST-AB, 18, 010701 (2015)

- The presence of higher-order modes is evident in GENESIS simulations as well.

main mode sequence

00
10 01
20 11 02
30 21 12 03
……

n
m

- A measure of transverse coherence - ζ_{coh} - can be constructed from the mode gain factors $\exp(z/L_g^i)$.

- The degree of transverse coherence can be improved by ***reducing the dispersion*** -and the aspect ratio - at a small cost in gain.

Conclusions

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- ✓ We have developed a versatile theory-based approach that enables us to study a broad range of FEL concepts.
- ✓ The semi-analytical solution of the 3D IVP allows us to rigorously describe FEL systems with variable beam (or undulator) parameters.
- ✓ For constant parameter systems, we have applied standard methods (eigenmode analysis) to the study of novel concepts such as the TGU-based FEL.
- ✓ The results obtained compare favorably with simulation and can be used in fast optimization studies involving key parameters like the gain length and the degree of transverse coherence.

Thank you for your attention!