# RF SUPERCONDUCTIVITY

## Jean Delayen

**Center for Accelerator Science
Old Dominion University
and
Thomas Jefferson National Accelerator Facility**

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# Outline

- Fundamentals of rf superconductivity
  - Underlying physics
  - New developments

- Superconducting cavities
  - Various shapes
  - How they are made

- Limitations
  - Why SRF cavities aren't behaving (yet) according to theory

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# Why SRF?

- Normal Conductors
  - Skin depth proportional to $\omega^{-1/2}$
  - Surface resistance proportional to $\omega^{1/2 \to 2/3}$
  - Surface resistance independent of temperature (at low T)
  - For Cu at 300K and 1 GHz, $R_s = 8.3$ m$\Omega$

- Superconductors
  - Penetration depth independent of $\omega$
  - Surface resistance proportional to $\omega^2$
  - Surface resistance strongly dependent of temperature
  - For Nb at 2 K and 1 GHz, $R_s \approx 7$ n$\Omega$

## However: do not forget Carnot

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# Why are SC and NC Accelerators Different?

$$\frac{P}{L} \propto \frac{1}{\frac{R}{Q} Q R_S} \frac{E^2 R_S}{\omega}$$

For normal conductors $\qquad R_S \propto \omega^{1/2}$

$$\frac{P}{L} \propto \omega^{-\frac{1}{2}}$$

$$\frac{P}{A} \propto \omega^{\frac{1}{2}}$$

For superconductors $\qquad R_S \propto \omega^2$

$$\frac{P}{L} \propto \omega$$

$$\frac{P}{A} \propto \omega^2$$

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# SUPERCONDUCTIVITY FUNDAMENTALS

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# Historical Overview

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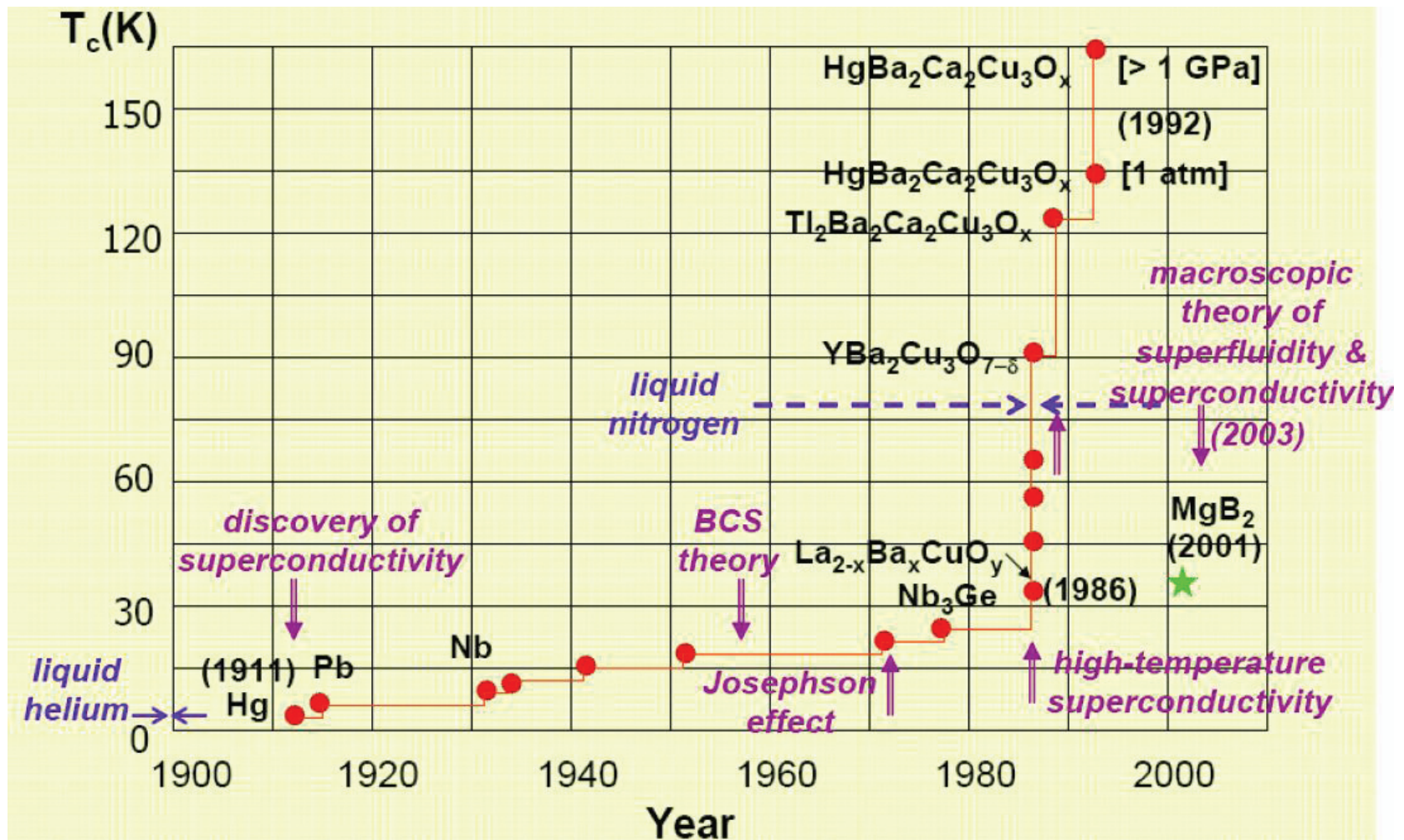
# Perfect Conductivity
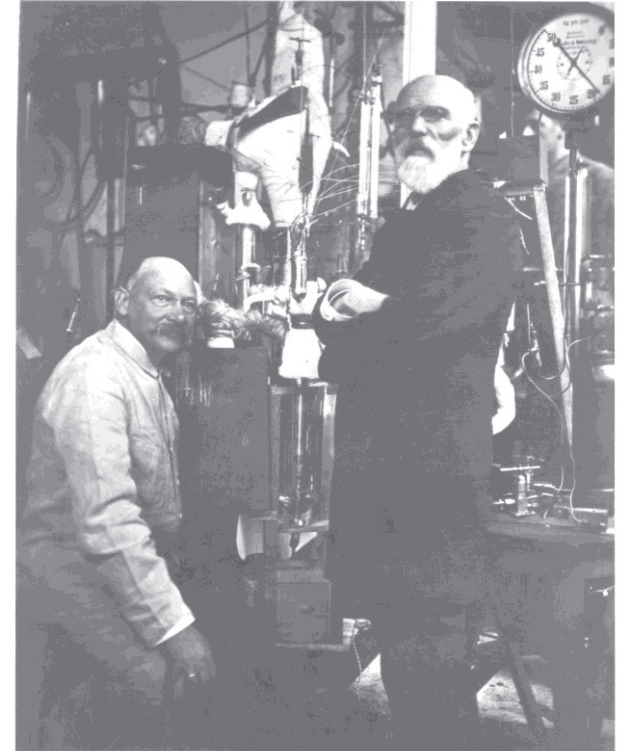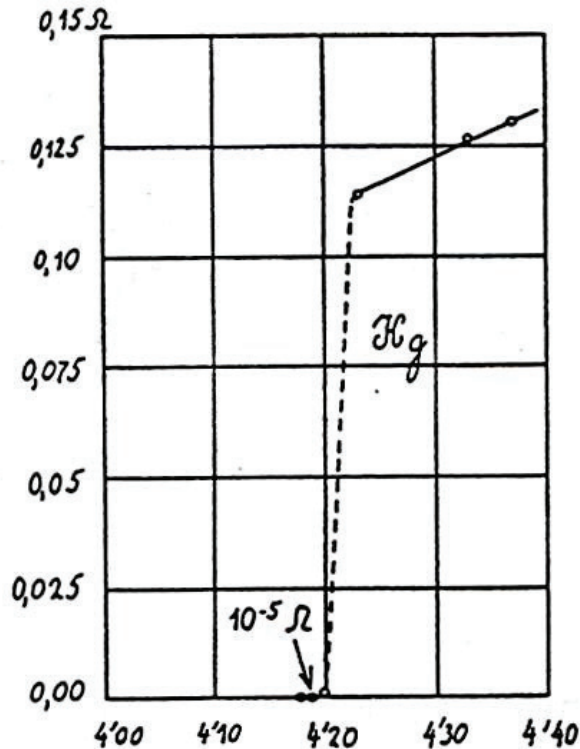




**Kamerlingh Onnes and van der Waals in Leiden with the helium 'liquefactor' (1908)**

# Perfect Conductivity

**Persistent current experiments on rings have measured**

$$\frac{\sigma_s}{\sigma_n} > 10^{15}$$

**Resistivity < $10^{-23}$ Ω.cm**

**Decay time > $10^5$ years**

**Perfect conductivity is not superconductivity**

**Superconductivity is a phase transition**

**A perfect conductor has an infinite relaxation time L/R**

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# Perfect Diamagnetism (Meissner & Ochsenfeld 1933)

**Perfect conductor**

**Superconductor**



*Case I.* The specimen is first cooled below its transition temperature

and then brought into a magnetic field.

*Case II.* The specimen is brought into a magnetic field while it is in the normal state

and subsequently cooled below its transition temperature.
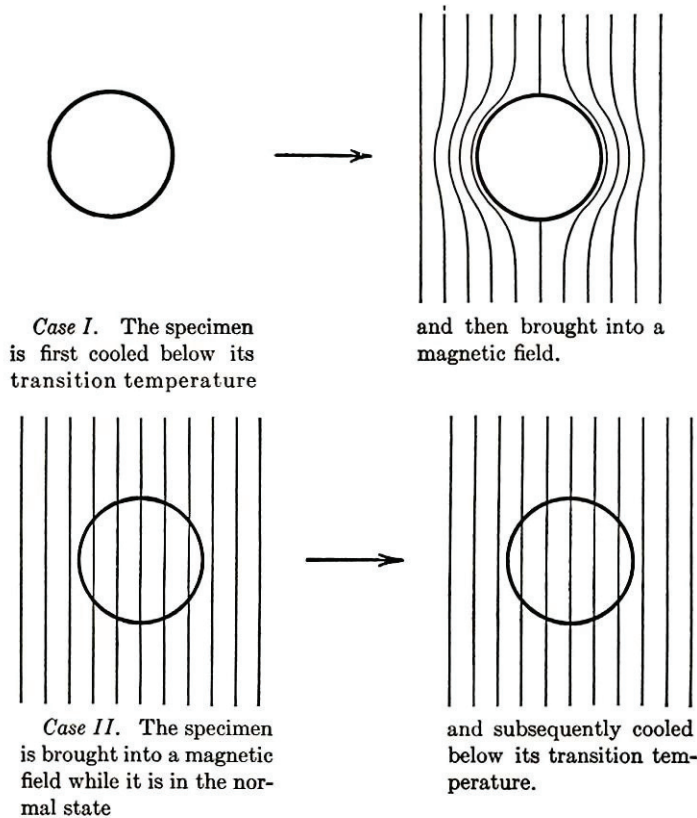
FIG. 3. The behavior expected for a transition into a state of *perfect conductivity*. The final state would depend on the *serial order* in which the specimen is brought into the same external conditions.

*Case I.* The specimen is first cooled below its transition temperature

and then brought into a magnetic field.

The magnetic field is applied while the specimen is in the normal state;

the field is pushed out when the specimen is cooled below its transition temperature.
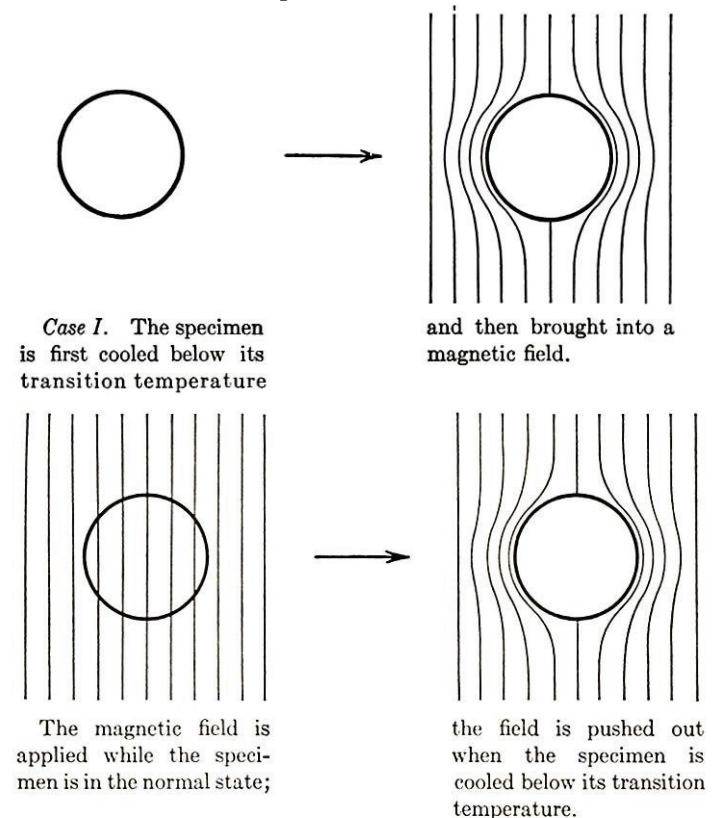
FIG. 4. Case II of Fig. 3 according to Meissner. The *superconductor*, in contrast to the perfect conductor, has zero magnetic induction independently of the way in which the superconducting state has been reached.

$$\frac{\partial B}{\partial t} = 0$$

$$B = 0$$

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# Critical Field (Type I)

**Superconductivity is destroyed by the application of a magnetic field**

$$H_c(T) \simeq H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$
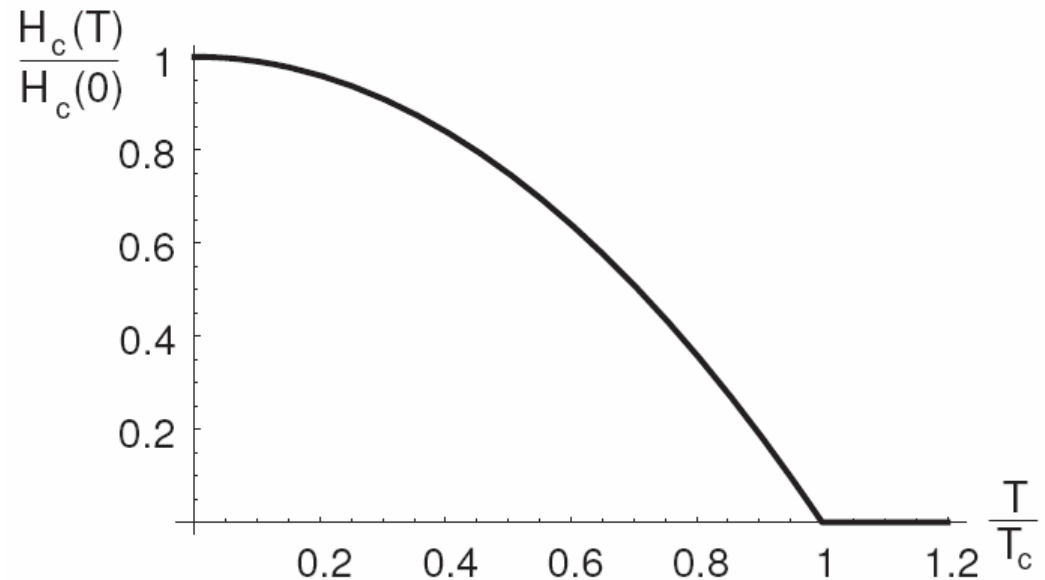


**Type I or "soft" superconductors**

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# Critical Field (Type II or "hard" superconductors)



**Figure 3-1**
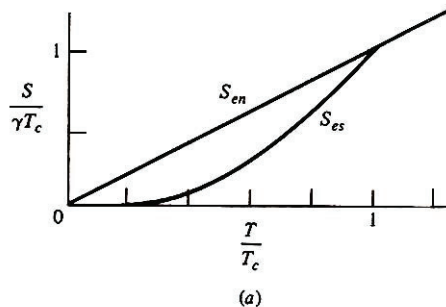Phase diagram for a long cylinder of a Type II super-conductor.

In the figure: $H$ (vertical axis), $T$ (horizontal axis), $T_0$ marked on horizontal axis. Curves labeled $H_{c3}$, $H_{c2}$, $H_{c1}$. Regions labeled: surface superconductivity, Schubnikov phase, complete Meissner effect $B = 0$.
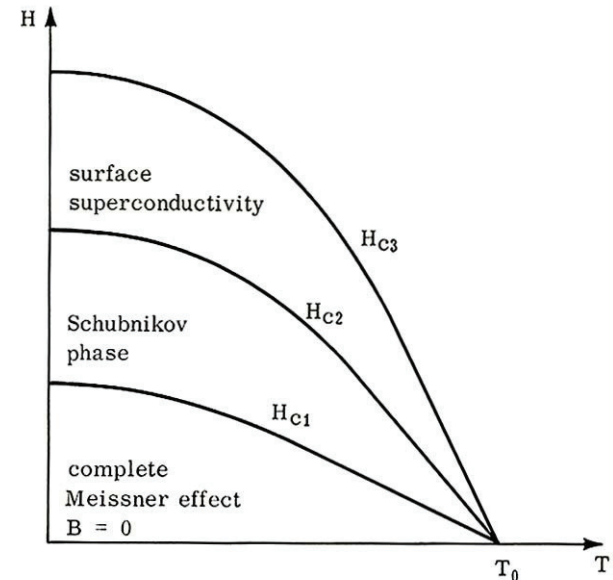
**Expulsion of the magnetic field is complete up to $H_{c1}$, and partial up to $H_{c2}$**

**Between $H_{c1}$ and $H_{c2}$ the field penetrates in the form if quantized vortices or fluxoids**

$$\phi_0 = \frac{\pi\hbar}{e}$$

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# Thermodynamic Properties

**Entropy**

**Specific Heat**
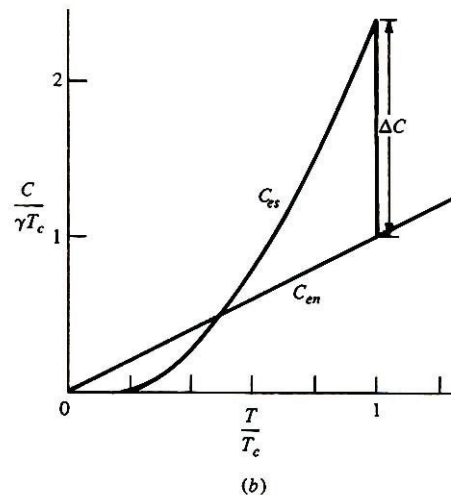
**Energy**
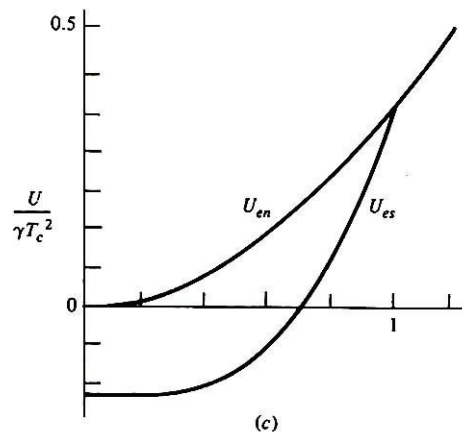
**Free Energy**
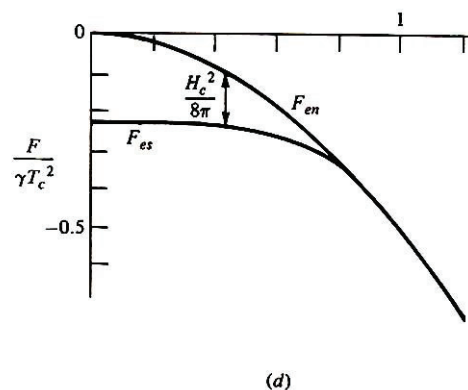
**FIGURE 2-3**

Comparison of thermodynamic quantities in superconducting and normal states. $U_{en}(0)$ is chosen as the zero of ordinates in (c) and (d). Because the transition is of second order, the quantities $S$, $U$, and $F$ are continuous at $T_c$. Moreover, the slope of $F_{es}$ joins continuously to that of $F_{en}$ at $T_c$, since $\partial F/\partial T = -S$.
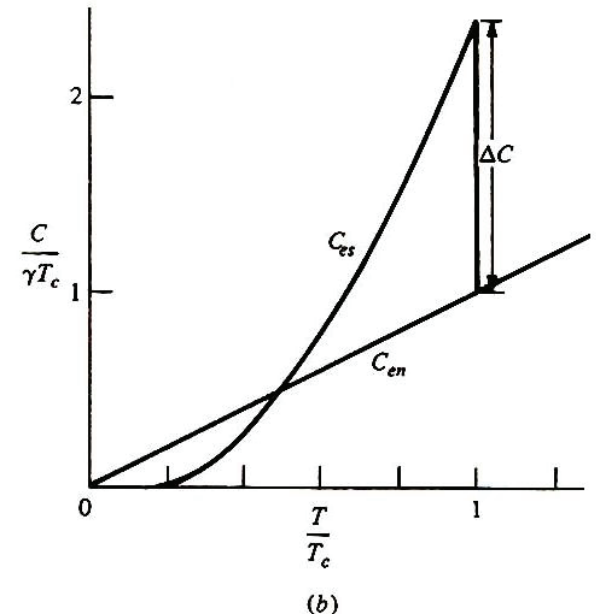
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# Thermodynamic Properties

When $T < T_c$ phase transition at $H = H_c(T)$ is of $1^{st}$ order $\Rightarrow$ latent heat

At $T = T_c$ transition is of $2^{nd}$ order $\Rightarrow$ no latent heat

jump in specific heat

$$C_{es}(T_c) \sim 3C_{en}(T_c)$$

$C_{en}(T) = \gamma T$    electronic specific heat

$C_{es}(T) \approx \alpha T^3$    reasonable fit to experimental data



(b)

# Isotope Effect (Maxwell 1950)

**The critical temperature and the critical field at 0K are dependent on the mass of the isotope**

$$T_c \sim H_c(0) \sim M^{-\alpha} \qquad \text{with } \alpha \simeq 0.5$$

Figure 26: The critical temperature of various tin isotopes.

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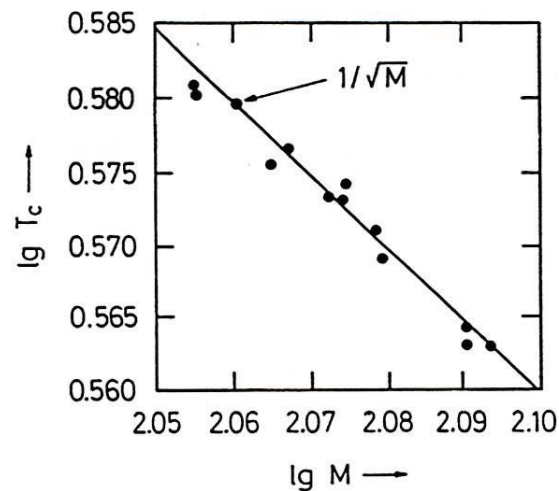
# Energy Gap (1950s)

**At very low temperature the specific heat exhibits an exponential behavior**

$$c_s \propto e^{-bT_c/T} \qquad \text{with } b \simeq 1.5$$

**Electromagnetic absorption shows a threshold**

**Tunneling between 2 superconductors separated by a thin oxide film shows the presence of a gap**

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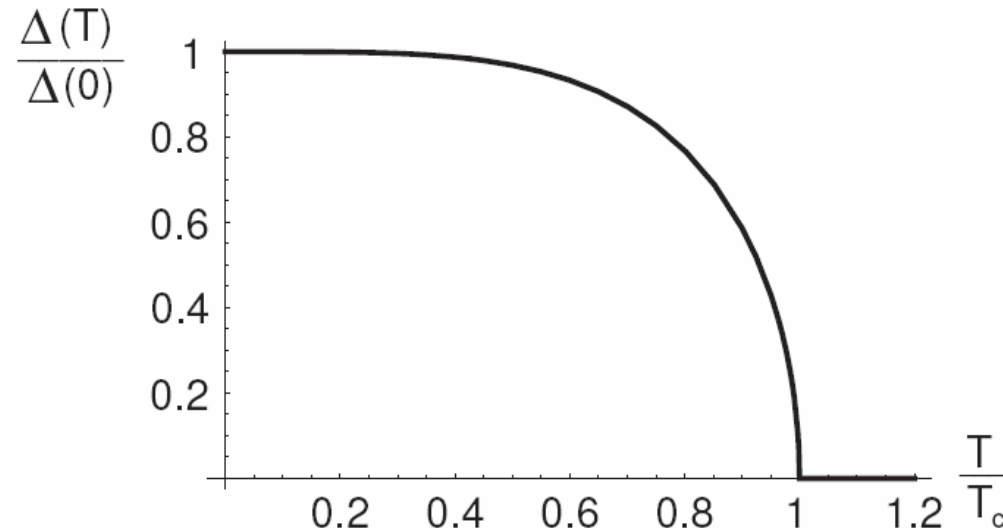
# Two Fundamental Lengths

- ## London penetration depth λ
  - Distance over which magnetic fields decay in superconductors

- ## Pippard coherence length ξ
  - Distance over which the superconducting state decays
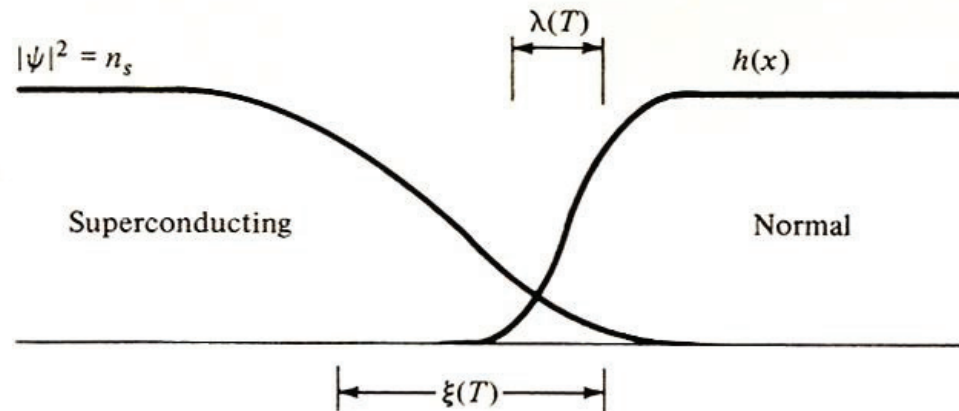


FIGURE 1-4
Interface between superconducting and normal domains in the intermediate sta

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# Two Types of Superconductors

- London superconductors (Type II)
  - $\lambda >> \xi$
  - Impure metals
  - Alloys
  - Local electrodynamics

- Pippard superconductors (Type I)
  - $\xi >> \lambda$
  - Pure metals
  - Nonlocal electrodynamics

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# Material Parameters for Some Superconductors

| Superconductor | $\lambda_L(0)$ (nm) | $\xi_0$ (nm) | $\kappa$ | $2\Delta(0)/kT_c$ | $T_c$(K) |
|---|---|---|---|---|---|
| Al | 16 | 1500 | 0.011 | 3.40 | 1.18 |
| In | 25 | 400 | 0.062 | 3.50 | 3.3 |
| Sn | 28 | 300 | 0.093 | 3.55 | 3.7 |
| Pb | 28 | 110 | 0.255 | 4.10 | 7.2 |
| Nb | 32 | 39 | 0.82 | 3.5-3.85 | 8.95-9.2 |
| Ta | 35 | 93 | 0.38 | 3.55 | 4.46 |
| $Nb_3Sn$ | 50 | 6 | 8.3 | 4.4 | 18 |
| NbN | 50 | 6 | 8.3 | 4.3 | $\leq 17$ |
| $Yba_2Cu_3o_x$ | 140 | 1.5 | 93 | 4.5 | 90 |

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# Phenomenological Models (1930s to 1950s)

**Phenomenological model:**
**Purely descriptive**
   **Everything behaves as though…..**

**A finite fraction of the electrons form some kind of condensate that behaves as a macroscopic system (similar to superfluidity)**

**At 0K, condensation is complete**

**At $T_c$ the condensate disappears**

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# Two Fluid Model – Gorter and Casimir

$T < T_c$    $x = $ fraction of "normal" electrons

$(1 - x):$ fraction of "condensed" electrons (zero entropy)

Assume:    $F(T) = x^{1/2} f_n(T) + (1 - x) f_s(T)$            free energy

Minimization of $F(T)$ gives    $\boxed{x = \left(\dfrac{T}{T_C}\right)^4}$        $C_{es} = 3\gamma \dfrac{\mathrm{T}^3}{\mathrm{T}_c^2}$

$$\boxed{\frac{H_c(T)}{H_c(0)} = 1 - \left(\frac{T}{T_C}\right)^2}$$

**The Gorter-Casimir model is an "ad hoc" model (there is no physical basis for the assumed expression for the free energy) but provides a fairly accurate representation of experimental results**

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# Model of F & H London (1935)

**Proposed a 2-fluid model with a normal fluid and superfluid components**

$n_s$ : density of the superfluid component of velocity $v_s$
$n_n$ : density of the normal component of velocity $v_n$

$$m \frac{\partial \vec{\upsilon}}{\partial t} = -e\vec{E}$$ superelectrons are accelerated by $E$

$$\overrightarrow{J_s} = -en_s \, \vec{\upsilon}$$

$$\frac{\partial \vec{J}_s}{\partial t} = \frac{n_s e^2}{m} \vec{E}$$ superelectrons

$$\vec{J}_n = \sigma_n \vec{E}$$ normal electrons

# Model of F & H London (1935)

$$\frac{\partial \vec{J}_s}{\partial t} = \frac{n_s e^2}{m} \vec{E}$$

Maxwell: 
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t}\left( \frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} \right) = 0 \qquad \Rightarrow \frac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} = \text{Constant}$$

F&H London postulated: $\boxed{\dfrac{m}{n_s e^2} \vec{\nabla} \times \vec{J}_s + \vec{B} = 0}$

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# Model of F & H London (1935)

**combine with** $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_s$

$$\boxed{\nabla^2 \vec{B} - \frac{\mu_0 n_s e^2}{m} \vec{B} = 0}$$

$$B(x) = B_o \exp\left[-x / \lambda_L\right]$$

$$\lambda_L = \left[\frac{m}{\mu_0 n_s e^2}\right]^{\frac{1}{2}}$$

$B_A$

$J_y = J_A \exp(-x/\lambda_L)$

$\otimes$

X
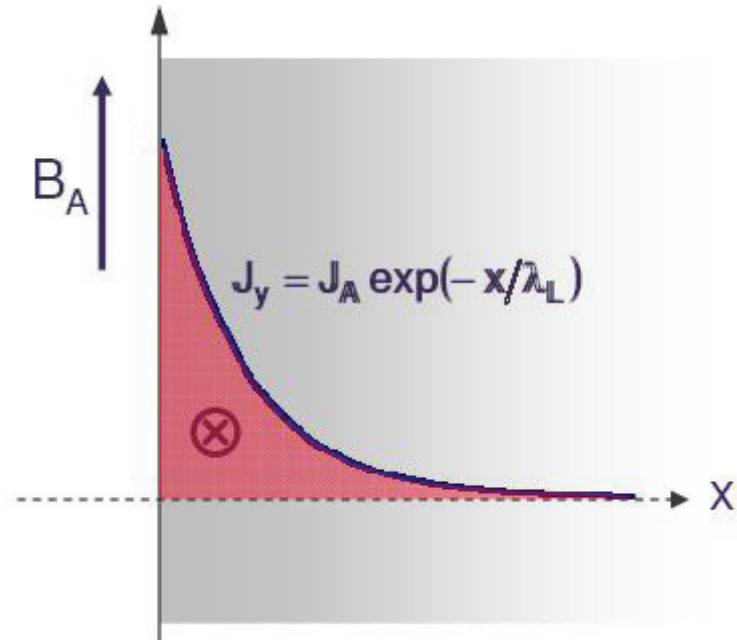
**The magnetic field, and the current, decay exponentially over a distance λ (a few 10s of nm)**

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# Model of F & H London (1935)

$$\lambda_L = \left[ \frac{m}{\mu_0 n_s e^2} \right]^{\frac{1}{2}}$$

From Gorter and Casimir two-fluid model

$$n_s \propto \left[ 1 - \left( \frac{T}{T_C} \right)^4 \right]$$

$$\lambda_L(T) = \lambda_L(0) \frac{1}{\left[ 1 - \left( \frac{T}{T_C} \right)^4 \right]^{\frac{1}{2}}}$$
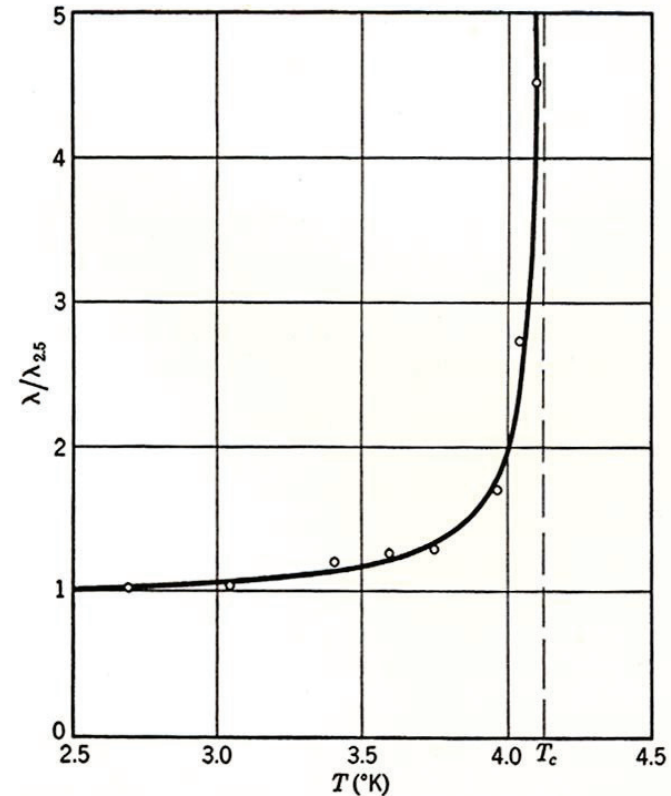


FIG. 21. Penetration depth as a function of temperature. (After Shoenberg, *Nature*, **43**, 433, 1939.)

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# Model of F & H London (1935)

London Equation: $\lambda^2 \nabla \times \vec{J}_s = -\dfrac{\vec{B}}{\mu_0} = -\vec{H}$

$$\nabla \times \vec{A} = \vec{H}$$

choose $\quad \nabla \cdot \vec{A} = 0, \quad\quad A_n = 0 \quad$ on sample surface $\quad$ (London gauge)

$$\boxed{\vec{J}_s = -\frac{1}{\lambda^2}\vec{A}}$$

Note: Local relationship between $\vec{J}_s$ and $\vec{A}$

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# Penetration Depth in Thin Films

**Very thin films**

**Very thick films**

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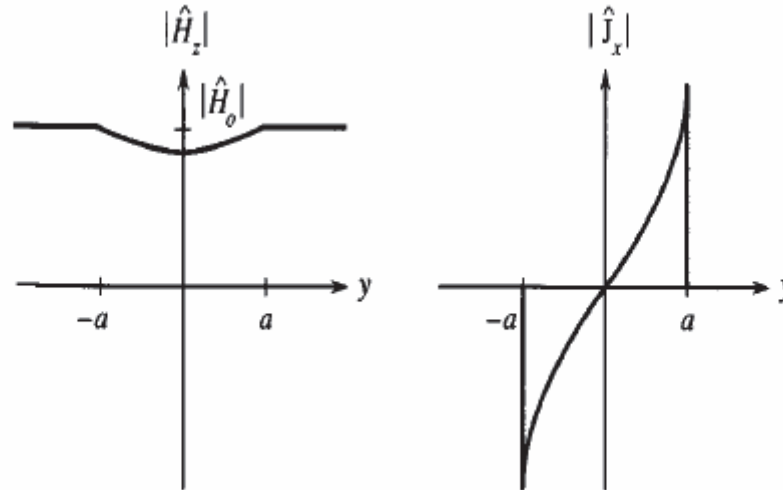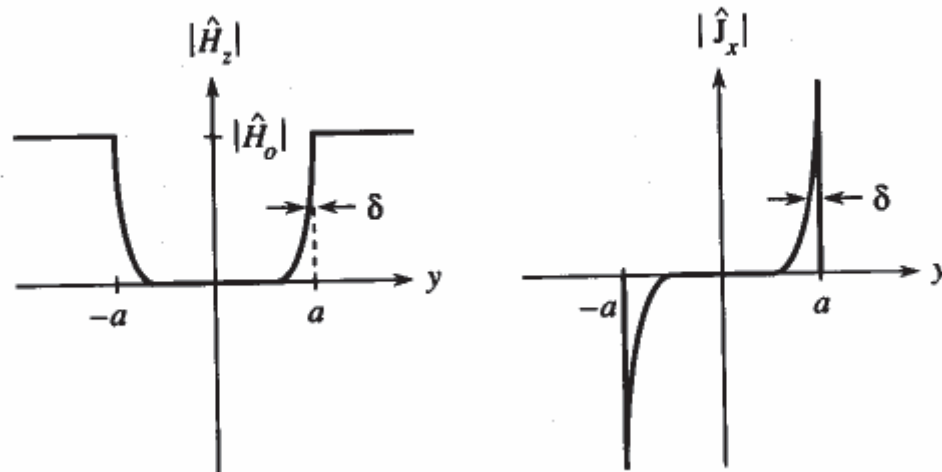
# Pippard's Extension of London's Model

**Observations:**

**-Penetration depth increased with reduced mean free path**

**- $H_c$ and $T_c$ did not change**

**-Need for a positive surface energy over $10^{-4}$ cm to explain existence of normal and superconducting phase in intermediate state**

**Non-local modification of London equation**

Local: 
$$\vec{J} = -\frac{1}{c\lambda}\vec{A}$$

Non local: 
$$\vec{J}(r) = -\frac{3\sigma}{4\pi\xi_0\lambda c}\int\frac{\vec{R}\left[\vec{R}\bullet\vec{A}(r')\right]e^{-\frac{R}{\xi}}}{R^4}d\upsilon$$

$$\frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{\ell}$$

# London Electrodynamics

Linear London equations

$$\frac{\partial \vec{J}_s}{\partial t} = -\frac{\vec{E}}{\lambda^2 \mu_0} \qquad \nabla^2 \vec{H} - \frac{1}{\lambda^2}\vec{H} = 0$$

together with Maxwell equations

$$\nabla \times \vec{H} = \vec{J}_s \qquad \nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

describe the electrodynamics of superconductors at all T if:

– The superfluid density $n_s$ is spatially uniform
– The current density $J_s$ is small

# Ginzburg-Landau Theory

- Many important phenomena in superconductivity occur because $n_s$ is not uniform
  - Interfaces between normal and superconductors
  - Trapped flux
  - Intermediate state

- London model does not provide an explanation for the surface energy (which can be positive or negative)

- GL is a generalization of the London model but it still retain the local approximation of the electrodynamics

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# Ginzburg-Landau Theory

- Ginzburg-Landau theory is a particular case of Landau's theory of second order phase transition

- Formulated in 1950, before BCS

- Masterpiece of physical intuition

- Grounded in thermodynamics

- Even after BCS it still is very fruitful in analyzing the behavior of superconductors and is still one of the most widely used theory of superconductivity

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# Ginzburg-Landau Theory

- Theory of second order phase transition is based on an order parameter which is zero above the transition temperature and non-zero below

- For superconductors, GL use a complex order parameter $\Psi(r)$ such that $|\Psi(r)|^2$ represents the density of superelectrons

- The Ginzburg-Landau theory is strictly valid close to $T_c$ but useful and applied over a wide range of temperature

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# Ginzburg-Landau Equation for Free Energy

- Assume that Ψ(r) is small and varies slowly in space

- Expand the free energy in powers of Ψ(r) and its derivative

$$f = f_{n0} + \alpha|\psi|^2 + \frac{\beta}{2}|\psi|^4 + \frac{1}{2m^*}\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\mathbf{A}\right)\psi\right|^2 + \frac{h^2}{8\pi}$$

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# Field-Free Uniform Case

**Identify the order parameter with the density of superelectrons**

$$n_s = \left|\Psi\right|^2 \sim \frac{1}{\lambda_L^2(T)} \Rightarrow \frac{\lambda_L^2(0)}{\lambda_L^2(T)} = \frac{\left|\Psi(T)\right|^2}{\left|\Psi(0)\right|^2} = -\frac{1}{n}\frac{\alpha(T)}{\beta}$$

The parameters $\alpha$ and $\beta$ are related to measurable quantities

$$n\alpha(T) = -\frac{H_c^2(T)}{4\pi}\frac{\lambda_L^2(T)}{\lambda_L^2(0)} \quad \text{and} \quad n^2\beta = \frac{H_c^2(T)}{4\pi}\frac{\lambda_L^4(T)}{\lambda_L^4(0)}$$

# Field-Free Nonuniform Case

**Equation of motion in the absence of electromagnetic field**

$$-\frac{1}{2m^*}\nabla^2\psi + \alpha(T)\psi + \beta|\psi|^2\psi = 0$$

**Look at solutions close to the constant one**

$$\psi = \psi_\infty + \delta \quad \text{where} \quad |\psi_\infty|^2 = -\frac{\alpha(T)}{\beta}$$

**To first order:**
$$\frac{1}{4m^*|\alpha(T)|}\nabla^2\delta - \delta = 0$$

**Which leads to** $\qquad \delta \approx e^{-\sqrt{2}\,r/\xi(T)}$

# Field-Free Nonuniform Case

$$\delta \approx e^{-\sqrt{2}\, r / \xi(T)} \quad \text{where} \quad \xi(T) = \frac{1}{\sqrt{2 m^* |\alpha(T)|}} = \sqrt{\frac{2\pi n}{m^* H_c^2(T)}} \frac{\lambda_L(0)}{\lambda_L(T)}$$

**is the Ginzburg-Landau coherence length.**

**It is different from, but related to, the Pippard coherence length.** $\quad \xi(T) \simeq \dfrac{\xi_0}{\left(1 - t^2\right)^{1/2}}$

**GL parameter:** $\qquad \kappa(T) = \dfrac{\lambda_L(T)}{\xi(T)}$

Both $\lambda_L(T)$ and $\xi(T)$ diverge as $T \to T_c$ but their ratio remains finite

$\kappa(T)$ is almost constant over the whole temperature range

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# 2 Fundamental Lengths

**London penetration depth: length over which magnetic field decay**

$$\lambda_L(T) = \left(\frac{m^*\beta}{2e^2\alpha'}\right)^{1/2} \sqrt{\frac{T_c}{T_c - T}}$$

**Coherence length: scale of spatial variation of the order parameter (superconducting electron density)**

$$\xi(T) = \left(\frac{\hbar^2}{4m^*\alpha'}\right)^{1/2} \sqrt{\frac{T_c}{T_c - T}}$$

**The critical field is directly related to those 2 parameters**

$$H_c(T) = \frac{\phi_0}{2\sqrt{2}\,\xi(T)\,\lambda_L(T)}$$

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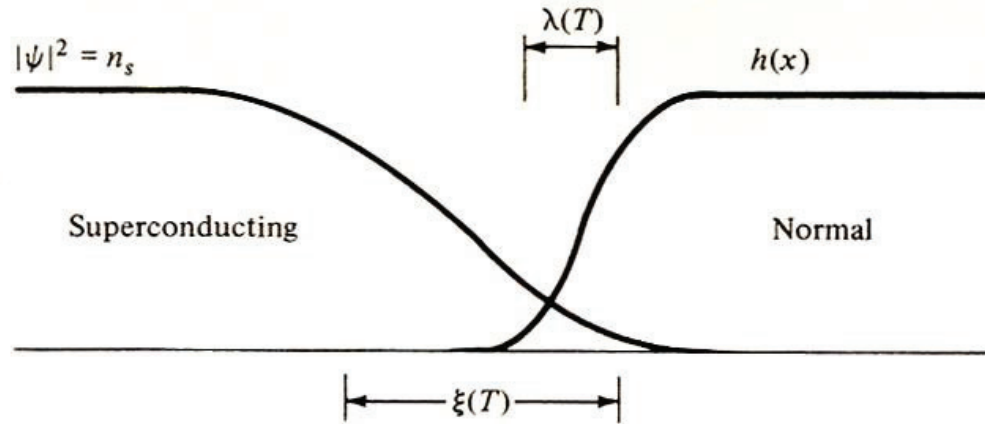
# Surface Energy



$$\sigma \simeq \frac{1}{8\pi}\left[H_c^2\xi - H^2\lambda\right]$$

$\dfrac{H^2\lambda}{8\pi}$ :   Energy that can be gained by letting the fields penetrate

$\dfrac{H_c^2\xi}{8\pi}$ :   Energy lost by "damaging" superconductor

# **Surface Energy**    $\sigma \simeq \dfrac{1}{8\pi}\left[ H_c^2 \xi - H^2 \lambda \right]$

Interface is stable if $\sigma$>0

    If $\xi \gg \lambda$         $\sigma$>0

         Superconducting up to $H_c$ where superconductivity is destroyed globally

    If $\lambda \gg \xi$    $\sigma$<0   for $H^2 > H_c^2 \dfrac{\xi}{\lambda}$

Advantageous to create small areas of normal state with large area to volume ratio
$\rightarrow$ quantized fluxoids

More exact calculation (from Ginzburg-Landau):

$$\kappa = \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}} \qquad : \text{Type I}$$

$$\kappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}} \qquad : \text{Type II}$$

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# Critical Fields

**Even though it is more energetically favorable for a type I superconductor to revert to the normal state at H$_c$, the surface energy is still positive up to a superheating field H$_{sh}$>H$_c$ → metastable superheating region in which the material may remain superconducting for short times.**

Type I      $H_c$      Thermodynamic critical field

$$H_{sh} \simeq \frac{H_c}{\sqrt{\kappa}} \text{ Superheating critical field}$$

Field at which surface energy is

Type II      $H_c$      Thermodynamic critical field

$$H_{c2} = \sqrt{2}\ \kappa\ H_c$$

$$H_{c1} \simeq \frac{H_c^2}{H_{c2}}$$

$$\simeq \frac{1}{2\kappa}(\ln \kappa + .008)H_c \qquad \text{(for } \kappa \gg 1\text{)}$$
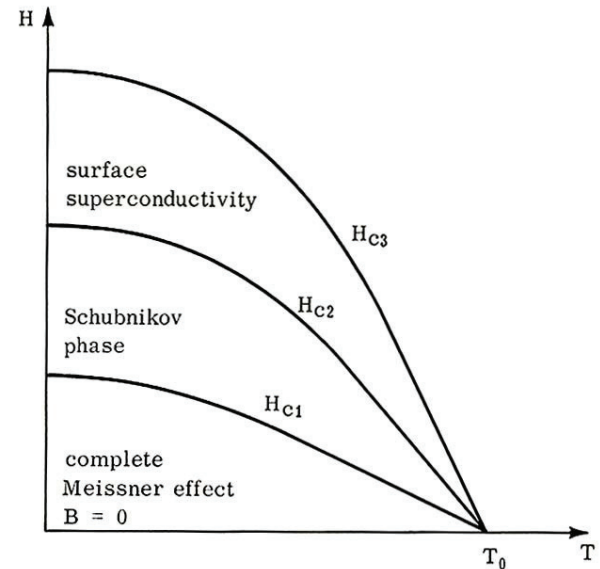


**Figure 3–1**
Phase diagram for a long cylinder of a Type II superconductor.

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# Superheating Field

Ginsburg-Landau:

$$H_{sh} \sim \frac{0.9 H_c}{\sqrt{\kappa}} \quad \text{for } \kappa \ll 1$$

$$\sim 1.2 \; H_c \quad \text{for } \kappa \sim 1$$

$$\sim 0.75 \; H_c \quad \text{for } \kappa \gg 1$$

**The exact nature of the rf critical field of superconductors is still an open question**
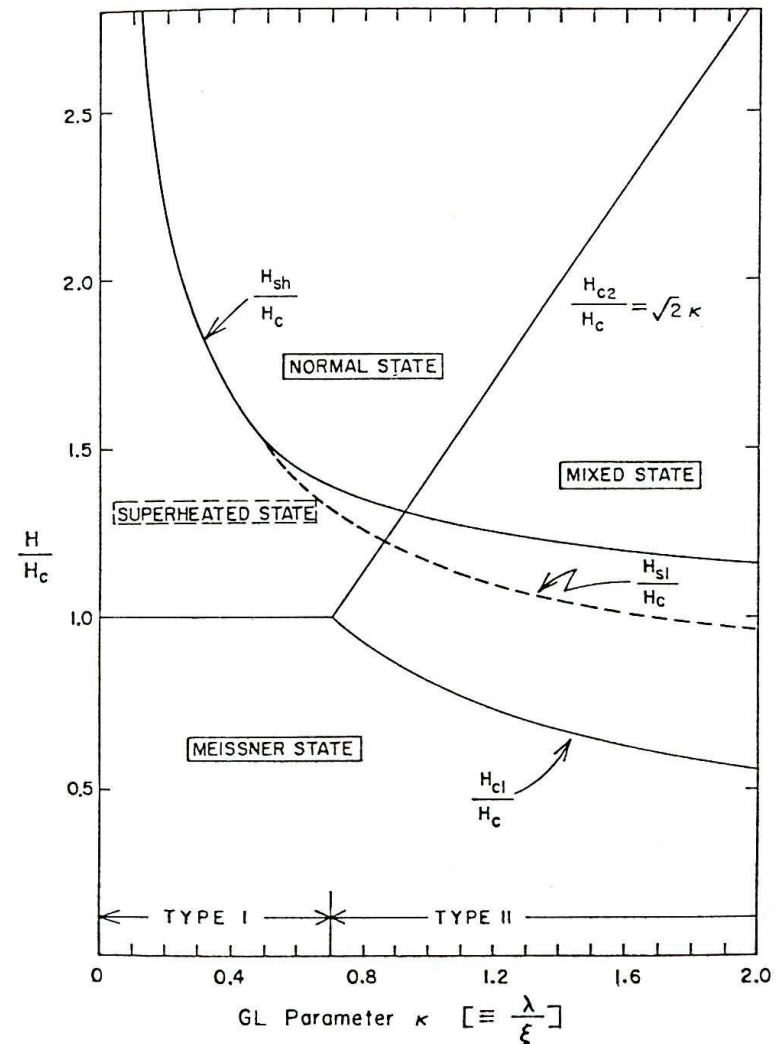


Fig. 13: Phase diagram of superconductors[42] in the transition regime of type I and II. The normalized critical fields are shown as a function of $\kappa$.

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# Material Parameters for Some Superconductors

| Superconductor | $\lambda_L(0)$ (nm) | $\xi_0$ (nm) | $\kappa$ | $2\Delta(0)/kT_c$ | $T_c$(K) |
|---|---|---|---|---|---|
| Al | 16 | 1500 | 0.011 | 3.40 | 1.18 |
| In | 25 | 400 | 0.062 | 3.50 | 3.3 |
| Sn | 28 | 300 | 0.093 | 3.55 | 3.7 |
| Pb | 28 | 110 | 0.255 | 4.10 | 7.2 |
| Nb | 32 | 39 | 0.82 | 3.5-3.85 | 8.95-9.2 |
| Ta | 35 | 93 | 0.38 | 3.55 | 4.46 |
| $Nb_3Sn$ | 50 | 6 | 8.3 | 4.4 | 18 |
| NbN | 50 | 6 | 8.3 | 4.3 | $\leq 17$ |
| $Yba_2Cu_3o_x$ | 140 | 1.5 | 93 | 4.5 | 90 |

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# BCS

- What needed to be explained and what were the clues?

  - Energy gap (exponential dependence of specific heat)

  - Isotope effect (the lattice is involved)

  - Meissner effect



Figure 26: The critical temperature of various tin isotopes.

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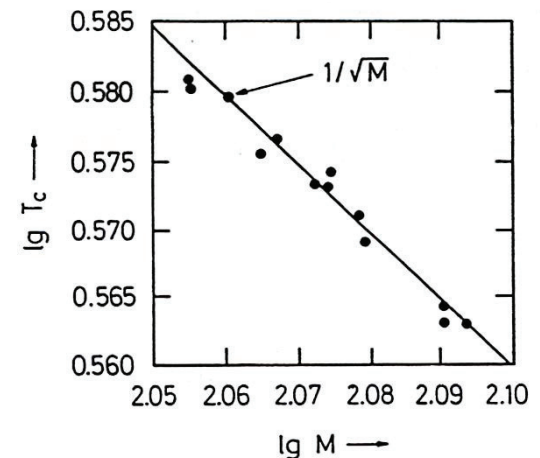
# Cooper Pairs

Assumption: Phonon-mediated attraction between electron of equal and opposite momenta located within $\hbar\omega_D$ of   Fermi surface

Moving electron distorts lattice and leaves behind a trail of positive charge that attracts another electron moving in opposite direction
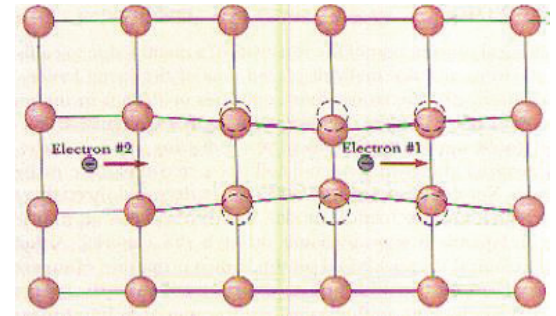


Fermi ground state is unstable

Electron pairs can form bound states of lower energy

Bose condensation of overlapping Cooper pairs into a coherent Superconducting state
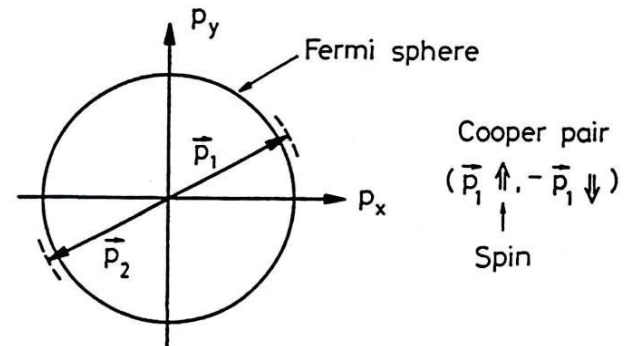


Figure 20: A pair of electrons of opposite momenta added to the full Fermi sphere.

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# BCS
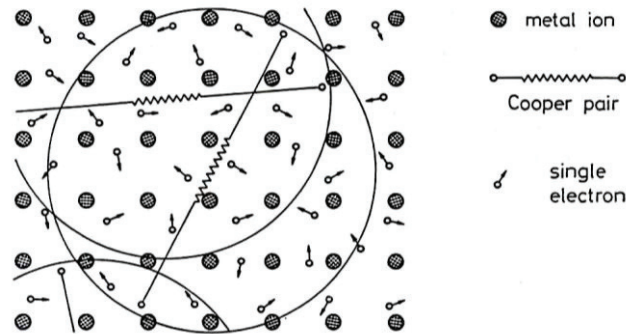


Figure 22: Cooper pairs and single electrons in the crystal lattice of a superconductor. (After Essmann and Träuble [12]).

metal ion

Cooper pair

single electron

$P_y$

Fermi sphere, $\vec{p}^2/2m = E_F$

$\stackrel{\frown}{=} E_F \pm \hbar\omega_D$

$P_x$

possible Cooper pairs:
$(\vec{p}, -\vec{p}), (\vec{p}', -\vec{p}')$
$(\vec{p}'', -\vec{p}''), \ldots\ldots$
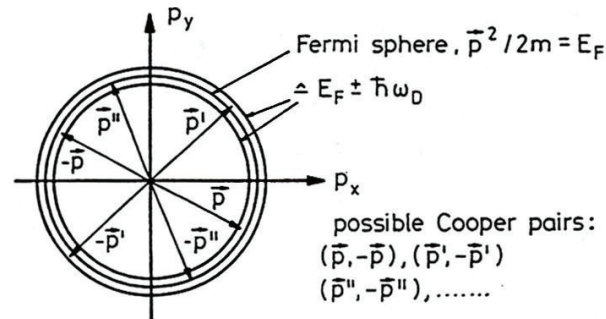
Figure 23: Various Cooper pairs $(\vec{p}, -\vec{p})$, $(\vec{p}', -\vec{p}')$, $(\vec{p}'', -\vec{p}'')$, ... in momentum space.

The size of the Cooper pairs is much larger than their spacing

They form a coherent state

# BCS

- The BCS model is an extremely simplified model of reality
  - The Coulomb interaction between single electrons is ignored
  - Only the term representing the scattering of pairs is retained
  - The interaction term is assumed to be constant over a thin layer at the Fermi surface and 0 everywhere else
  - The Fermi surface is assumed to be spherical

- Nevertheless, the BCS results (which include only a very few adjustable parameters) are amazingly close to the real world

# BCS

**Critical temperature**

$$kT_c = 1.14\,\hbar\omega_D \; \exp\left[-\frac{1}{VN(E_F)}\right]$$

$$\Delta(0) = 1.76\,kT_c$$

| element | Sn | In | Tl | Ta | Nb | Hg | Pb |
|---------|------|-----|-----|------|------|-----|------|
| $\Delta(0)/k_B T_c$ | 1.75 | 1.8 | 1.8 | 1.75 | 1.75 | 2.3 | 2.15 |

**Coherence length (the size of the Cooper pairs)**

$$\xi_0 = .18\frac{\hbar\upsilon_F}{kT_c}$$

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# BCS Energy Gap

**At finite temperature:**

**Implicit equation for the temperature dependence of the gap:**

$$\frac{1}{V\rho(0)} = \int_0^{\hbar\omega_D} \frac{\tanh\left[(\varepsilon^2 + \Delta^2)^{1/2} / 2kT\right]}{(\varepsilon^2 + \Delta^2)^{1/2}} d\varepsilon$$

$\Delta(T)$

$\Delta(0) = 1.76k_B T_0$

$\Delta \sim 3.2k_B T_0 [1 - (T/T_0)]^{1/2}$

$1 \quad T/T_0$
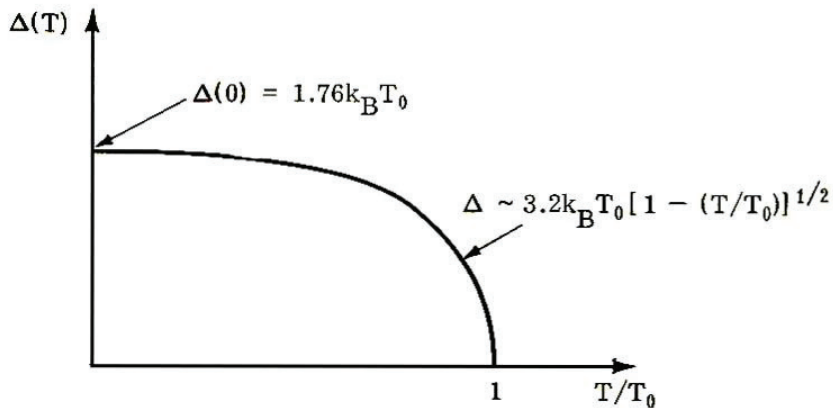
**Figure 4-4**

Variation of the order parameter $\Delta$ with temperature in the BCS approximation.

BCS theory

In
Sn
Pb

$T/T_c$

$\Delta(T)/\Delta(0)$
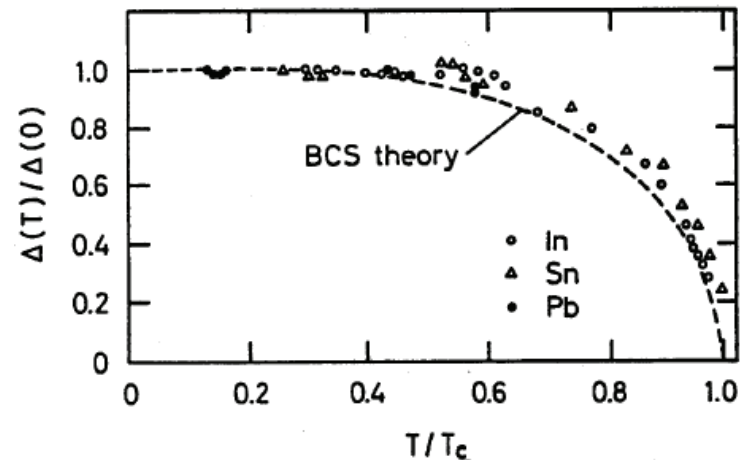
# BCS Specific Heat

Specific heat

$$C_{es} \simeq \quad \exp\left(-\frac{\Delta}{kT}\right) \ \text{for} \ T < \frac{T_c}{10}$$
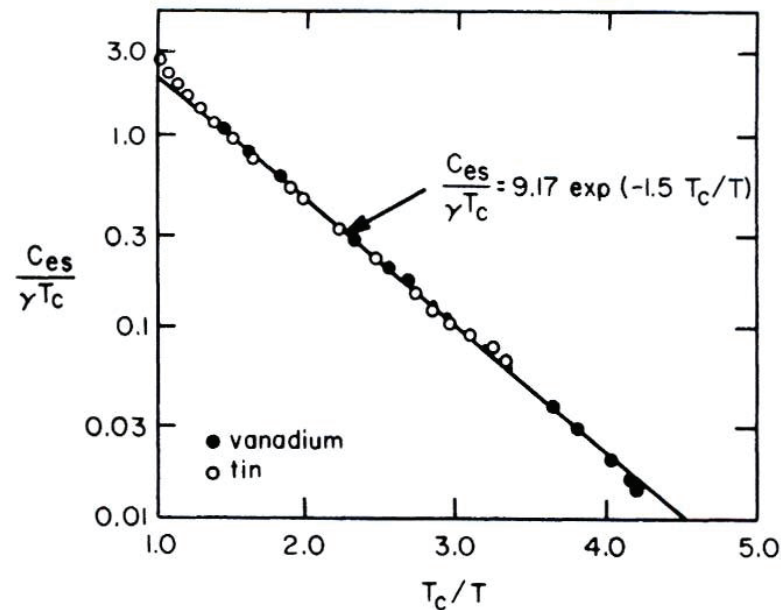


Fig. 22.   Reduced electronic specific heat in superconducting vanadium and tin.
[From Biondi et al., (150).]

# Electrodynamics and Surface Impedance in BCS Model

$$H_0\phi + H_{ex}\ \phi = i\hbar\frac{\partial\phi}{\partial t}$$

$$H_{ex} = \frac{e}{mc}\sum A(r_i,t)p_i$$

$H_{ex}$ is treated as a small perturbation

$$H_{rf} << H_c$$

There is, at present, no model for superconducting surface resistance at high rf field

$$J \propto \int \frac{R[R\cdot A]I(\omega,R,T)e^{-\frac{R}{l}}}{R^4}dr \qquad \text{similar to Pippard's model}$$

$$J(k) = -\frac{c}{4\pi}K(k)A(k)$$

$$K(0) \neq 0: \quad \text{Meissner effect}$$

# Penetration Depth

$$\lambda = \frac{2}{\pi} \int \frac{dk}{K(k)+k^2}\,dk \qquad \text{(specular)}$$

Represented accurately by $\lambda \sim \dfrac{1}{\sqrt{1-\left(\dfrac{T_c}{T}\right)^4}}$    near $T_c$
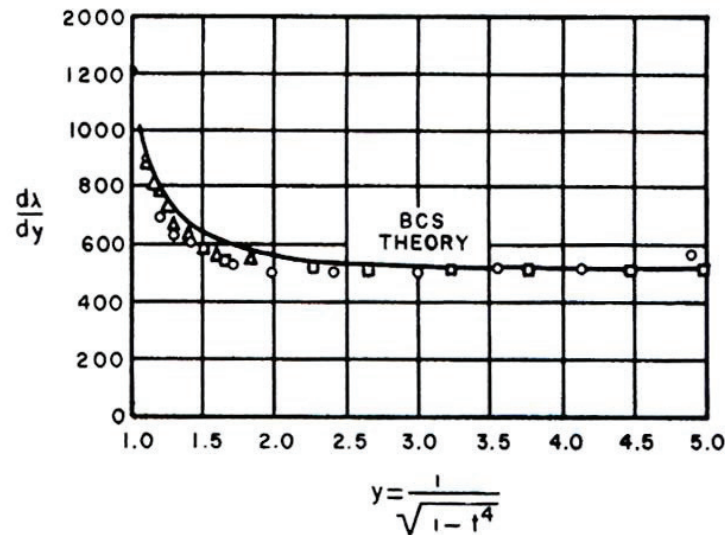
**Fig. 30.** Temperature dependence of $d\lambda/dy$ for tin obtained by Schawlow and Devlin (*207*) compared with the theoretical curve obtained from the BCS theory.

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# Surface Resistance of Superconductors

Superconductors are free of power dissipation in static fields.

In microwave fields, the time-dependent magnetic field in the penetration depth will generate an electric field.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

The electric field will induce oscillations in the normal electrons, which will lead to power dissipation

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# Surface Impedance in the Two-Fluid Model

$$R_s \simeq \frac{1}{\lambda_L} \frac{\sigma_n}{\sigma_s^2}$$

$$\sigma_n = \frac{n_n e^2 l}{m_e v_F} \propto l \ \exp\left[-\frac{\Delta(T)}{kT}\right] \qquad \sigma_s = \frac{1}{\mu_0 \lambda_L^2 \omega}$$

$$\boxed{R_s \propto \lambda_L^3 \ \omega^2 \ l \ \exp\left[-\frac{\Delta(T)}{kT}\right]}$$

This assumes that the mean free path is much larger than the coherence length

For niobium we need to replace the London penetration depth with

$$\Lambda = \lambda_L \sqrt{1 + \xi / l}$$

As a result, the surface resistance shows a minimum when $\xi \approx l$

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# Surface Resistance of Niobium



Surface Resistance - Nb - 1500 MHz
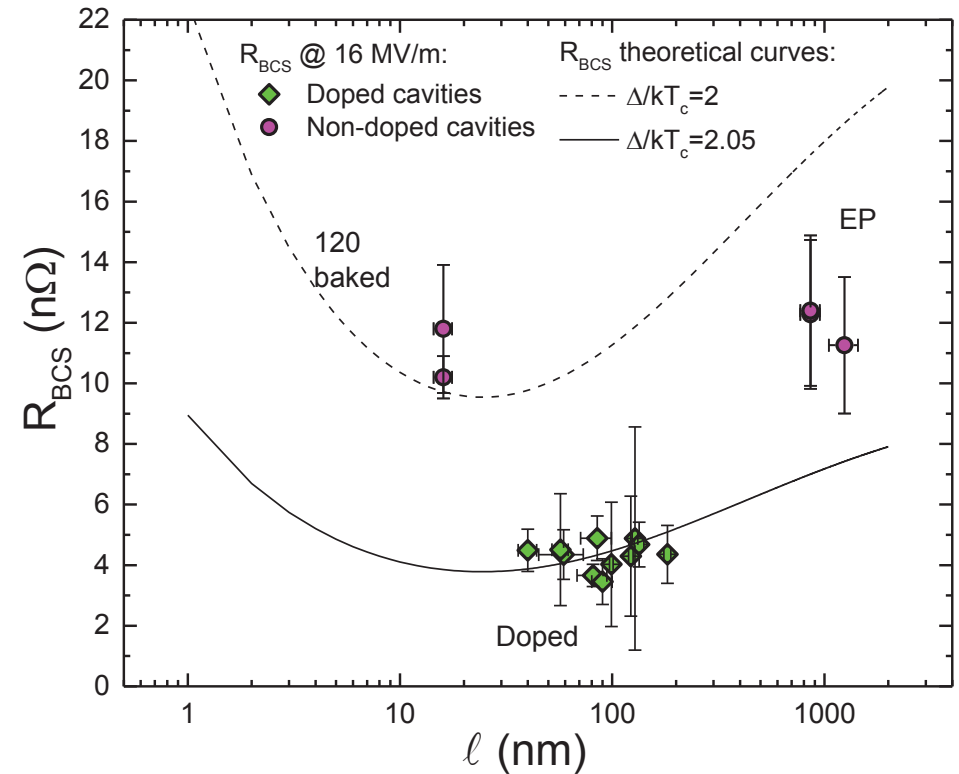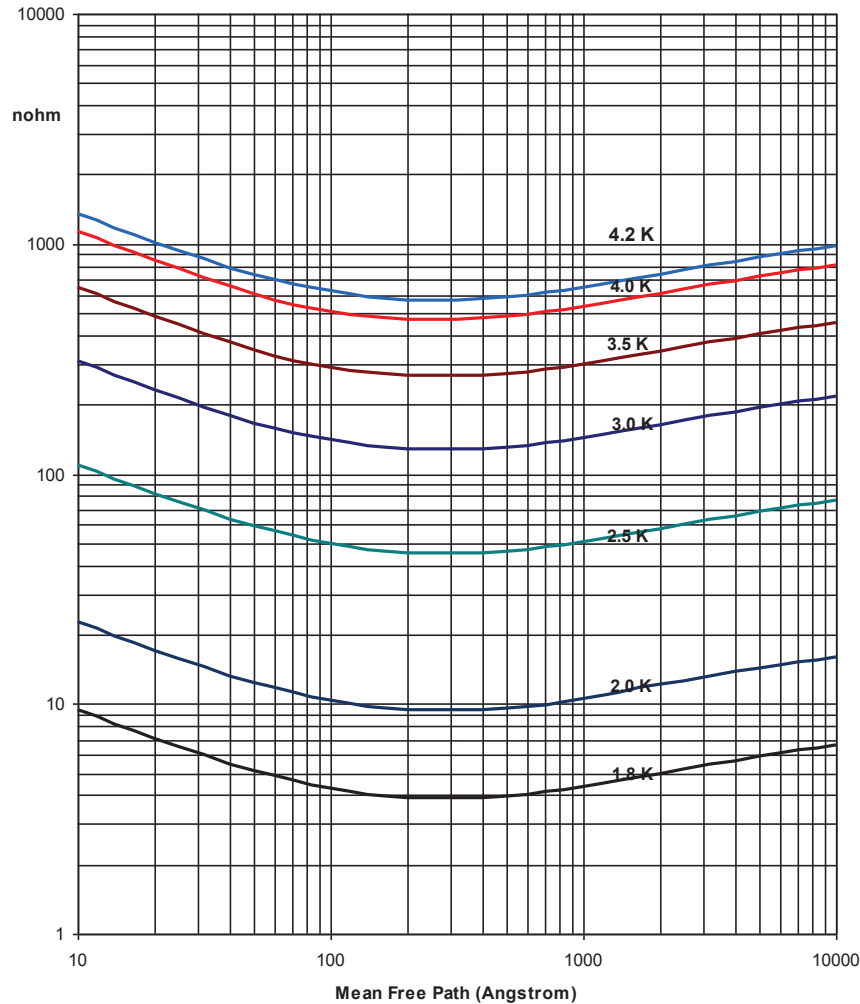
# Surface Resistance of Superconductors

Temperature dependence

–close to $T_c$ :

dominated by change in $\lambda(t)$  $\dfrac{t^4}{\left(1-t^2\right)^{3/2}}$

–for $T < \dfrac{T_c}{2}$ :

dominated by density of excited states $\sim e^{-\Delta/kT}$

$$R_s \sim \frac{A}{T}\omega^2 \exp\left(-\frac{\Delta}{kT}\right)$$

Frequency dependence

$\omega^2$ is a good approximation

A reasonable formula for the BCS surface resistance of niobium is

$$R_{BCS} = 9\times10^{-5}\,\frac{f^2\,(\text{GHz})}{T}\exp\left(-1.83\frac{T_c}{T}\right)$$
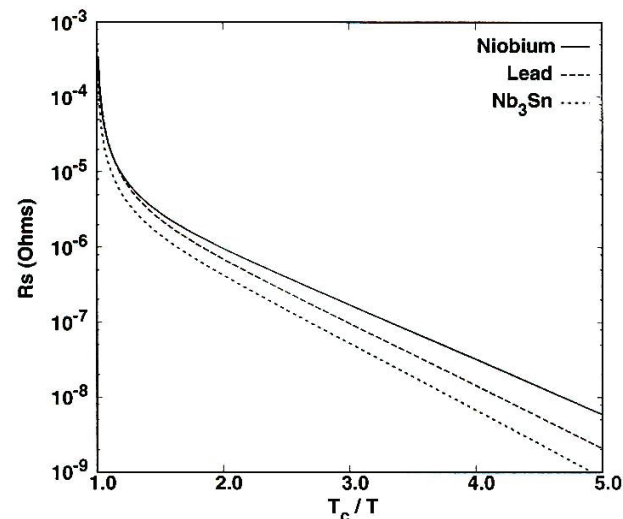


Figure 4.5: Theoretical surface resistance at 1.5 GHz of lead, niobium and Nb$_3$Sn as calculated from program [94]. The values given in Table 4.1 were used for the material parameters.

# Surface Resistance of Superconductors

- The surface resistance of superconductors depends on the frequency, the temperature, and a few material parameters
  - Transition temperature
  - Energy gap
  - Coherence length
  - Penetration depth
  - Mean free path

- A good approximation for $T < T_c/2$ and $\omega \ll \Delta/h$ is

$$R_s \sim \frac{A}{T}\omega^2 \exp\left(-\frac{\Delta}{kT}\right) + R_{res}$$

# Surface Resistance of Superconductors
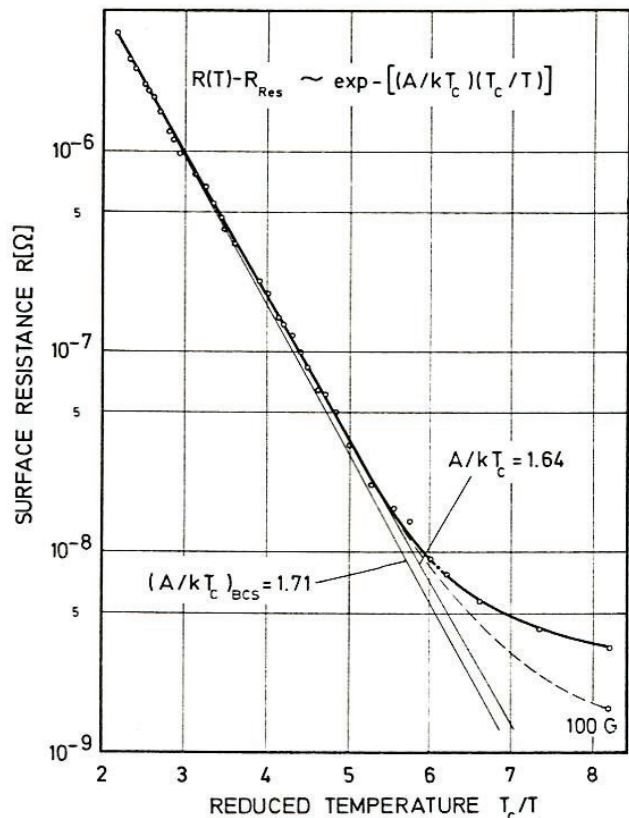


Fig. 2. Temperature dependence of surface resistance of niobium at 3.7 GHz measured in the $TE_{011}$ mode at $H_{rf} \simeq 10$ G. The values computed with the BCS theory used the following material parameters:

$T_c = 9.25$ K; $\quad \lambda_L(T=0, l=\infty) = 320$ Å;

$\Delta(0)/kT = 1.85$; $\quad \xi_F(T=0, l=\infty) = 620$ Å; $\quad l = 1\,000$ Å or 80 Å.
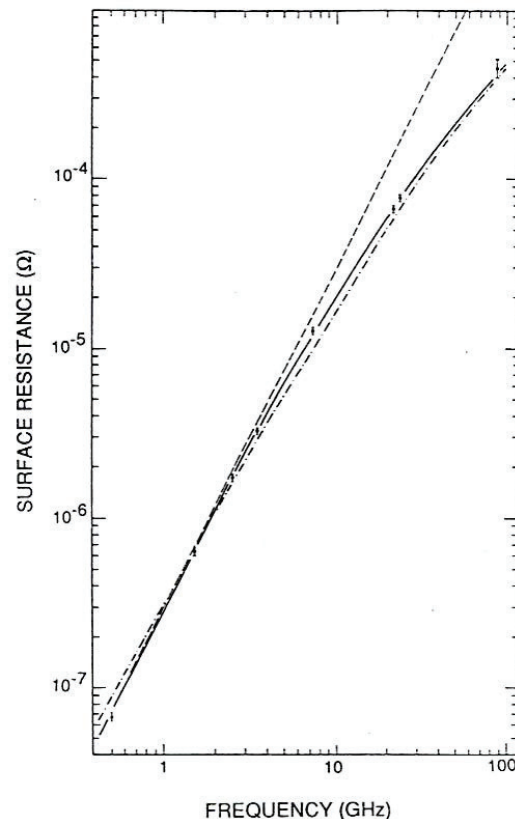


Fig. 5. The surface resistance of Nb at 4.2 K as a function of frequency [62,63]. Whereas the isotropic BCS surface resistance $(-\cdot-\cdot)$ resulted in $R \propto \omega^{1.8}$ around 1 GHz, the measurements fit better to $\omega^2$ $(---)$. The solid curve, which fits the data over the entire range, is a calculation based on the smearing of the BCS density-of-states singularity by the energy gap anisotropy in the presence of impurity scattering [61]. The authors thank G. Müller for providing this figure.

# Super and Normal Conductors

- Normal Conductors
  - Skin depth proportional to $\omega^{-1/2}$
  - Surface resistance proportional to $\omega^{1/2 \to 2/3}$
  - Surface resistance independent of temperature (at low T)
  - For Cu at 300K and 1 GHz, $R_s$=8.3 m$\Omega$

- Superconductors
  - Penetration depth independent of $\omega$
  - Surface resistance proportional to $\omega^2$
  - Surface resistance strongly dependent of temperature
  - For Nb at 2 K and 1 GHz, $R_s \approx$7 n$\Omega$

## However: do not forget Carnot

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# SRF CAVITIES

## Jean Delayen

**Center for Accelerator Science
Old Dominion University
and
Thomas Jefferson National Accelerator Facility**

# Design Considerations

$$\frac{H_{s,\max}}{E_{acc}}$$  minimum  critical field

$$\frac{E_{s,\max}}{E_{acc}}$$  minimum  field emission

$$\frac{<H_s^2>}{E_{acc}^2}$$  minimum  shunt impedance, current losses

$$\frac{<E_s^2>}{E_{acc}^2}$$  minimum  dielectric losses

$$\frac{U}{E_{acc}^2}$$  minimum  control of microphonics

maximum  voltage drop for high charge per bunch

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# TM-CLASS CAVITIES

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# 350 MHz, 4-cell, Nb on Cu

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# 1500 MHz, 5-cell

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# 1300 MHz 9-cell

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# Pill Box Cavity

**Hollow right cylindrical enclosure**

**Operated in the TM$_{010}$ mode** $\quad H_z = 0$

$$\frac{\partial^2 E_z}{\partial^2 r} + \frac{1}{r}\frac{\partial E_z}{\partial r} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial^2 t} \qquad \omega_0 = \frac{2.405c}{R}$$

$$E_z(r,z,t) = E_0 J_0\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$

$$H_\varphi(r,z,t) = -i\frac{E_0}{\mu_0 c}J_1\left(2.405\frac{r}{R}\right)e^{-i\omega_0 t}$$



$H_\phi$

$E_z = E_o$

$R$

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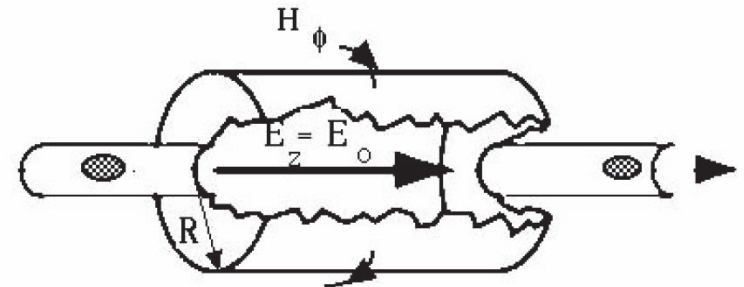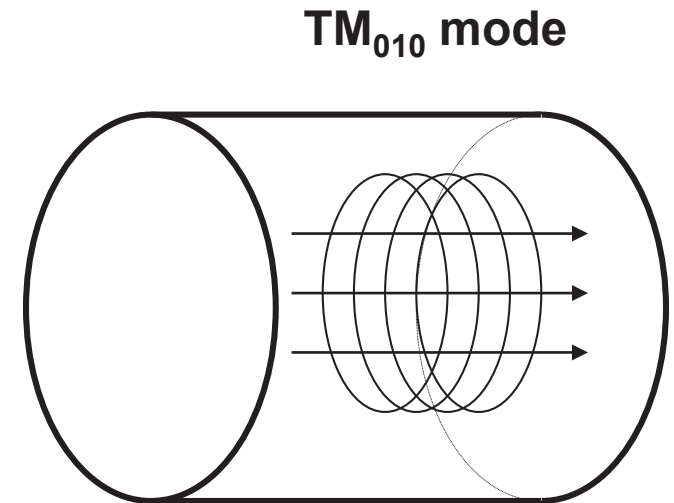
# TM$_{010}$ Mode in a Pill Box Cavity

$$E_r = E_\varphi = 0 \qquad\qquad E_z = E_0 J_0\left(x_{01}\frac{r}{R}\right)$$

$$H_r = H_z = 0 \qquad\qquad H_\varphi = -i\omega\varepsilon E_0 \frac{R}{x_{01}} J_1\left(x_{01}\frac{r}{R}\right)$$

$$\omega = x_{01}\frac{c}{R} \qquad\qquad x_{01} = 2.405$$

$$R = \frac{x_{01}}{2\pi}\lambda = 0.383\lambda$$

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# TM$_{010}$ Mode in a Pill Box Cavity

**Energy content**

$$U = \varepsilon_0 E_0^2 \frac{\pi}{2} J_1^2(x_{01}) L R^2$$

**Power dissipation**

$$P = E_0^2 \frac{R_s}{\eta^2} \pi J_1^2(x_{01})(R+L)R$$

$$x_{01} = 2.40483$$

$$J_1(x_{01}) = 0.51915$$

**Geometrical factor**

$$G = \eta \frac{x_{01}}{2} \frac{L}{(R+L)}$$

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# TM$_{010}$ Mode in a Pill Box Cavity

## Energy Gain

$$\Delta W = E_0 \frac{\lambda}{\pi} \sin \frac{\pi L}{\lambda}$$

## Gradient

$$E_{acc} = \frac{\Delta W}{\lambda / 2} = E_0 \frac{2}{\pi} \sin \frac{\pi L}{\lambda}$$

## Shunt impedance

$$R_{sh} = \frac{\eta^2}{R_s} \frac{1}{\pi^3 J_1^2(x_{01})} \frac{\lambda^2}{R(R+L)} \sin^2\left(\frac{\pi L}{\lambda}\right)$$

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# Real Cavities

**Beam tubes reduce the electric field on axis**

> **Gradient decreases**
>
> **Peak fields increase**
>
> **R/Q decreases**

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# Real Cavities



TM010

$f = 1323$ MHz

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# Single Cell Cavities



**Electric field high at iris**

**Magnetic field high at equator**

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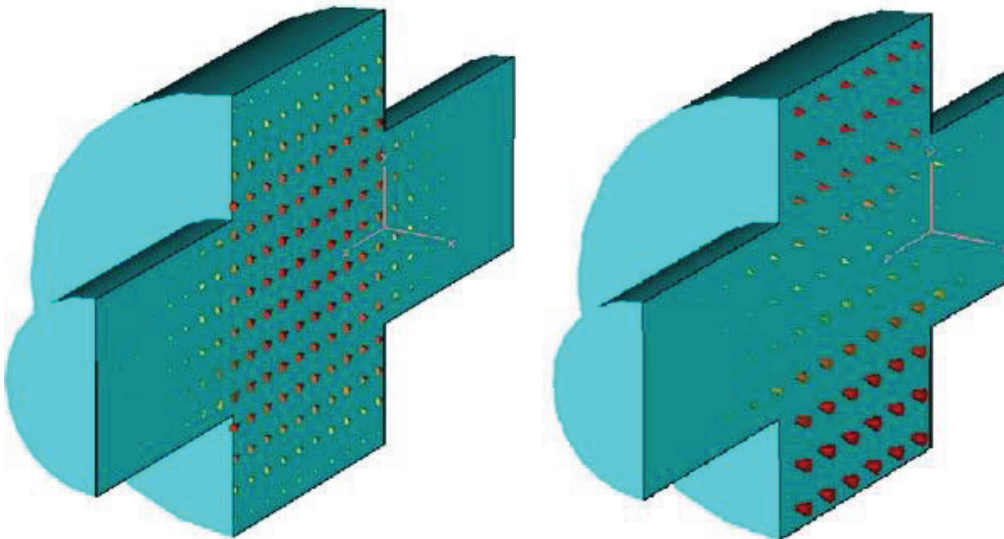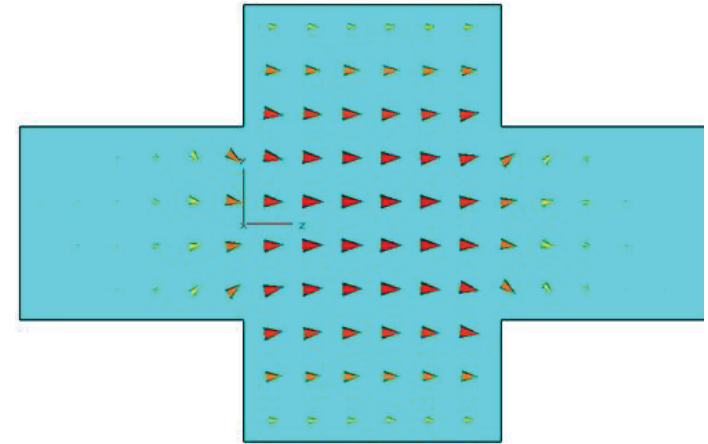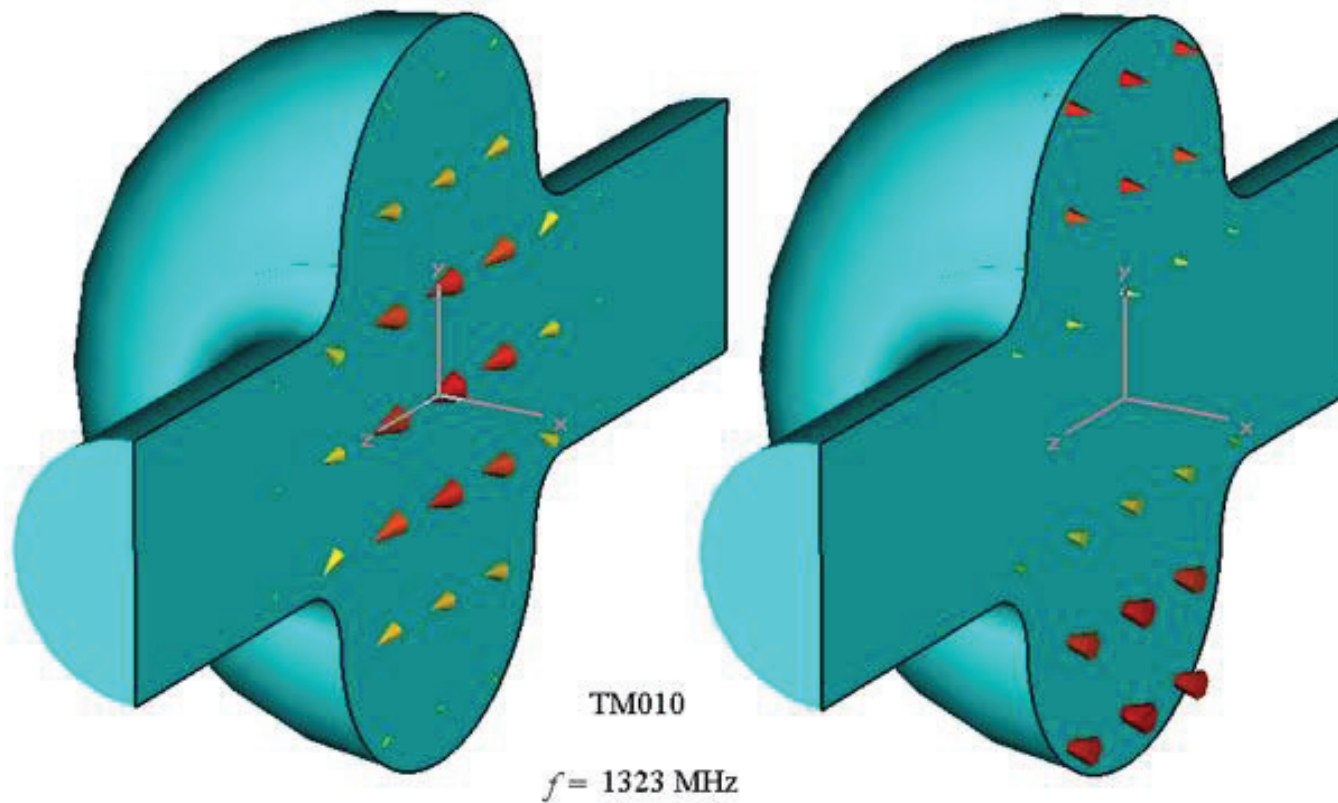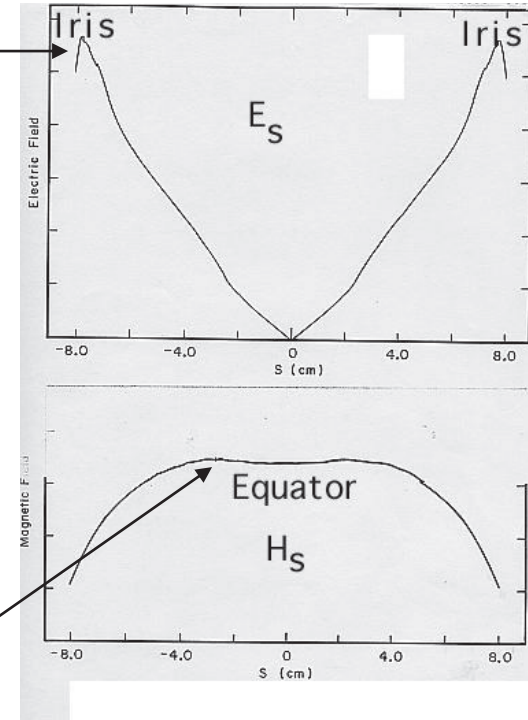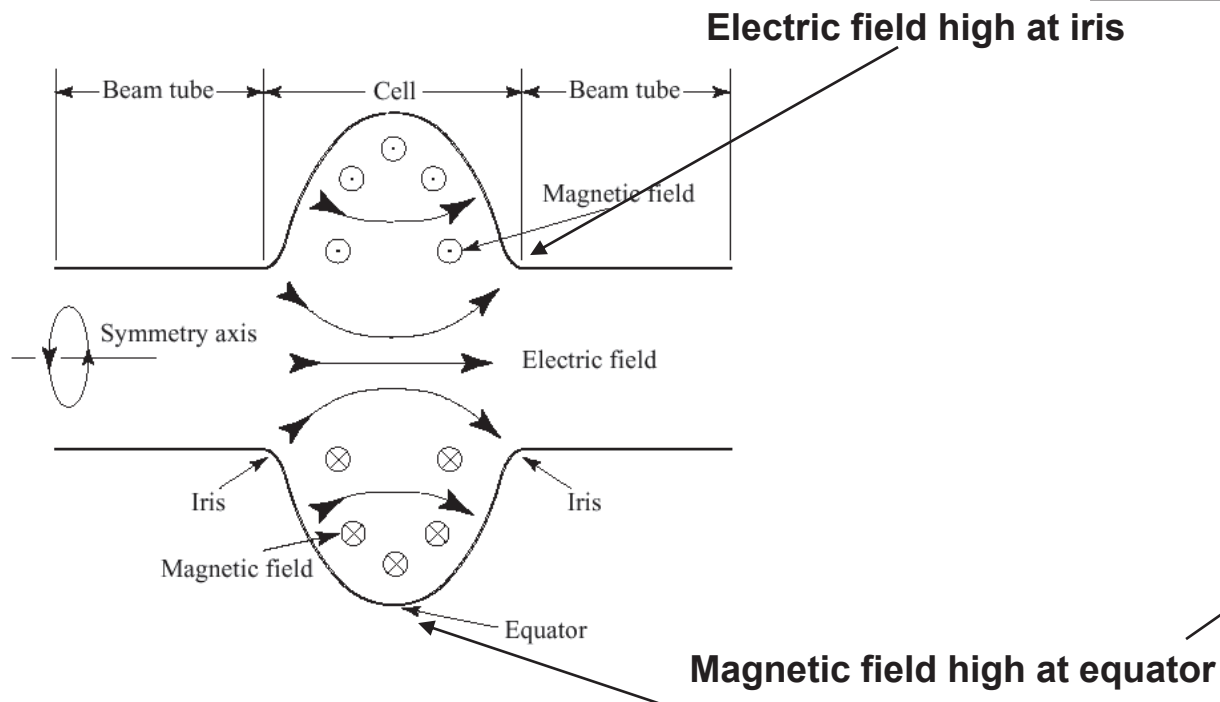
# Cell Shape Design

- What is the purpose of the cavity?
- What EM parameters should be optimized to meet the design specs?

## The "perfect" shape does not exist, it all depends on your application

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# New Trend in TM-Cavity Design

• The **field emission is not a hard limit** in the performance of sc cavities if the surface preparation is done in the right way.

• **Magnetic flux on the wall** limits performance of a sc cavity ($Q_0$ decreases or/and quench). Hard limit ~**180 mT** for Nb.

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# "Rule of thumb" for Optimizing Peak Surface Fields

**Add "*magnetic volume*" at the equator to reduce B<sub>peak</sub>**

Add "*magnetic volume*" at the equator to reduce $B_{peak}$

**Add "*electric volume*" at the iris to reduce E<sub>peak</sub>**

Add "*electric volume*" at the iris to reduce $E_{peak}$

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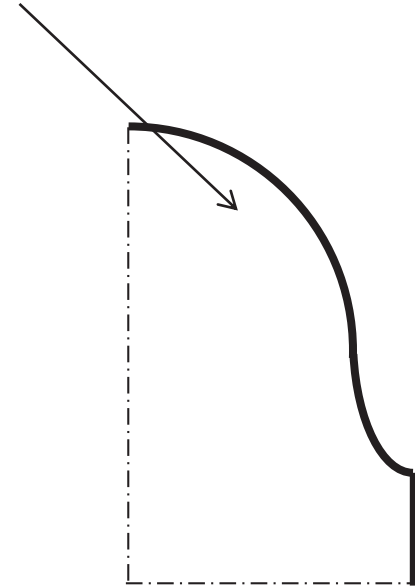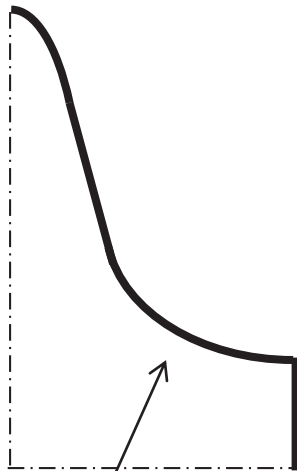
# New Shapes for ILC
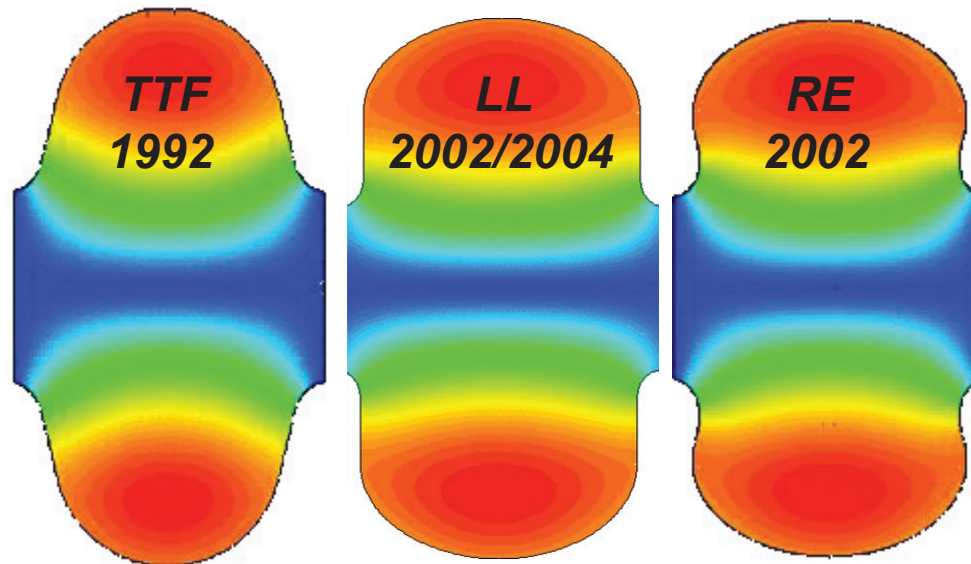


| | | TTF 1992 | LL 2002/2004 | RE 2002 |
|---|---|---|---|---|
| $r_{iris}$ | [mm] | 35 | 30 | 33 |
| $k_{cc}$ | [%] | 1.9 | 1.52 | 1.8 |
| $E_{peak}/E_{acc}$ | - | 1.98 | 2.36 | 2.21 |
| $B_{peak}/E_{acc}$ | [mT/(MV/m)] | 4.15 | 3.61 | 3.76 |
| $R/Q$ | [Ω] | 113.8 | 133.7 | 126.8 |
| $G$ | [Ω] | 271 | 284 | 277 |
| $R/Q*G$ | [Ω*Ω] | 30840 | 37970 | 35123 |

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# SUPERFISH

- Free, 2D finite-difference code to design cylindrically symmetric structures (monopole modes only)
- Use symmetry planes to reduce number of mesh points
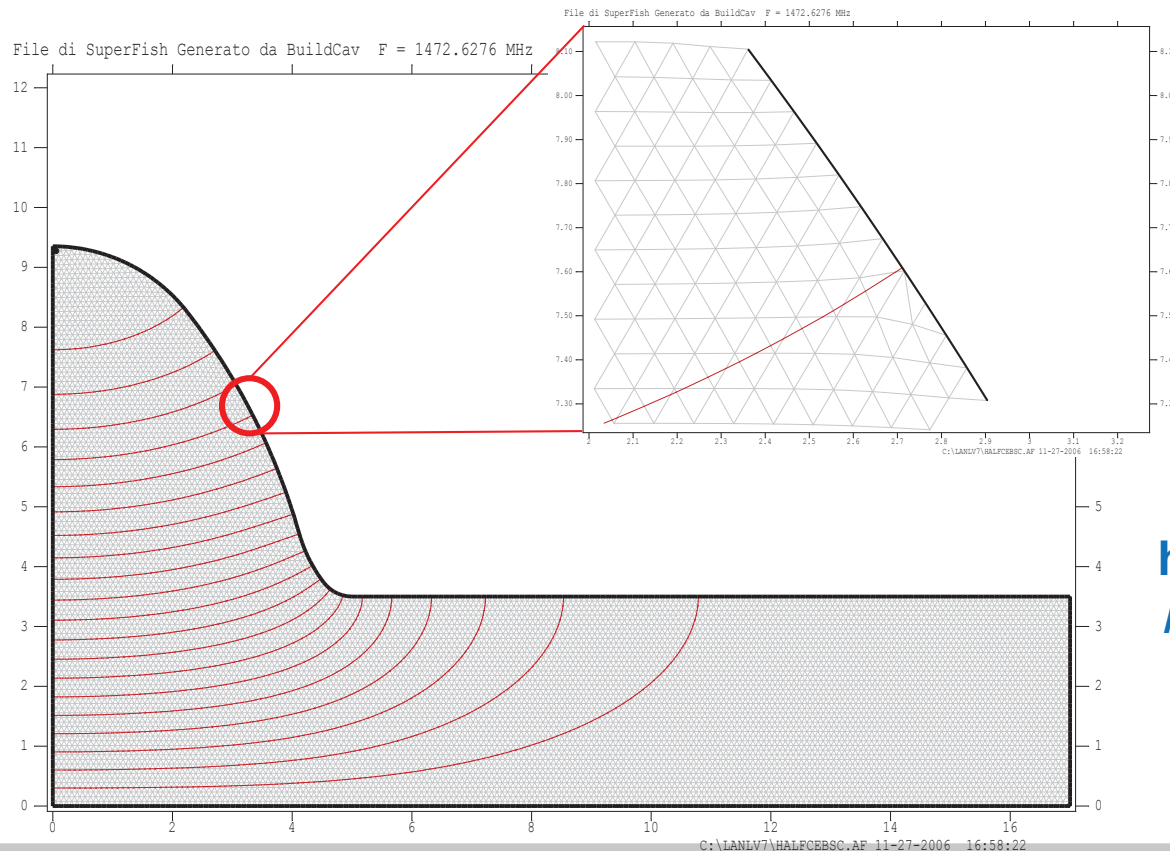


**http://laacg1.lanl.gov/laacg /services/download_sf.pht ml**

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# CST Microwave Studio

- Expensive, 3D finite-element code, used to design complex RF structure.

http://www.cst.com/Content/Products/MWS/Overview.aspx

- Runs on PC
- Perfect Boundary Approximation

U/m

1.07e7
9.10e6
7.68e6
6.46e6
5.42e6
4.51e6
3.73e6
3.07e6
2.49e6
1.99e6
1.57e6
1.20e6
8.82e5
6.10e5
3.76e5
1.74e5
0

**Hexahedral mesh**

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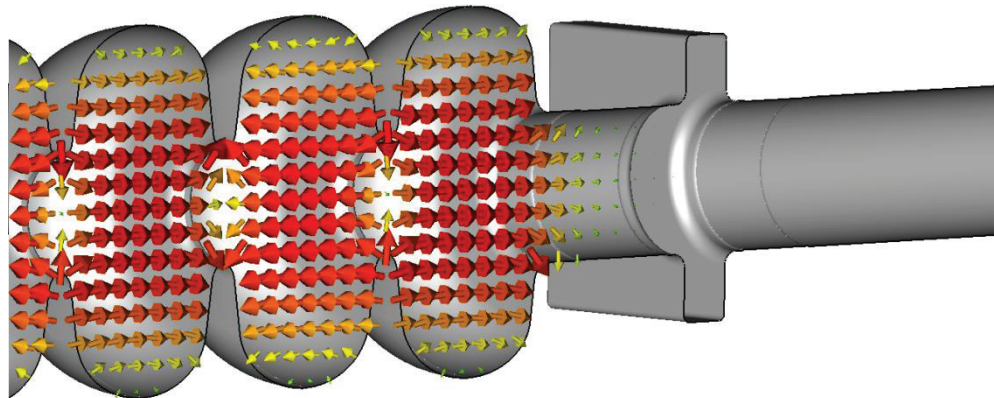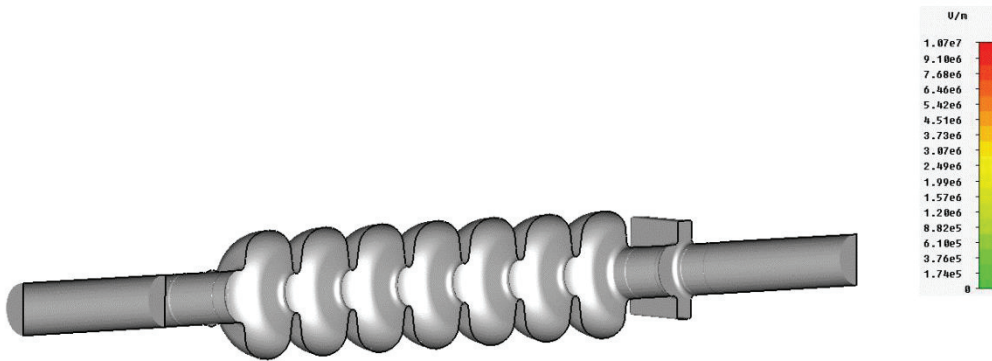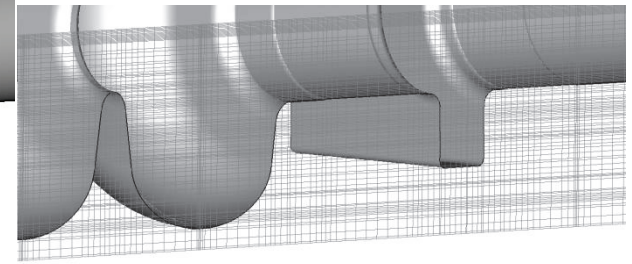
# Omega3P

- SLAC, 3D code, high-order Parallel Finite Element (PFE) method
- Runs on Linux
- Tetrahedral conformal mesh
- High order finite elements (basis order p =1 – 6
- Separate software for user interface (CuBit)

Example:
LL end cell with
Input Coupler Only

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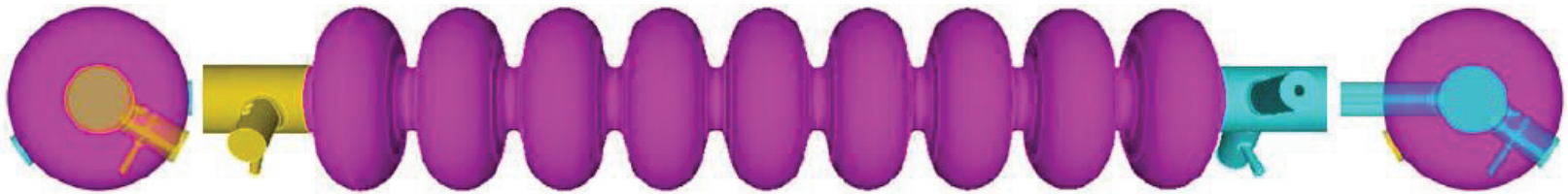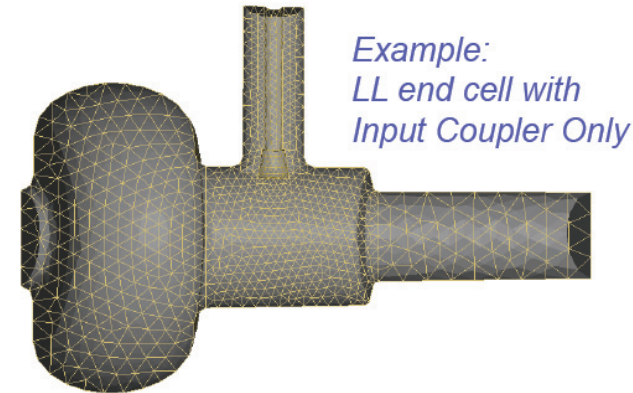
# Multicell Cavities
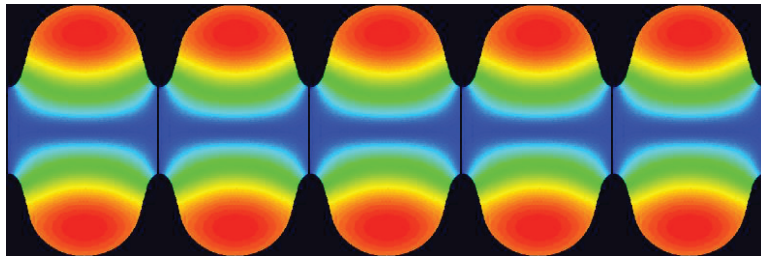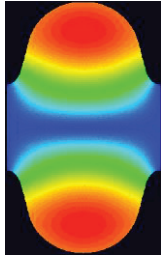
**Single-cell is attractive from the RF-point of view:**
- **Easier to manage HOM damping**
- **No field flatness problem.**
- **Input coupler transfers less power**
- **Easy for cleaning and preparation**

- *But it is expensive to base even a small linear accelerator on the single cell. We do it only for very high beam current machines.*

**A multi-cell structure is less expensive and offers higher real-estate gradient but:**
  *Field flatness (stored energy) in cells becomes sensitive to frequency errors of individual cells*
- *Other problems arise: HOM trapping…*

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# Velocity Acceptance



Velocity Acceptance for Sinusoidal Field Profile
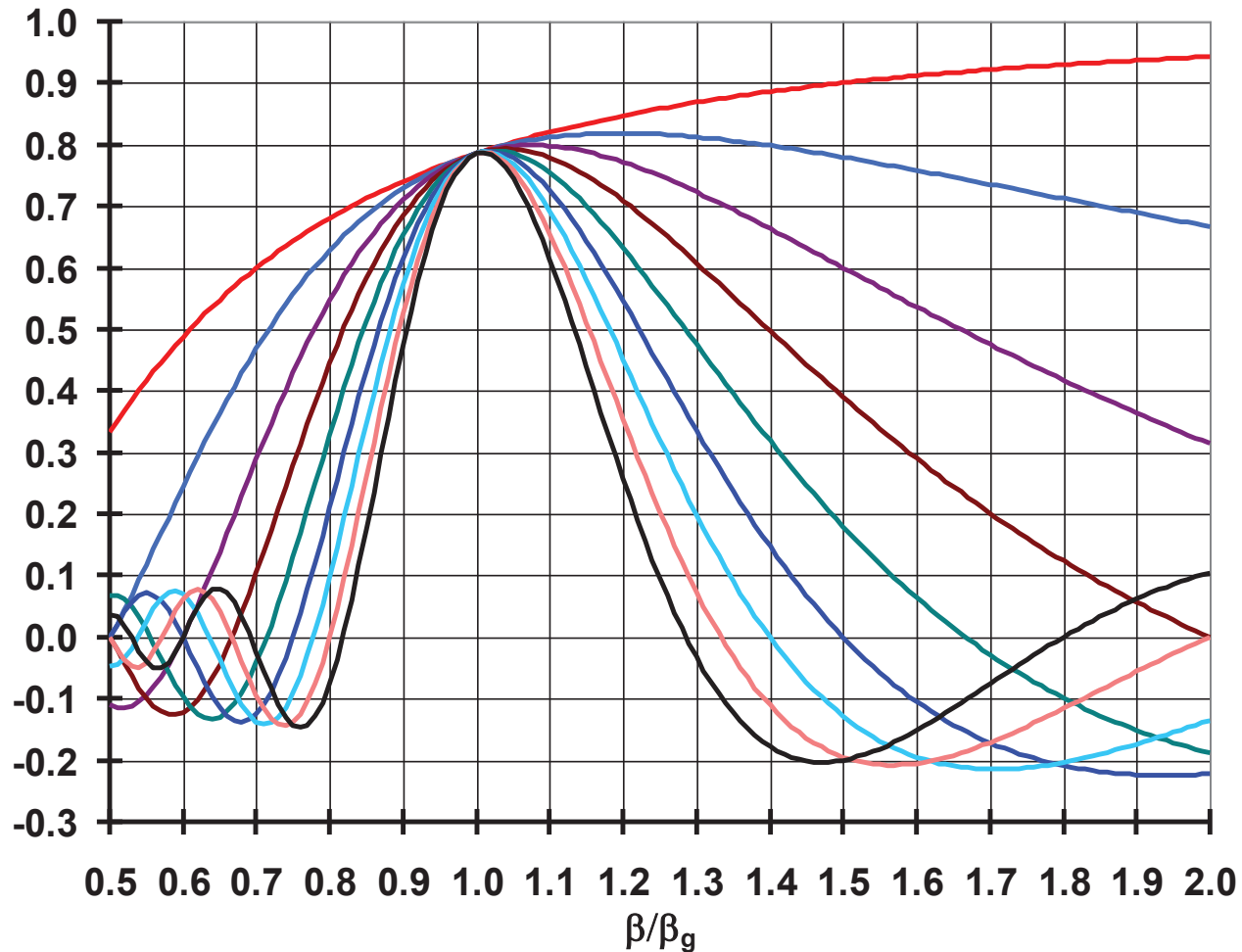
# Pros and Cons of Multicells

- Cost of accelerators are lower (less auxiliaries: LHe vessels, tuners, fundamental power couplers, control electronics)
- Higher real-estate gradient (better fill factor)

- Field flatness vs. N
- HOM trapping vs. N
- Power capability of fundamental power couplers  vs. N
- Chemical treatment and final preparation become more complicated
- The worst performing cell limits whole multi-cell structure

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# Coupling between cells

$\omega_o$

$\omega_\pi$
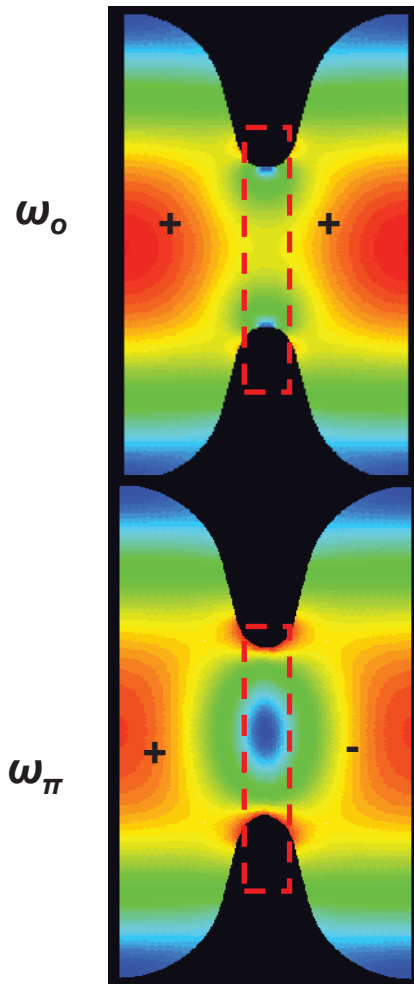
*Symmetry plane for the H field*

*Symmetry plane for the E field which is an additional solution*

**The normalized difference between these frequencies is a measure of the energy flow via the coupling region**

$$k_{cc} = \frac{\omega_\pi - \omega_0}{\dfrac{\omega_\pi + \omega_0}{2}}$$

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# Multi-Cell Cavities
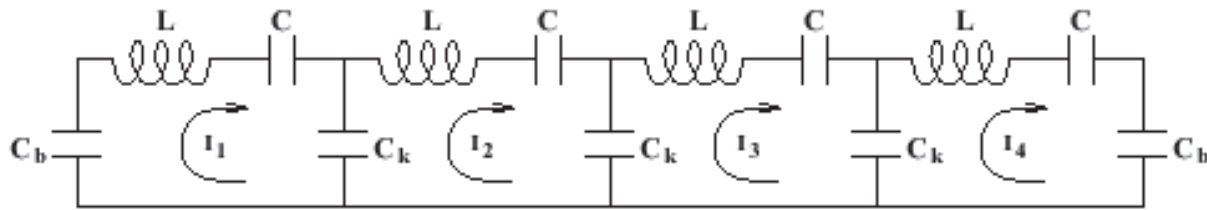


$$k = \frac{C}{C_k} \qquad C_b = C_k / 2$$

**Mode frequencies:**

$$\frac{\omega_m^2}{\omega_0^2} = 1 + 2k\left(1 - \cos\frac{\pi m}{n}\right)$$

$$\frac{\omega_n - \omega_{n-1}}{\omega_0} \simeq k\left(1 - \cos\frac{\pi}{n}\right) \simeq \frac{k}{2}\left(\frac{\pi}{n}\right)^2$$

**Voltages in cells:**

$$V_j^m = \sin\left(\pi m \frac{2j-1}{2n}\right)$$

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# Pass-Band Modes Frequencies

**9-cell cavity**
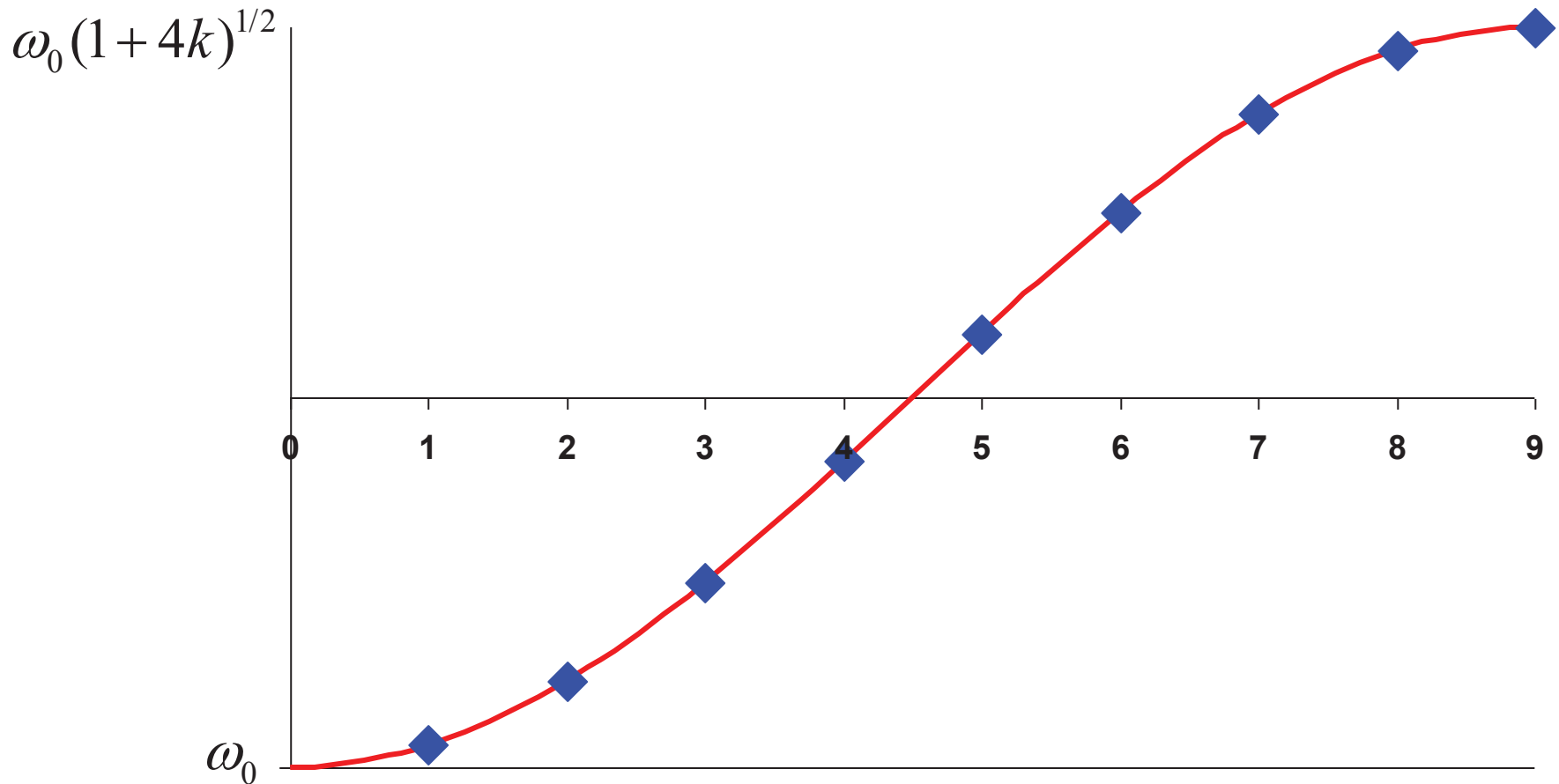
# Cell Excitations in Pass-Band Modes
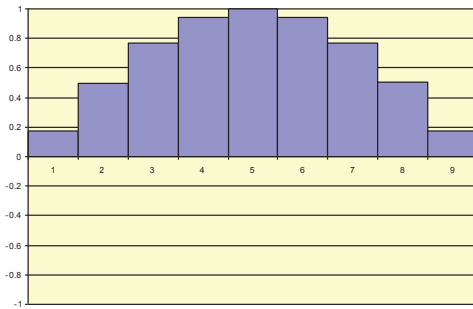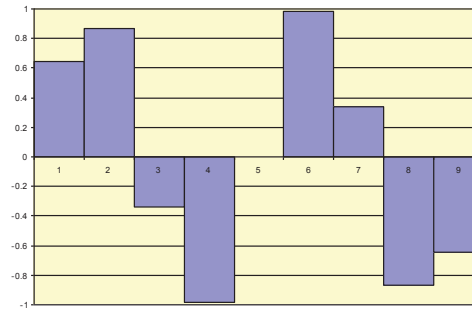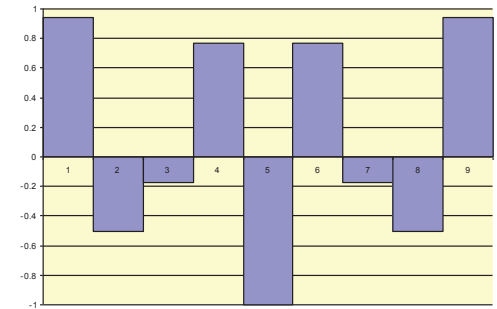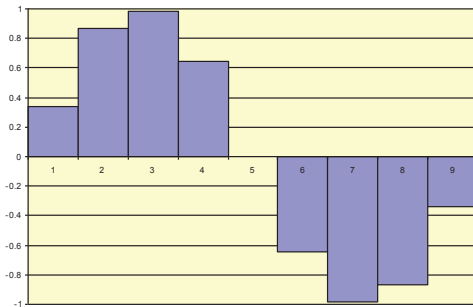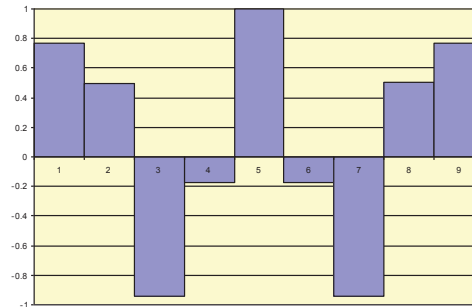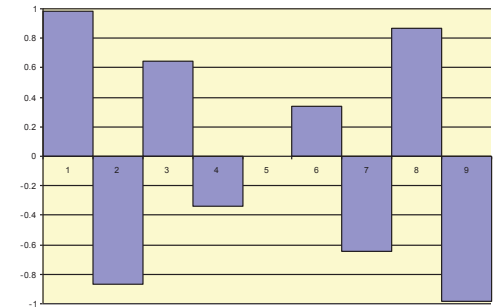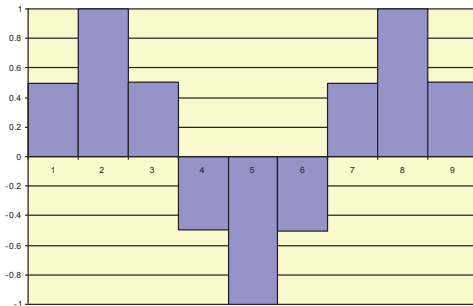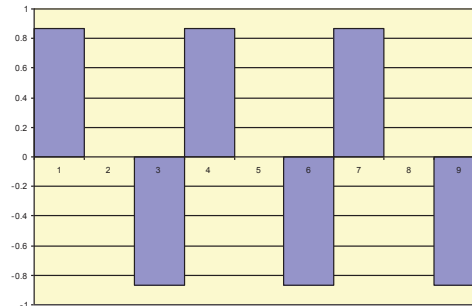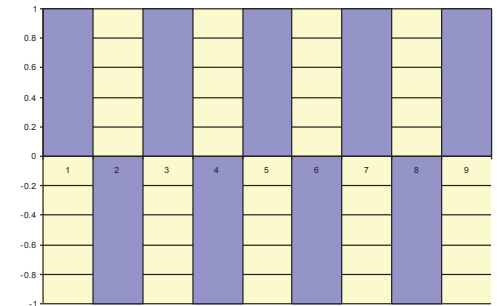
# Multipacting Simulations

Once the cavity shape has been designed, multipacting simulations have to be done:

• get the fields on the contour

• electrons are launched from given initial sites at given phases of the RF field

• for a fixed field level the electron trajectories are calculated by integrating the equations of motion, until the electrons hit the wall

• record the location, phase, and impact energy

• the number of secondary electrons is determined, given the SEY function

• the trajectory calculation is continued if the field phase is such as secondary electrons leave the wall

• after a given number of impacts N the No. of free electrons and their avg. impact energy and the No. of secondary electrons is calculated

Counter function

Enhanced counter function

Counter function: field levels at which resonant conditions are satisfied

At field levels where Enhanced counter function > No. initial electrons: Multipacting

# Example: Multipacting in SNS HOM Coupler



HOM2
HOM1
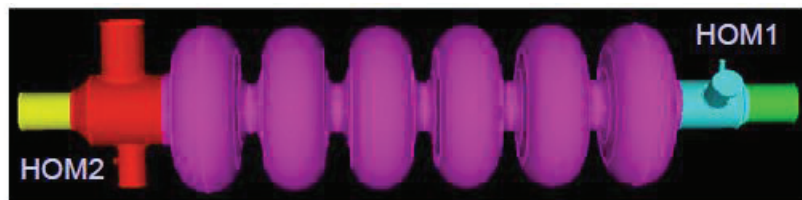
- SNS SCRF cavity experienced RF heating at HOM coupler
- 3D MP simulations showed MP barriers closed to measurements
- Similar analysis are carried out for ILC ICHIRO and crab cavity



HOM2

## Field level in HOM couplers



- HOM2 coupler
- HOM1 coupler

max E at HOM notch gap (MV/m)

HOM coupler notch gap (mm)



**Track3P**

Delta

**Expt. MP bands**

Field Level (MV/m)
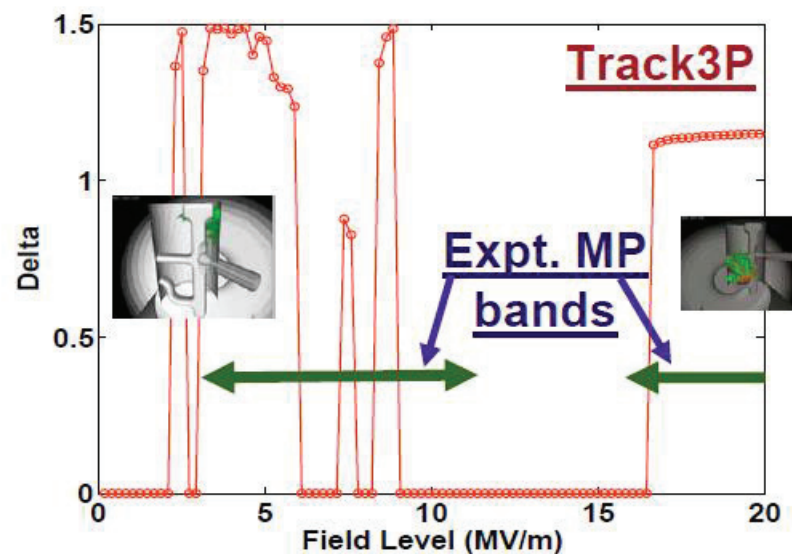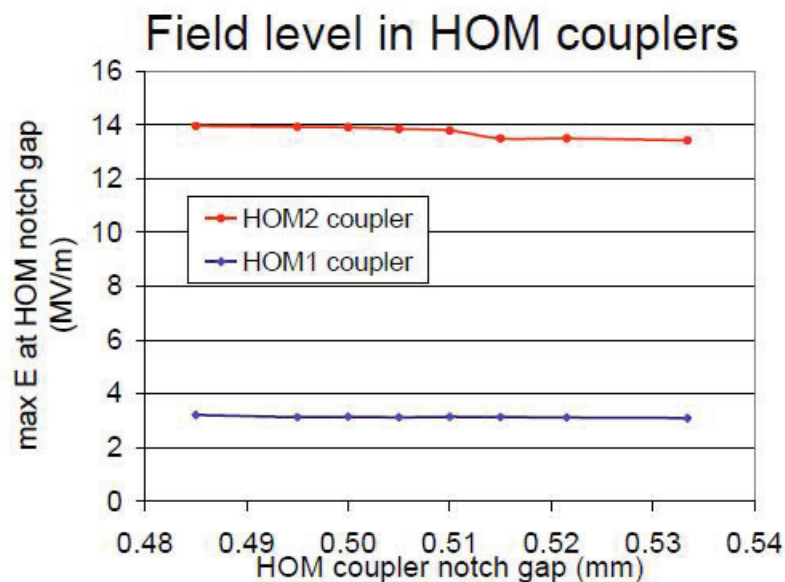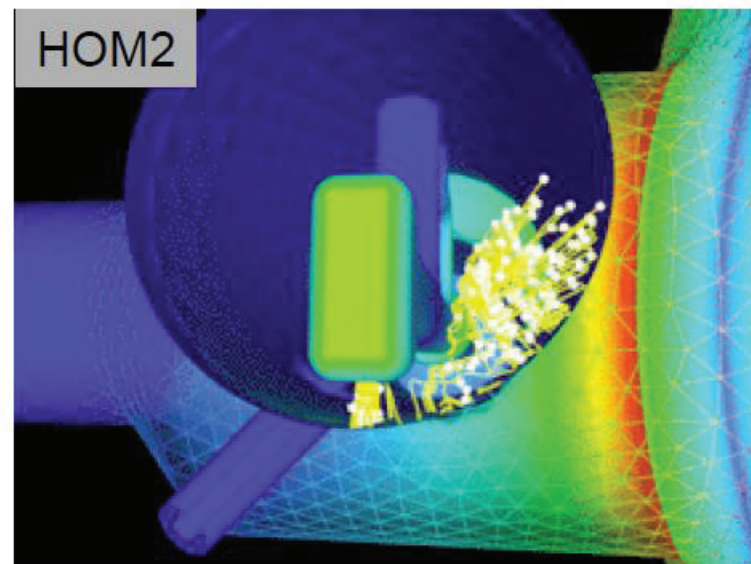
# Mechanical Design

**The mechanical design of a cavity follows its RF design:**

- **Lorentz Force Detuning**
- **Mechanical Resonances**

*Lorentz Force Detuning*

$$P = \frac{\mu_0 H_s^2 - \varepsilon_0 E_s^2}{4}$$

**92 kA/m**

**50 MV/m**
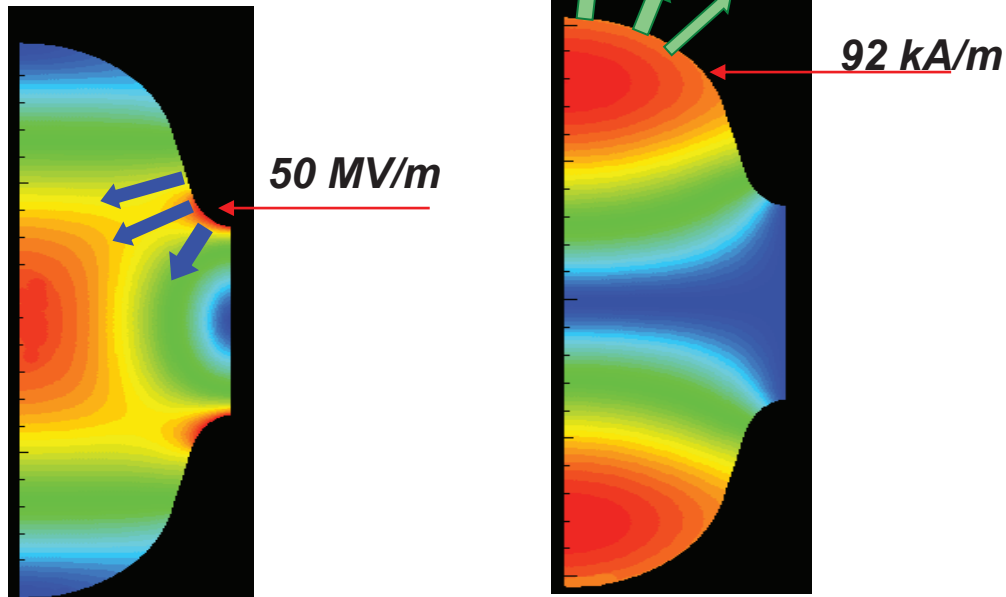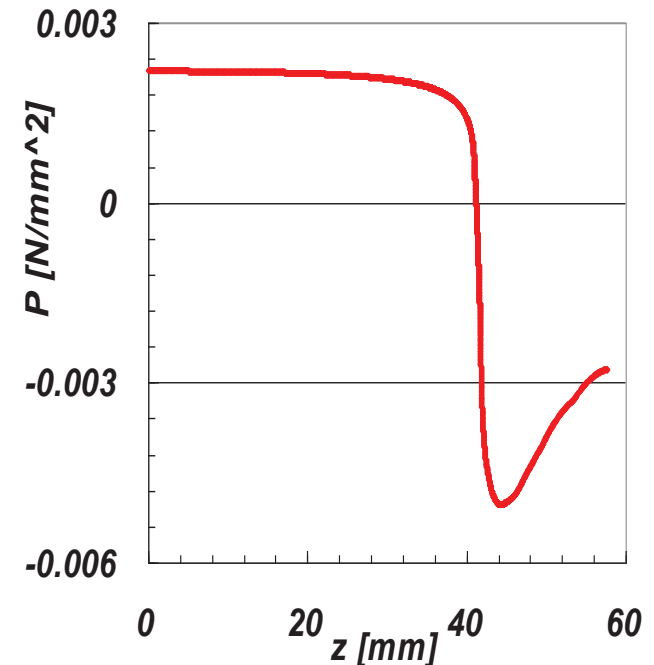
*E and H at $E_{acc}$ = 25 MV/m in TESLA inner-cup*

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# TEM-CLASS CAVITIES

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# Basic Structure Geometries

**Resonant Transmission Lines**

- $\lambda/4$
  - Quarter-wave
  - Split-ring
  - Twin quarter-wave
  - Lollipop

- $\lambda/2$
  - Coaxial half-wave
  - Spoke
  - H-types

    – **TM**
  - Elliptical
  - Reentrant

    – **Other**
  - Alvarez
  - Slotted-iris

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# A Word on Design Tools

TEM-class cavities are essentially 3D geometries



3D electromagnetic software is available

  MAFIA, Microwave Studio, HFSS, etc.

3D software is usually very good at calculating frequencies

  Not quite as good at calculating surface fields

    Use caution, vary mesh size

    Remember Electromagnetism 101

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# Design Tradeoffs

## Number of cells

    Voltage gain

    Velocity acceptance

## Frequency

    Size

    Voltage gain

    Rf losses

    Energy content, microphonics, rf control

    Acceptance, beam quality and losses
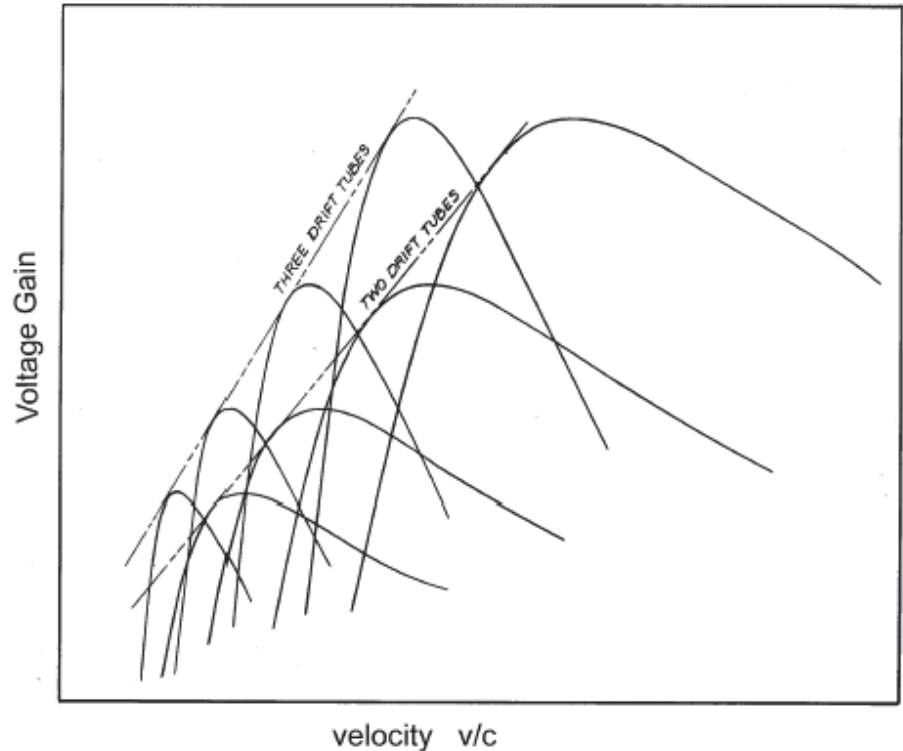
# A Simple Model:
# Loaded Quarter-wavelength Resonant Line

If characteristic length $<<\lambda$ ($\beta<0.5$), separate the problem in two parts:

Electrostatic model of high voltage region

Transmission line
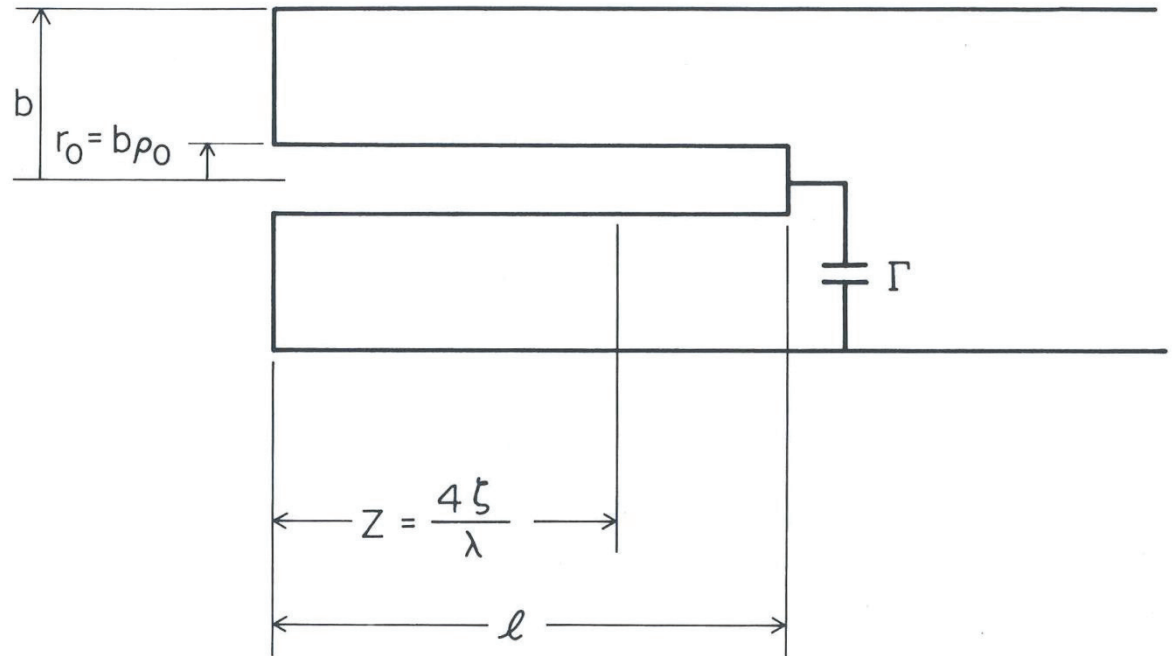
# A Simple Model:
# Coaxial Half-wave Resonator

2b

2a

L

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# Some Real Geometries ($\lambda/4$)

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# Some Real Geometries (λ/4)

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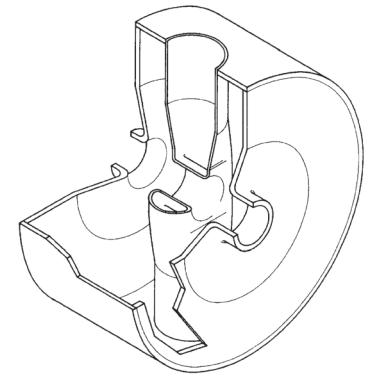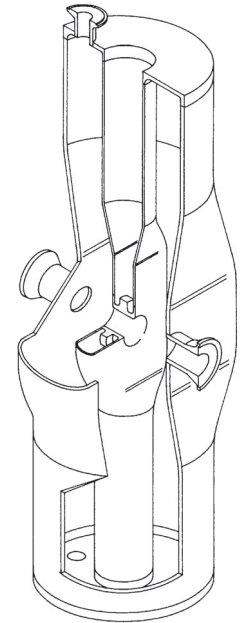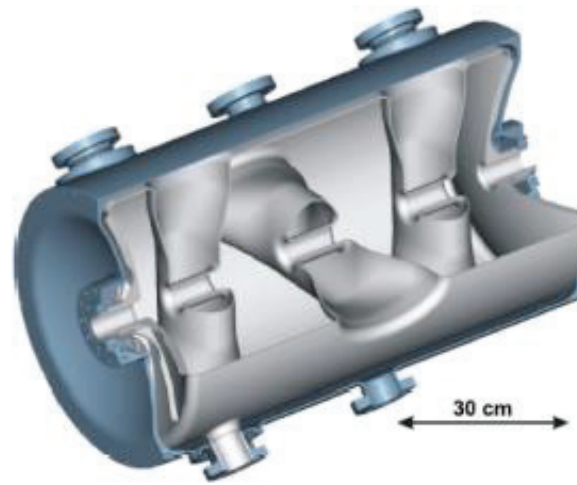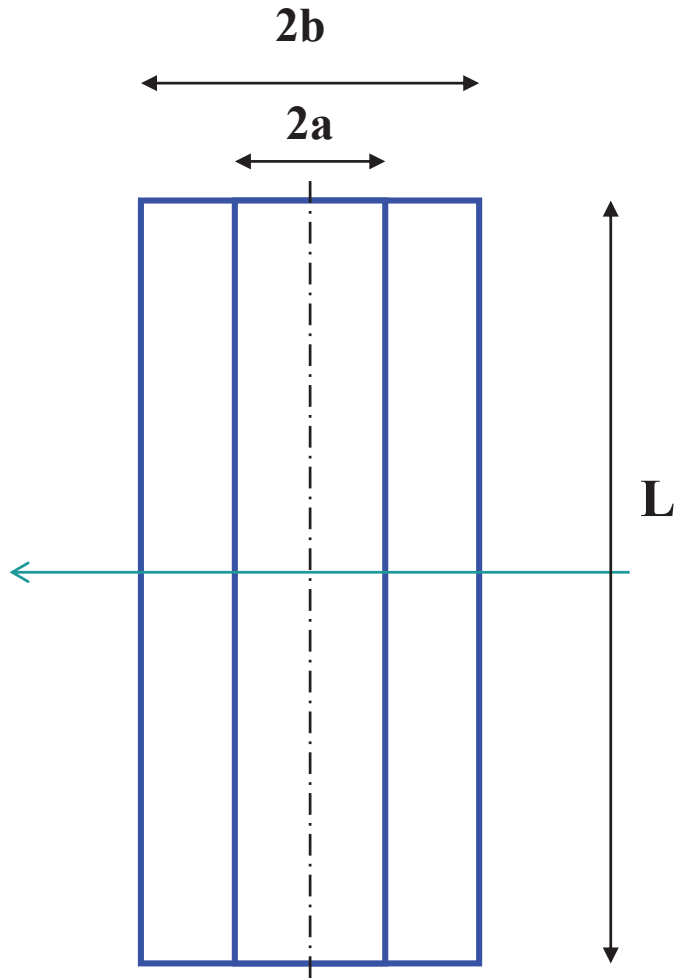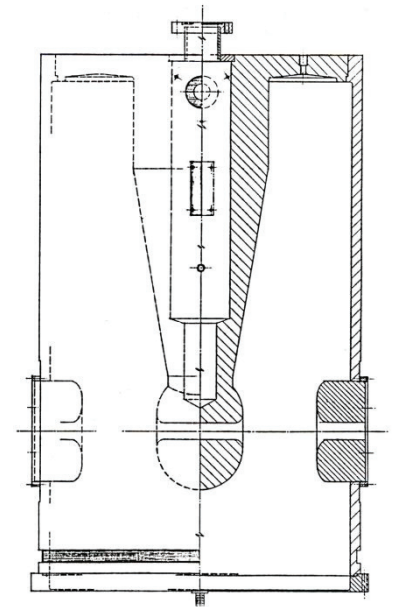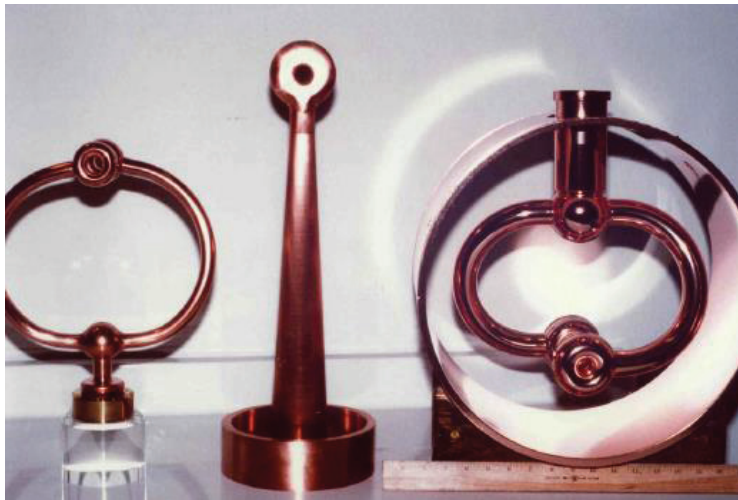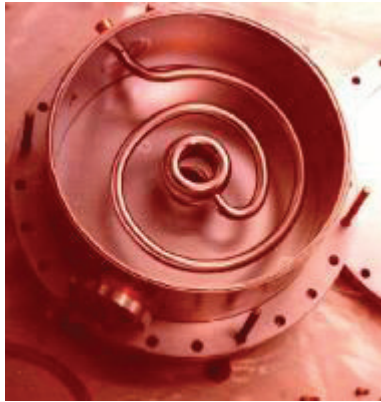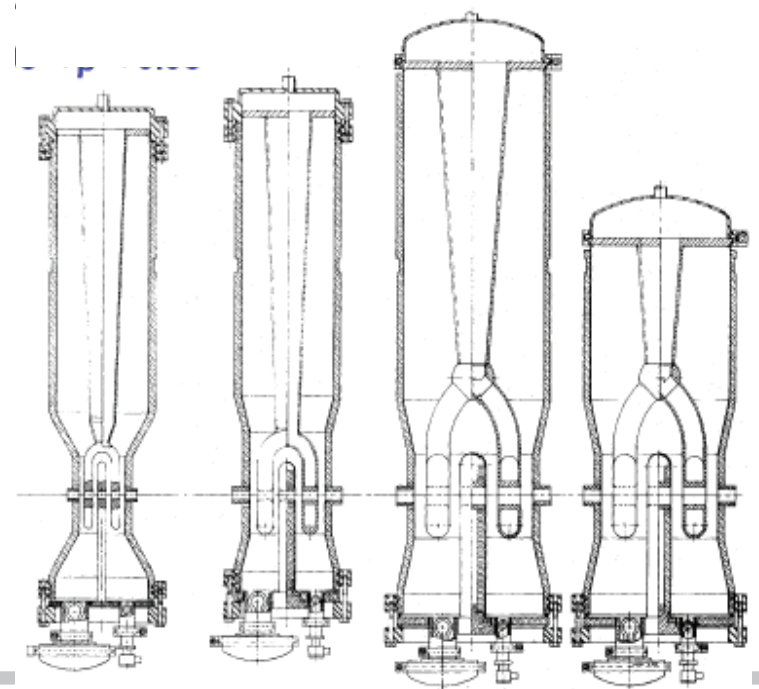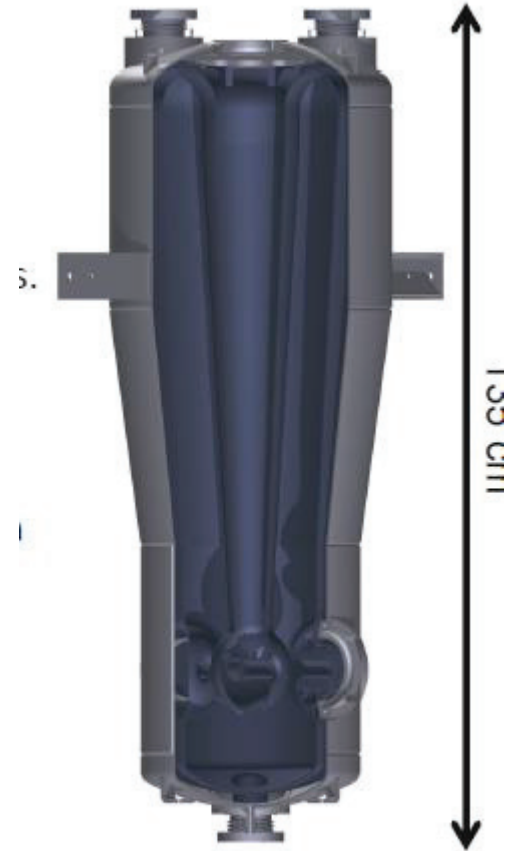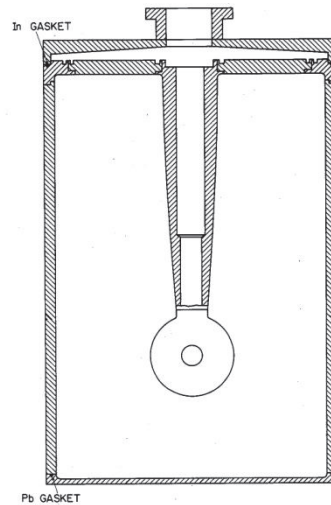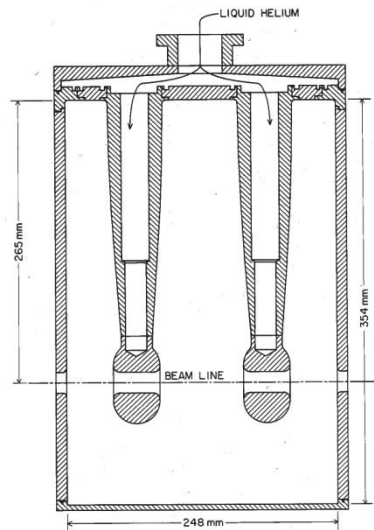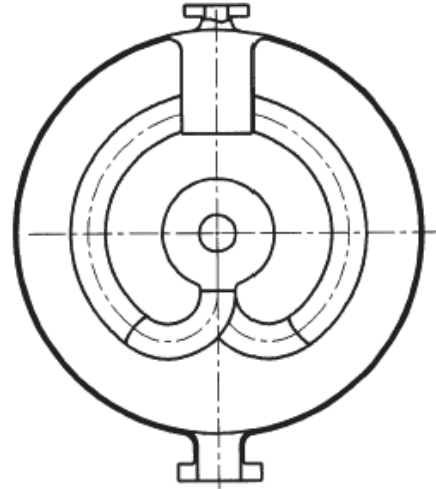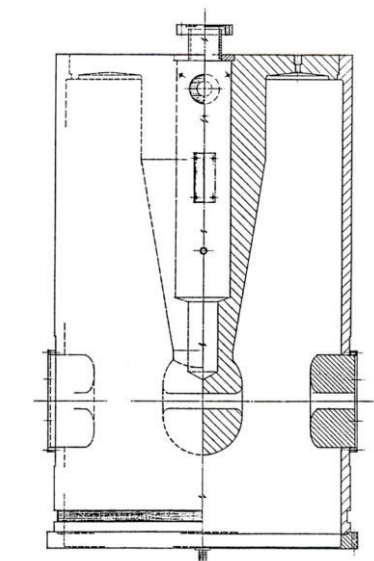
# λ/4 Resonant Lines



72.75 MHz, β = 0.077
Quarter-Wave Resonator

# λ/2 Resonant Lines

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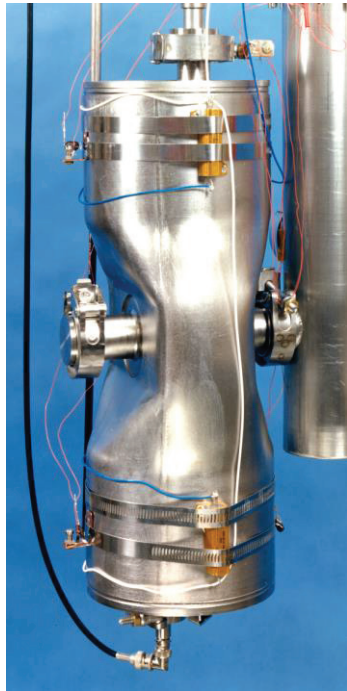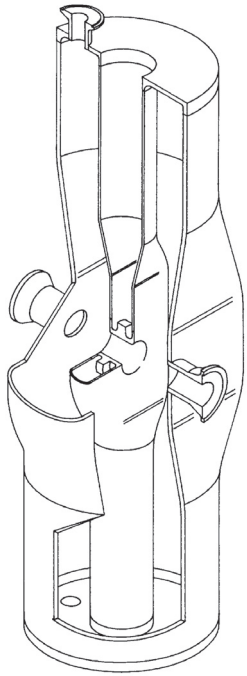
# λ/2 Resonant Lines – Single-Spoke
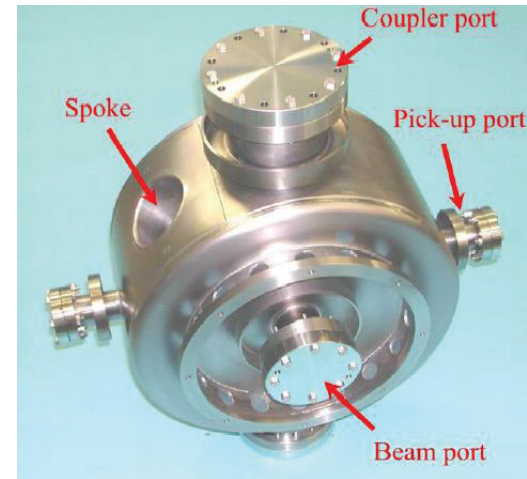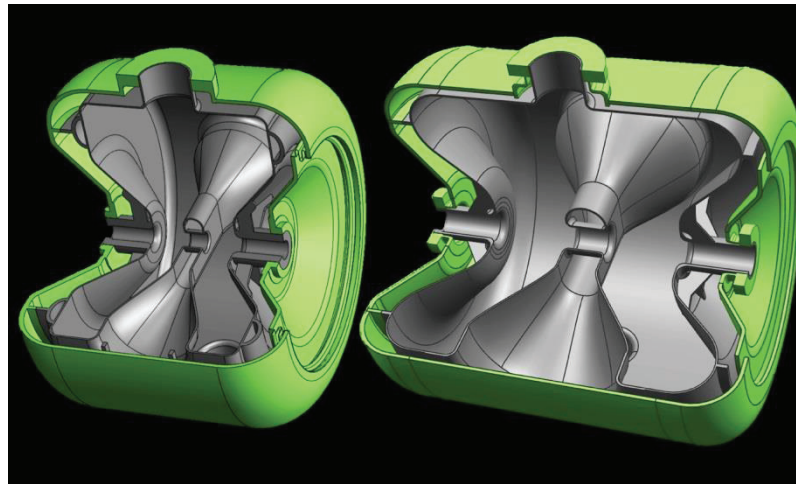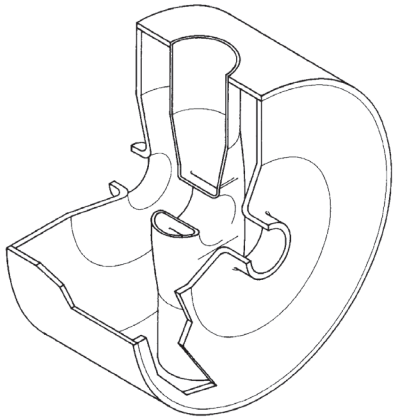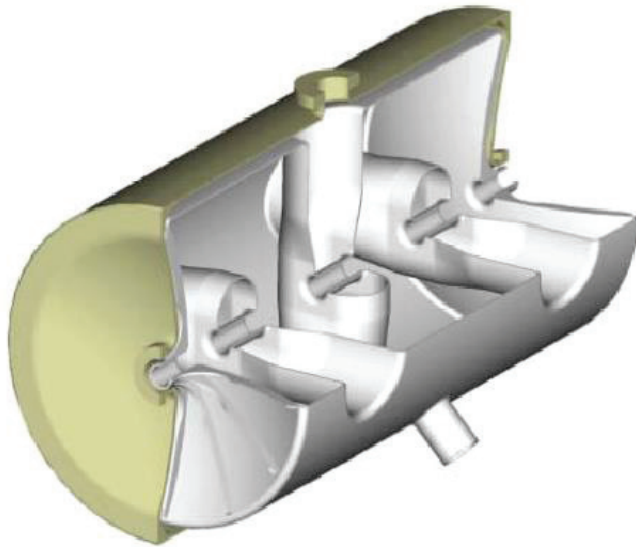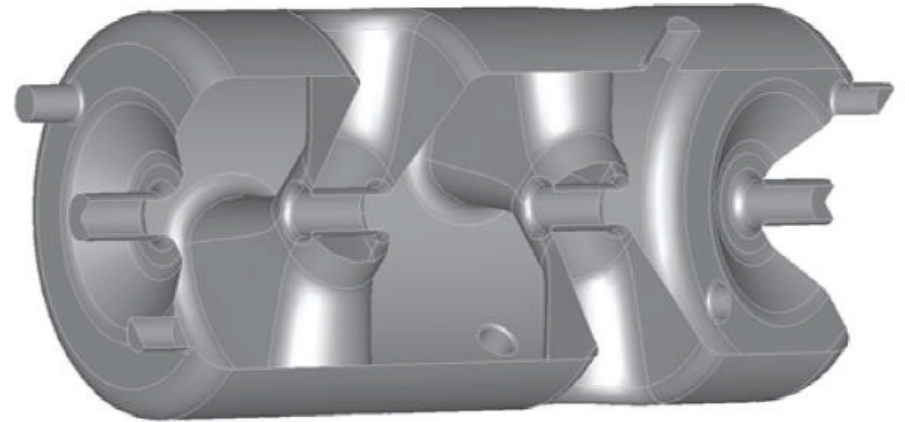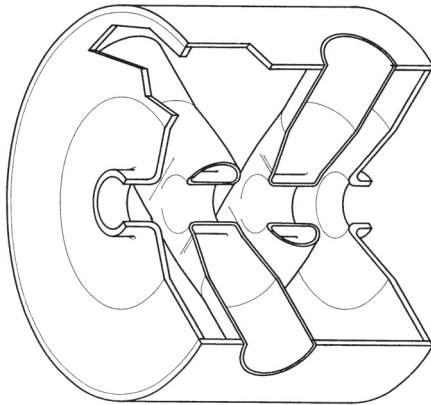
# λ/2 Resonant Lines – Multi-Spoke
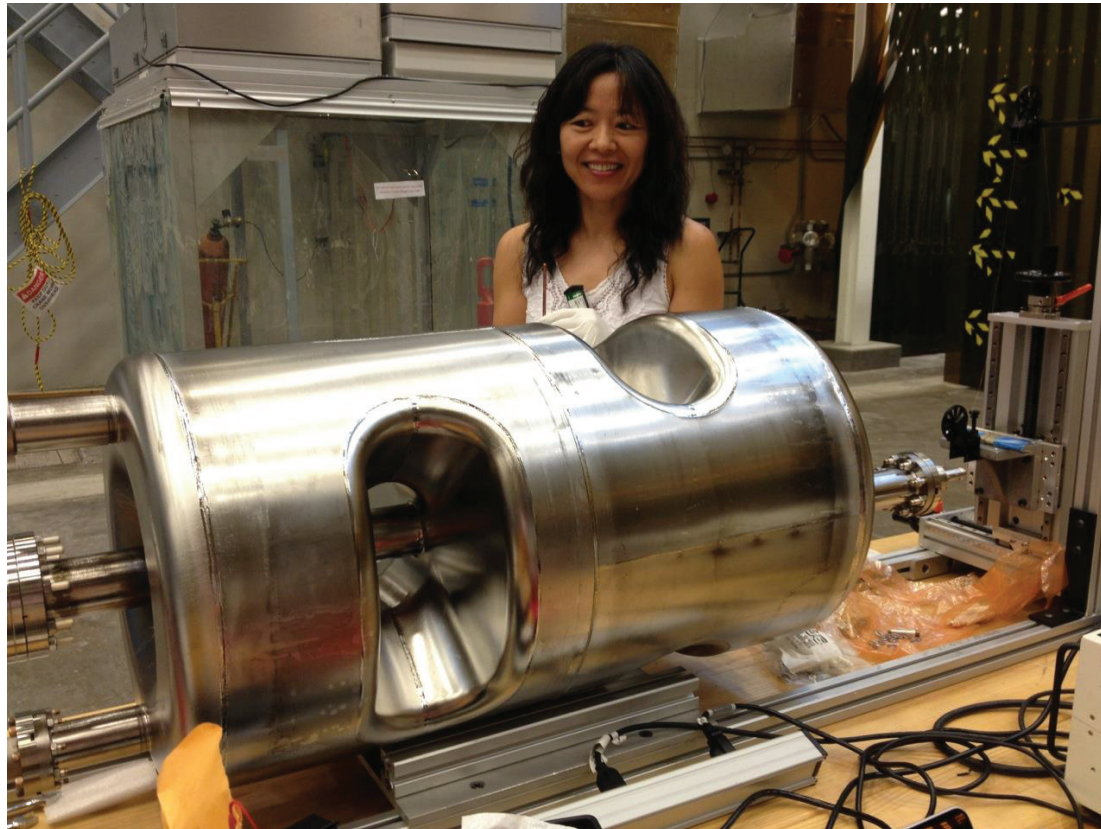
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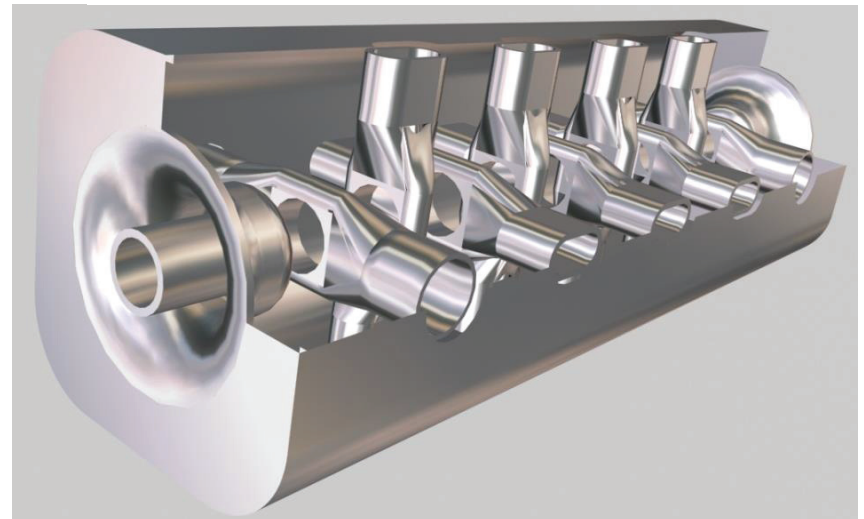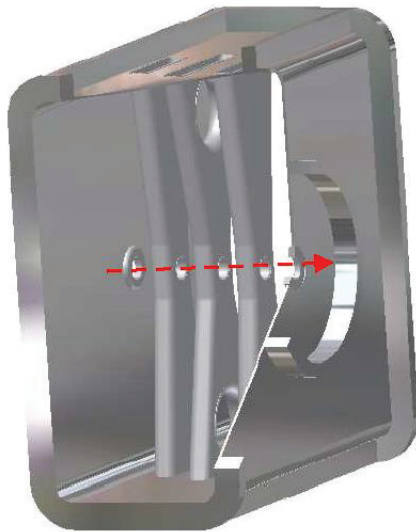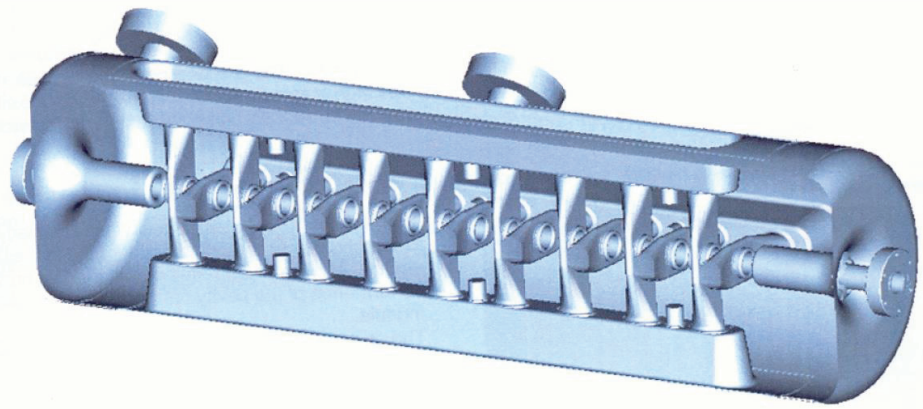
# ANL extended to TEM-class SC cavities the very high-performance techniques pioneered by TESLA



**Courtesy P. Ostroumov and K. Shepard**

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# New Electromagnetic Structures
# High-velocity Spoke Cavities

**Double-spoke Cavities**
**500 MHz, β=1**
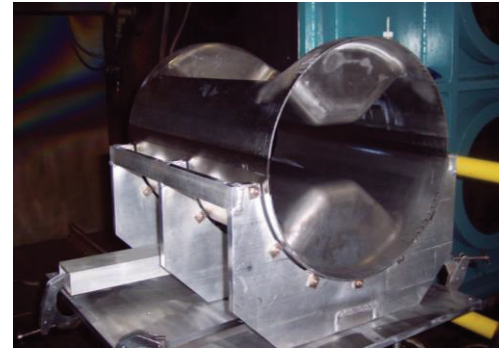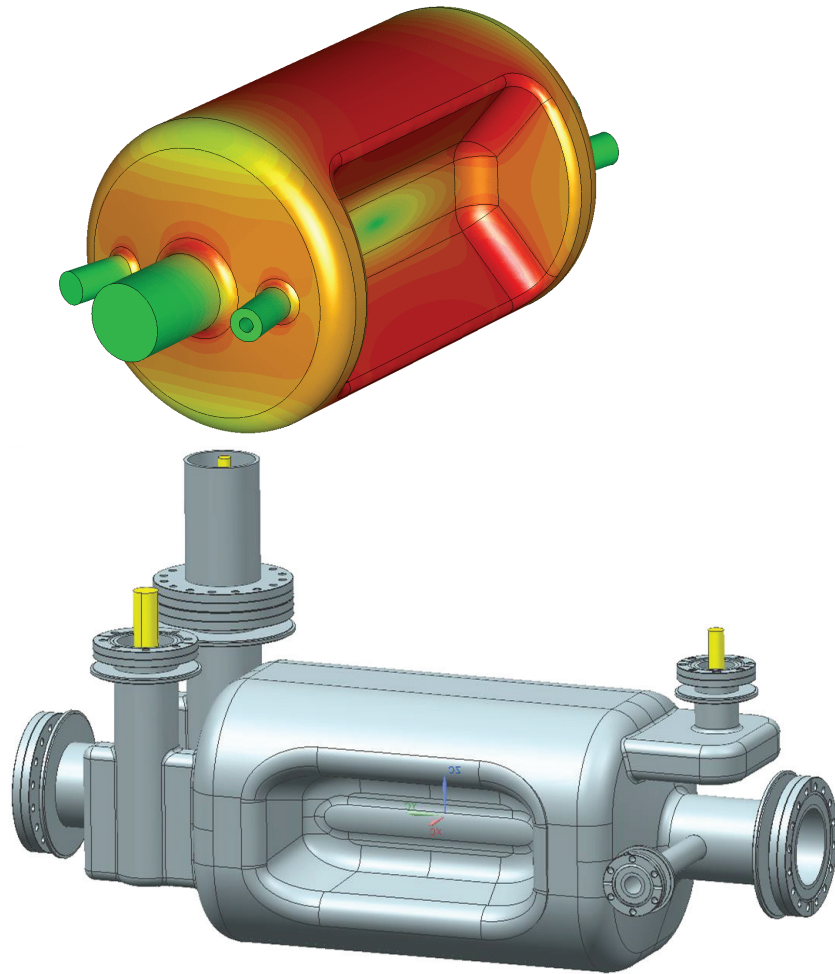
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# New Electromagnetic Structures
# Crabbing and Deflecting Cavities

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# New Electromagnetic Structures
# Crabbing and Deflecting Cavities



HOM coupler

Fundamental power coupler

Pick Up

48 cm

HOM coupler

66 cm

4 Rod

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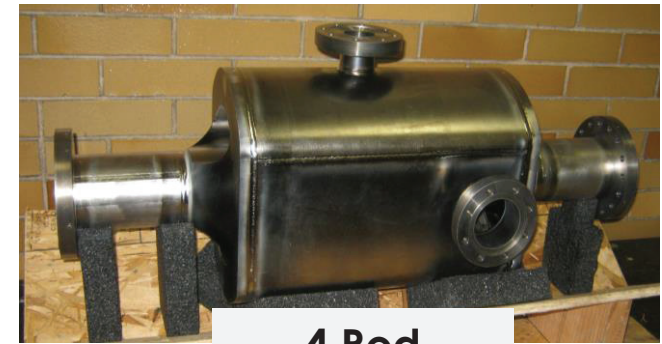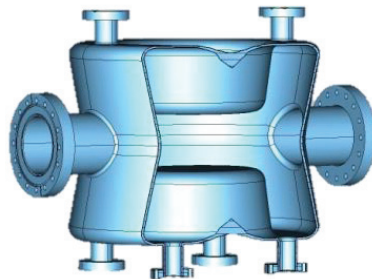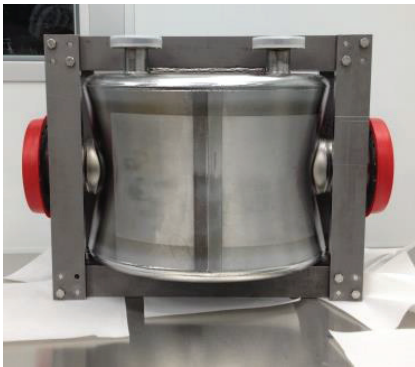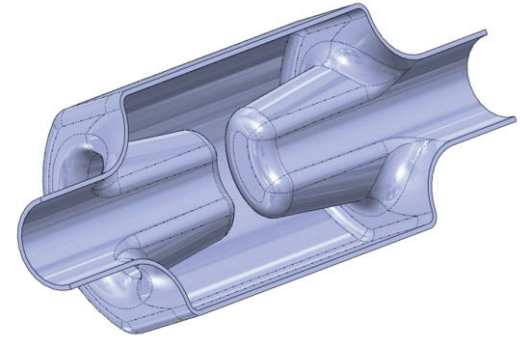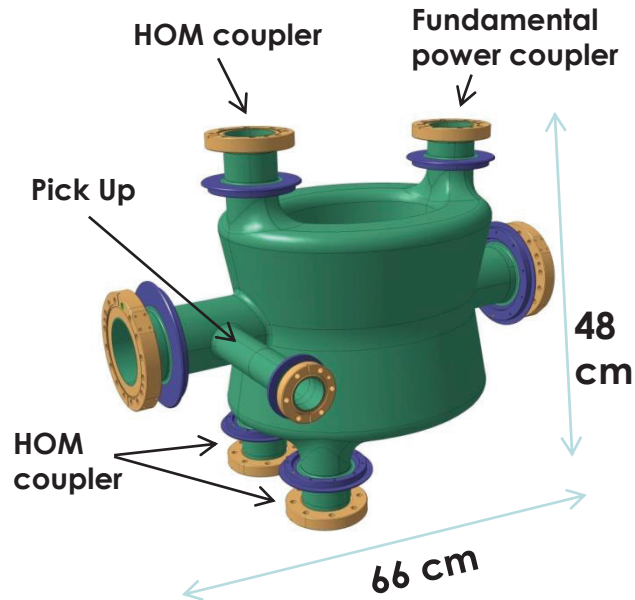
# Parting Words

In the last 30+ years, the development of superconducting cavities has been one of the richest and most imaginative area of srf

- The field has been in perpetual evolution and progress
- New geometries are constantly being developed
- The final word has not been said

The parameter, tradeoff, and option space available to the designer is large

- The design process is not, and probably will never be, reduced to a few simple rules or recipes
- There will always be ample opportunities for imagination, originality, and common sense

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# SRF LIMITATIONS

## Jean Delayen

**Center for Accelerator Science**
**Old Dominion University**
**and**
**Thomas Jefferson National Accelerator Facility**

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# Outline

- Residual resistance

- Multipacting

- Field emission

- Quench

- High-field Q-slope

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# The Real World

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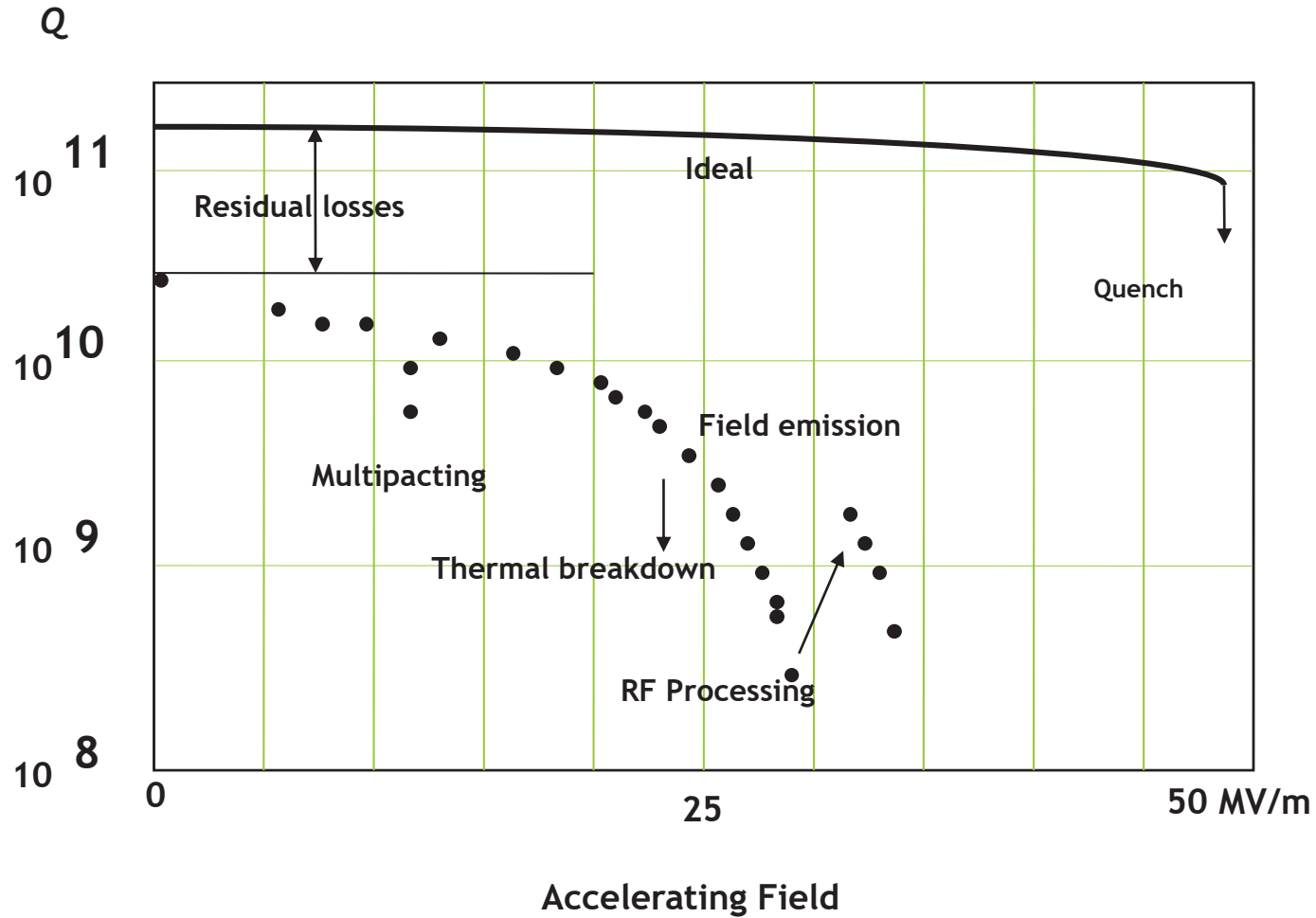
# Characteristics of Residual Surface Resistance

- No strong temperature dependence

- No clear frequency dependence

- Not uniformly distributed (can be localized)

- Not reproducible

- Can be as low as 1 n$\Omega$

- Usually between 5 and 30 n$\Omega$

- Often reduced by UHV heat treatment above 800C

# Origin of Residual Surface Resistance

- Dielectric surface contaminants (gases, chemical residues, dust, adsorbates)
- Normal conducting defects, inclusions
- Surface imperfections (cracks, scratches, delaminations)
- Trapped magnetic flux
- Hydride precipitation
- Localized electron states in the oxide (photon absorption)

**$R_{res}$ is typically 5-10 n$\Omega$ at 1-1.5 GHz**

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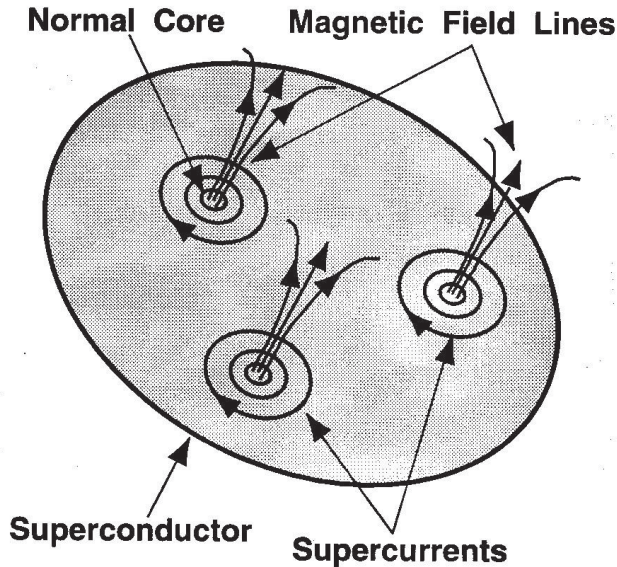
# Trapped Magnetic Field

**A parallel magnetic filed is expelled from a superconductor.**

**What about a perpendicular magnetic field?**
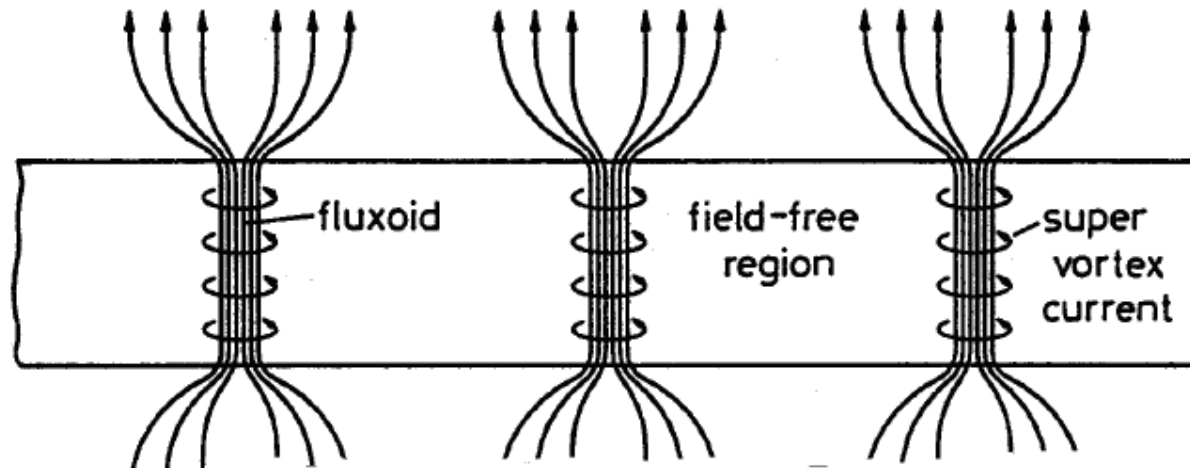


**The magnetic field will be concentrated in normal cores where
it is equal to the critical field.**

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# Trapped Magnetic Field

- Vortices are normal to the surface
- 100% flux trapping
- RF dissipation is due to the normal conducting core, of resistance $R_n$

$$R_{res} \cong R_n \frac{H_i}{H_{c2}}$$
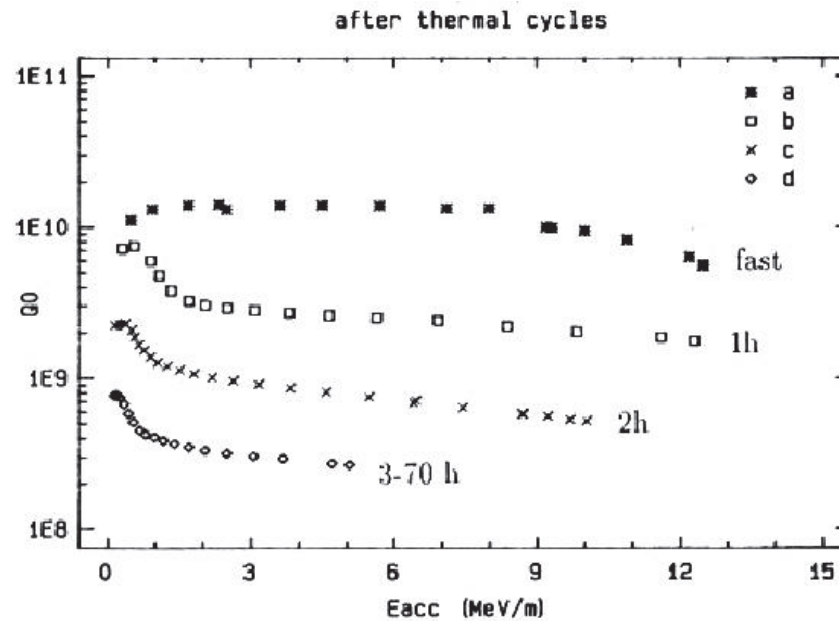
**$H_i$ = residual DC magnetic field**

• **For Nb:**   $R_{res} \approx 0.3$ to 1 nΩ/mG  around 1 GHz

**Depends on material treatment**

• While a cavity goes through the superconducting transition, the ambient magnetic filed cannot be more than a few mG.
• The earth's magnetic shield must be effectively shielded.
• Thermoelectric currents can cause trapped magnetic field, especially in cavities made of composite materials.

# $R_{res}$ Due to Hydrides (Q-Disease)

- **Cavities that remain at 70-150 K for several hours (or slow cool-down, < 1 K/min) experience a sharp increase of residual resistance**

- **More severe in cavities which have been heavily chemically etched**

after thermal cycles

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# Q Disease

At room temperature, the hydrogen moves freely through niobium

At lower temperature, H precipitates to form a hydride with poor superconducting properties: Tc=2.8 K, Hc=60 G

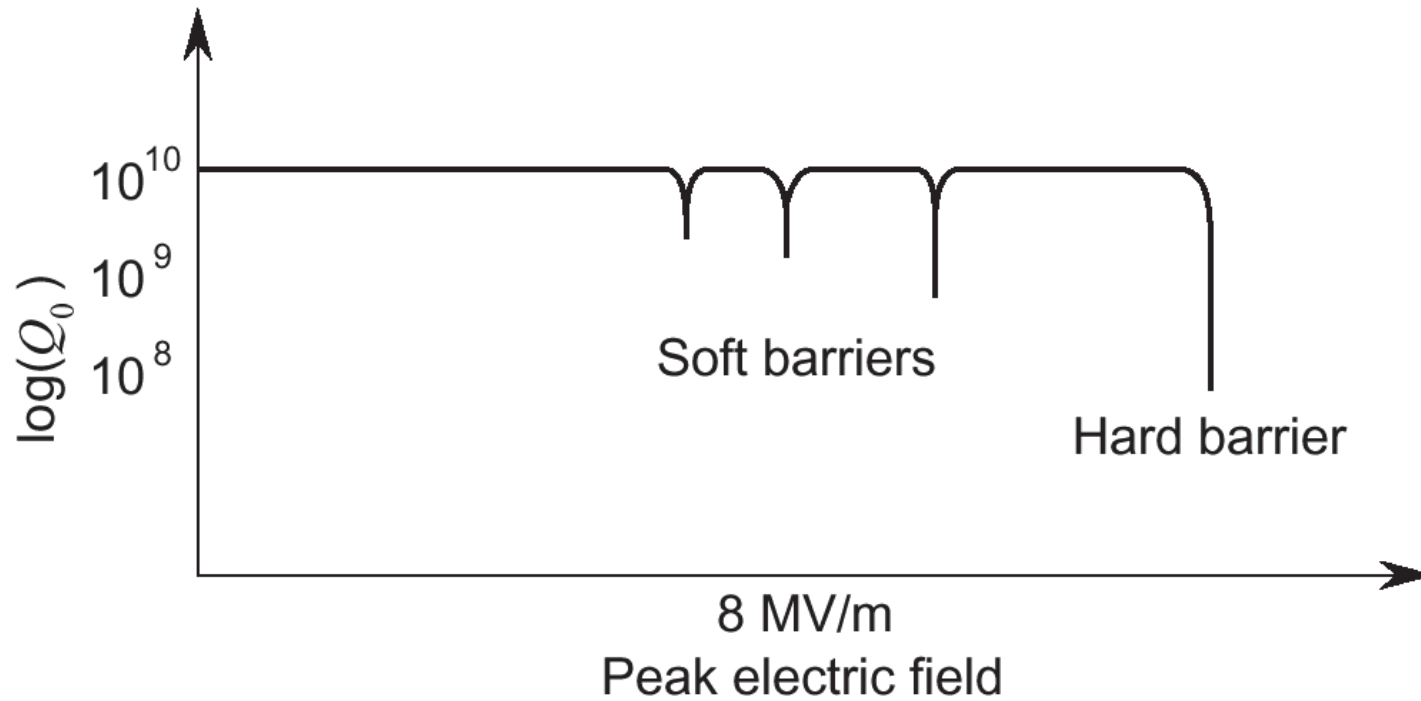At room temperature the required concentration to form a hydride is $10^3$-$10^4$ ppm

At 150K it is <10ppm

Can be eliminated by baking cavity at 600-800C

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# Cures for Q-disease

- Fast cool-down

- Maintain acid temperature below $\sim$ 20 °C during BCP

- "Purge" $H_2$ with $N_2$ "blanket" and cover cathode with Teflon cloth during EP

- "Degas" Nb in vacuum furnace at T > 600 °C

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# Multipacting



Soft barriers

Hard barrier

$\log(Q_0)$

$10^{10}$  $10^{9}$  $10^{8}$

8 MV/m
Peak electric field

- No increase of $P_t$ for increased $P_i$ during MP
- Can induce quenches and trigger field emission

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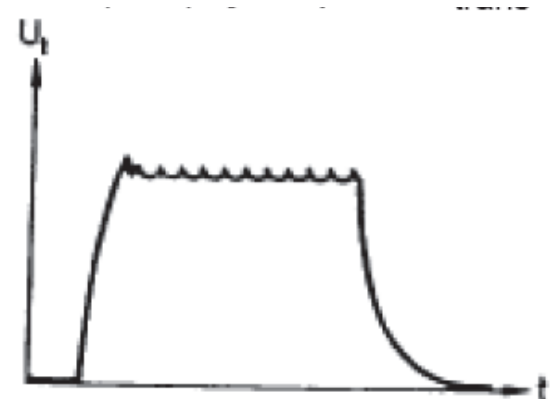
# Multipacting

Multipacting is characterized by an exponential growth in the number of electrons in a cavity

Common problems of RF structures (Power couplers, NC cavities…)

Multipacting requires 2 conditions:

- Electron motion is periodic (resonance condition)
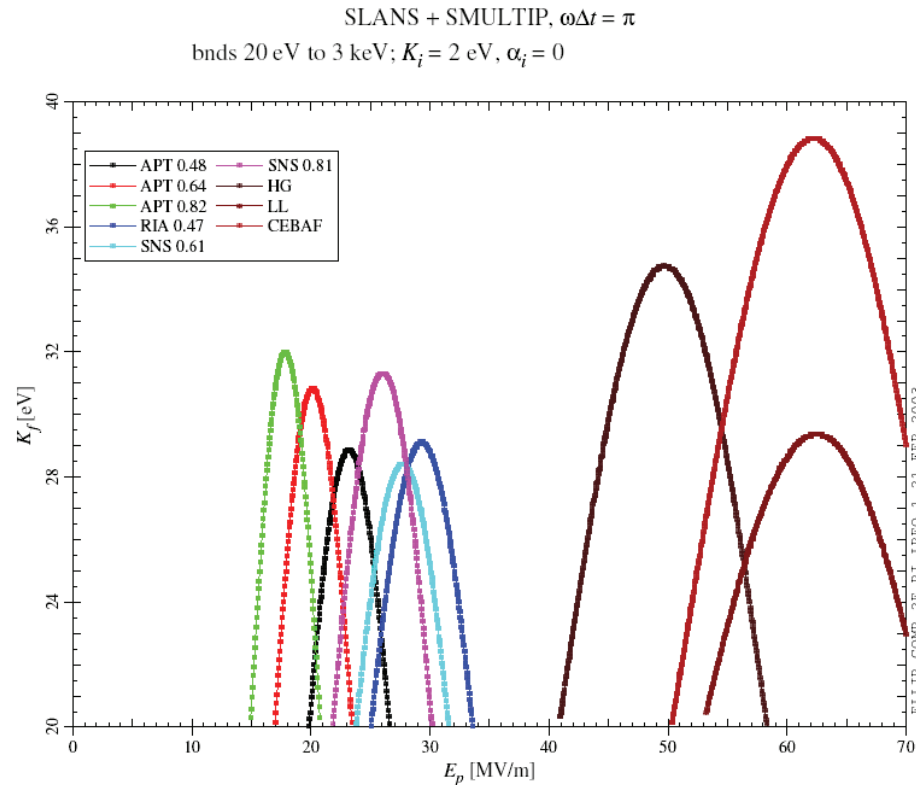- Impact energy is such that secondary emission coefficient is >1

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# Secondary Emission in Niobium
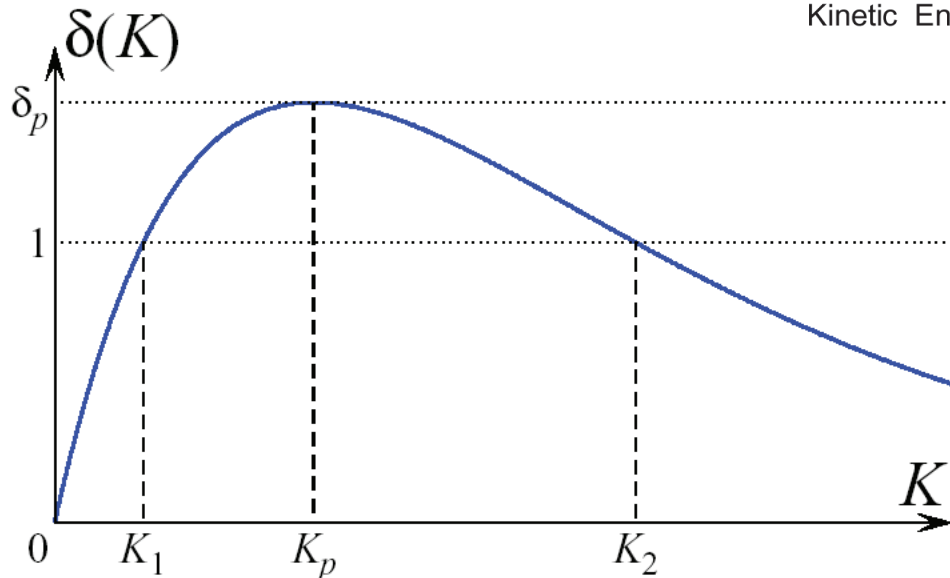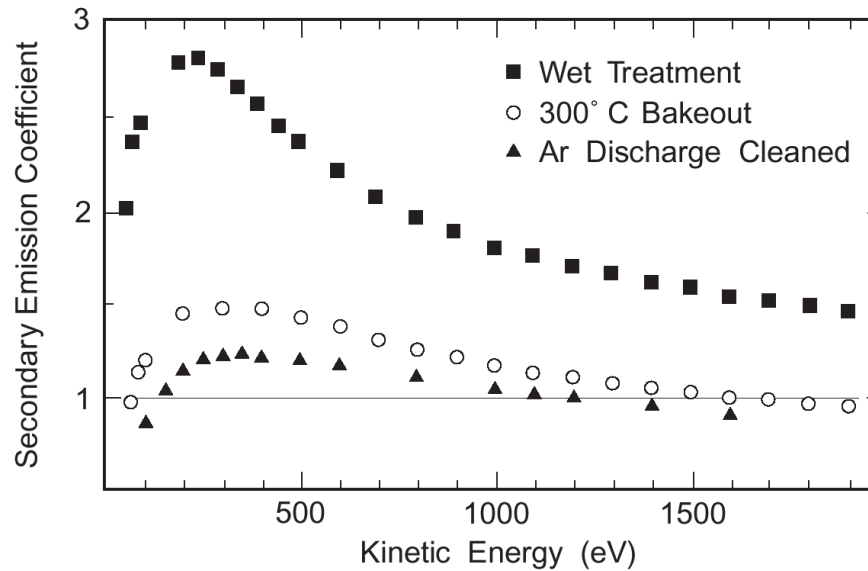


| Condition | $K_1$ | $K_2$ |
|---|---|---|
| high SEY | $\sim 27$ eV | $\gtrsim 2000$ eV |
| typical SEY | $\sim 40$ eV | $\sim 1000$ eV |
| low SEY | $\sim 150$ eV | $\sim 750$ eV |

# Cures for Multipacting

- Cavity design



SLANS + SMULTIP, $\omega\Delta t = \pi$
bnds 20 eV to 3 keV; $K_i = 2$ eV, $\alpha_i = 0$

Legend: APT 0.48, APT 0.64, APT 0.82, RIA 0.47, SNS 0.61, SNS 0.81, HG, LL, CEBAF

$K_f$ [eV] vs $E_p$ [MV/m]

ELLIP_COMP_2E_P1_LBEQ 1 21-FEB-2003
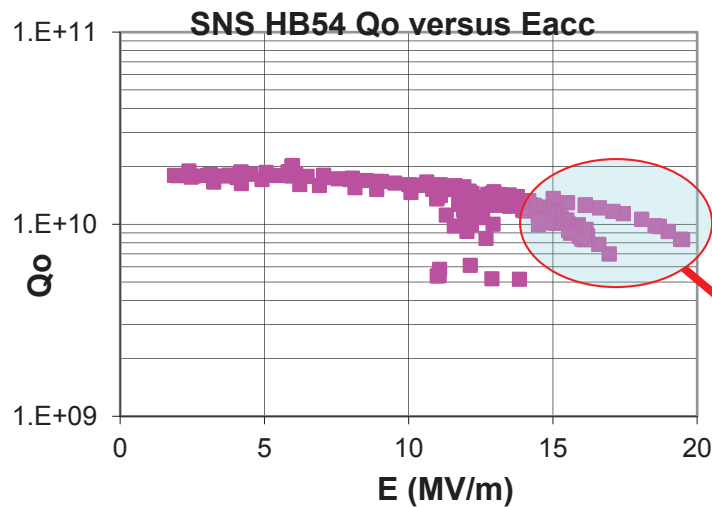
- Lower SEY: clean vacuum systems (low partial pressure of hydrocarbons, hydrogen and water), Ar discharge

- RF Processing: lower SEY by $e^-$ bombardment (minutes to several hours)

# Field Emission

- **Characterized by an exponential drop of the $Q_0$**

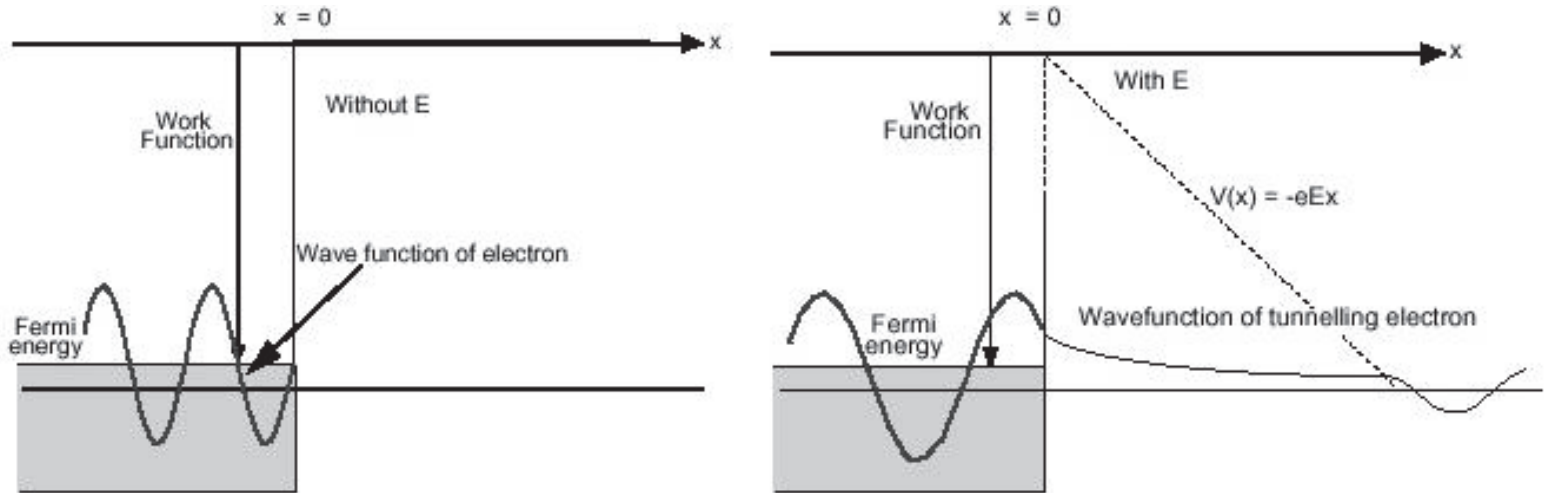- **Associated with production of x-rays and emission of dark current**



SNS HB54 Qo versus Eacc

SNS HTB 54 Radiation at top plate versus Eacc 5/16/08 cg

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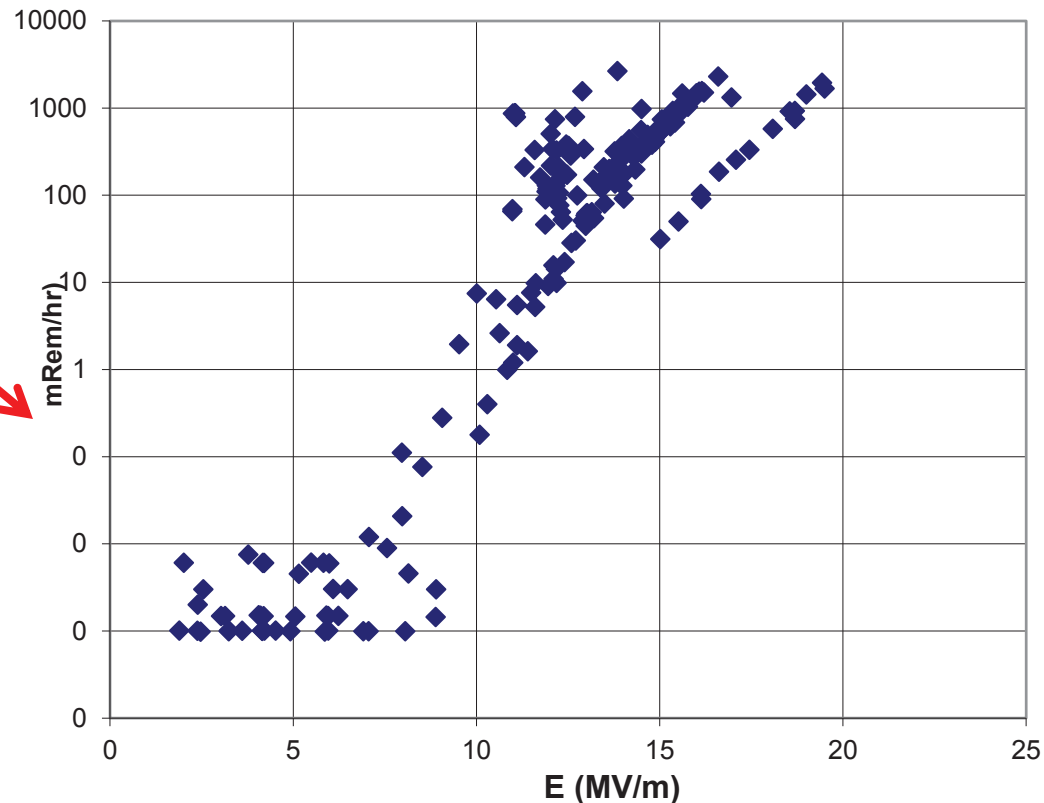
# DC Field Emission from Ideal Surface

**Fowler-Nordheim model**



$$J = \frac{1.54 \times 10^{-6} E^2}{\Phi} \exp\left( -\frac{6.83 \times 10^9 \, \Phi^{3/2}}{E} \right)$$

$J$ : Current density (A/m$^2$)

$E$ : Electric field (V/m)

$\Phi$ : Work function (eV)

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# Field Emission in rf Cavities

$$J = k \frac{1.54 \times 10^{-6} (\beta E)^{5/2}}{\Phi} \exp\left( -\frac{6.83 \times 10^{9} \Phi^{3/2}}{\beta E} \right)$$

$\beta$ :  Enhancement factor (10s to 100s)

$k$ :  Effective emitting surface

# Field Emission

Surface electric field is not a fundamental limitation

Surface fields above 100 MV/m over many $cm^2$ have been maintained cw in superconducting cavities (>200 MV/m for ms)

However field emission is still an important limitation

The main cause of field emission is particulate contamination

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# Cures for Field Emission

- ## Prevention:

  - Semiconductor grade acids and solvents

  - High-Pressure Rinsing with ultra-pure water

  - Clean-room assembly

  - Simplified procedures and components for assembly

  - Clean vacuum systems (evacuation and venting without re-contamination)

- ## Post-processing:

  - Helium processing

  - High Peak Power (HPP) processing

  - Plasma cleaning

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# Thermal Breakdown (Quench)

**Localized heating**

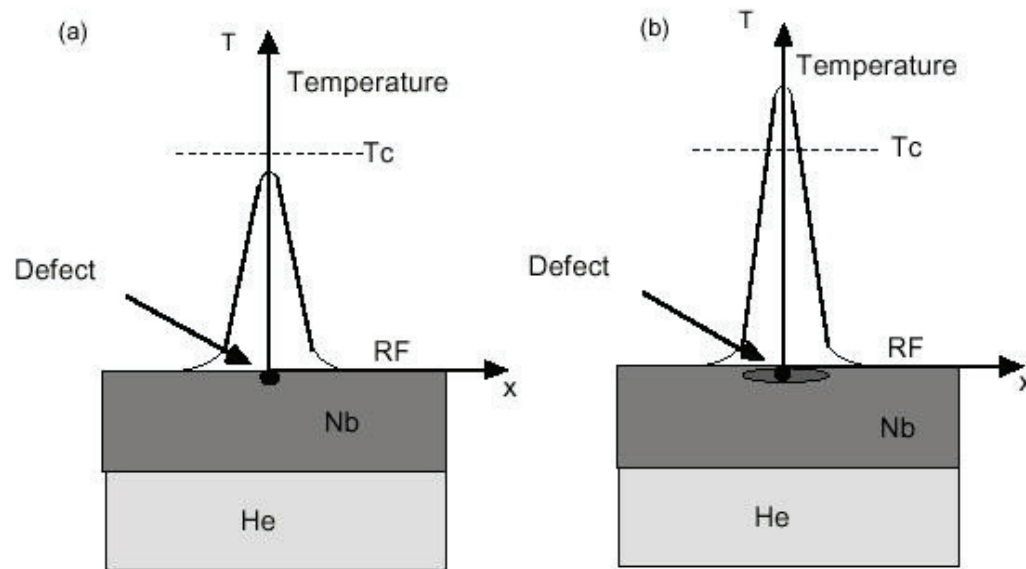**Hot area increases with field**

**At a certain field there is a thermal runaway, the field collapses**

- **sometimes displays a oscillator behavior**

- **sometimes settles at a lower value**

- **sometimes displays a hysteretic behavior**

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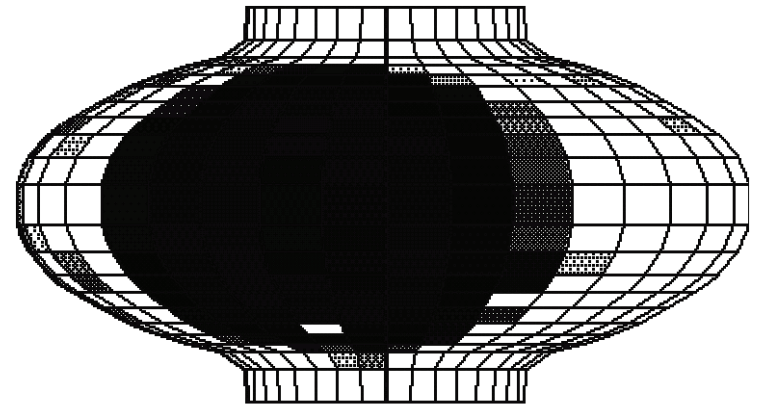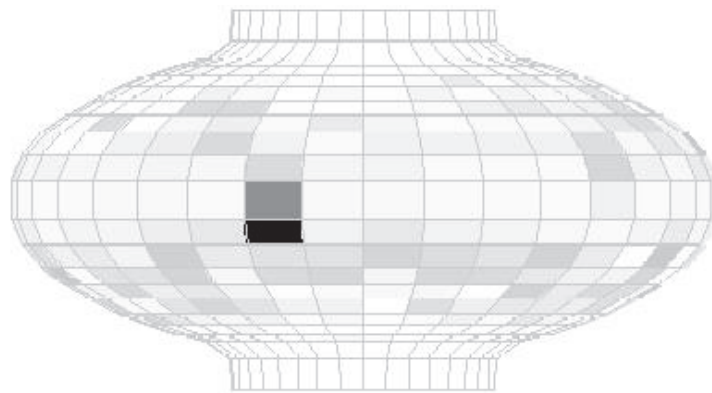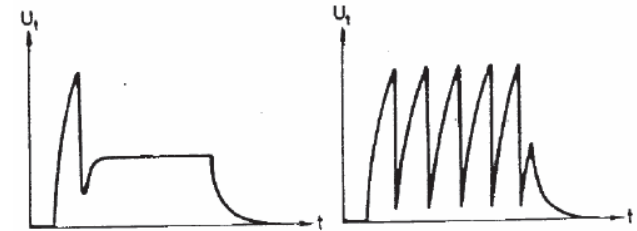
# Thermal Breakdown



**Thermal breakdown occurs when the heat generated at the hot spot is larger than that can be transferred to the helium bath causing $T > T_c$: "quench" of the superconducting state**
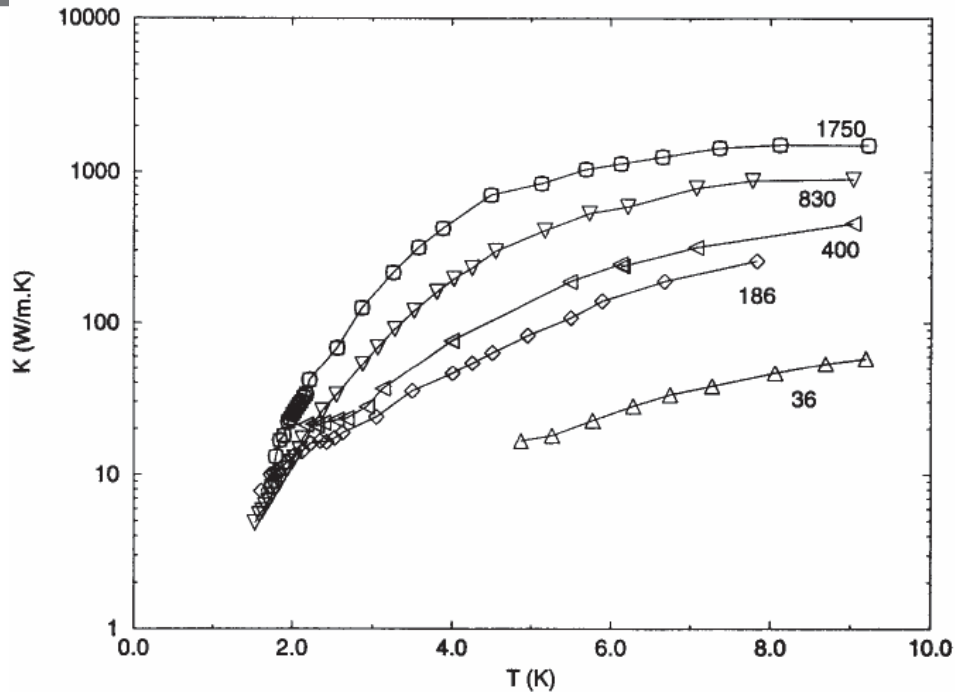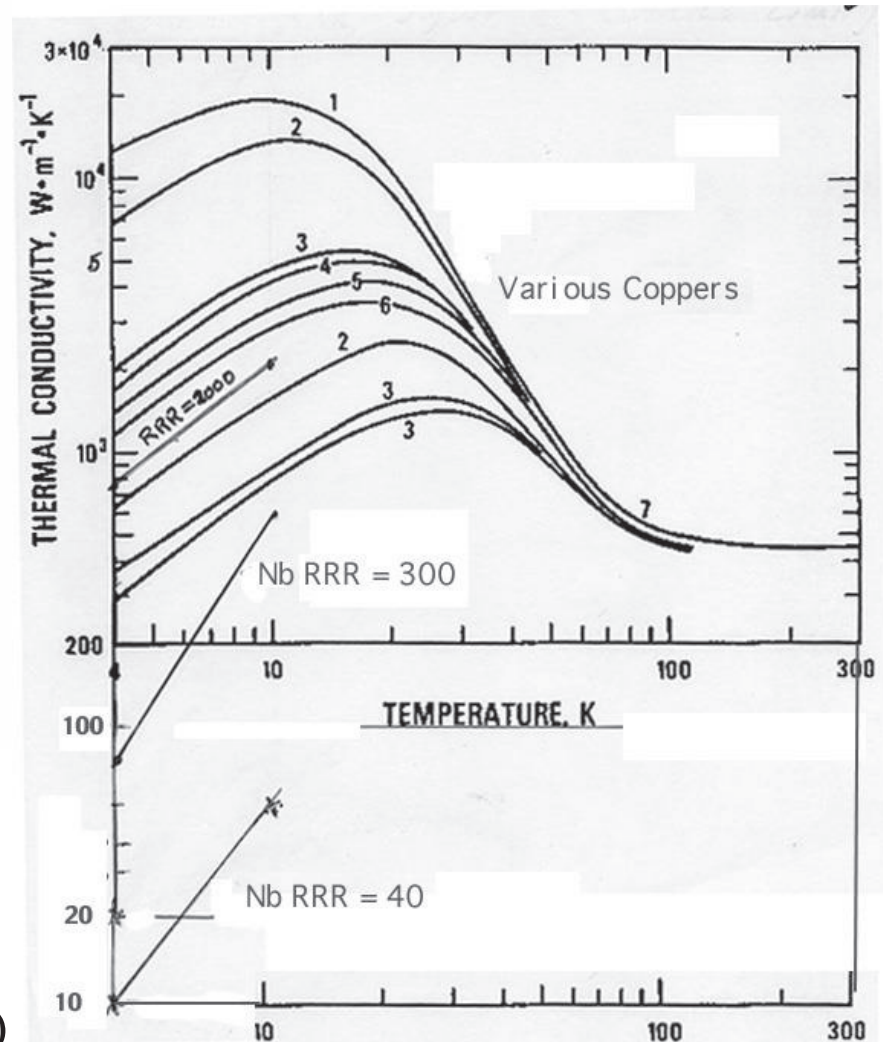
# Thermal Conductivity of Nb



Fig. 3 The thermal conductivity of niobium as a function of temperature, for various RRR values.



RRR is the ratio of the resistivity at 300K and 4.2K

$$RRR = \frac{\rho(300K)}{\rho(4.2K)}$$

RRR is related to the thermal conductivity
For Nb: $\lambda(T = 4.2K) \approx RRR / 4$ (W. m$^{-1}$. K$^{-1}$)

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# Summary on Quench

- Big improvement in Cavity fabrication and treatment
  less foreign materials found (at limitations <20MV/m only)

- Visual inspection systems are available

- Many irregularities in the cavity surface are found with this systems during and after fabrication and treatment
  pits and bumps, weld irregularities

- Often one defect limits the whole cavity

- Some correlations are found between defects and quench locations at higher fields.  But often no correlation between suspicious pits and bumps and quench location

- At gradient limitations in the range >30 MV/m defects are often not identified

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# High-Field Q-Slope ("Q-drop")
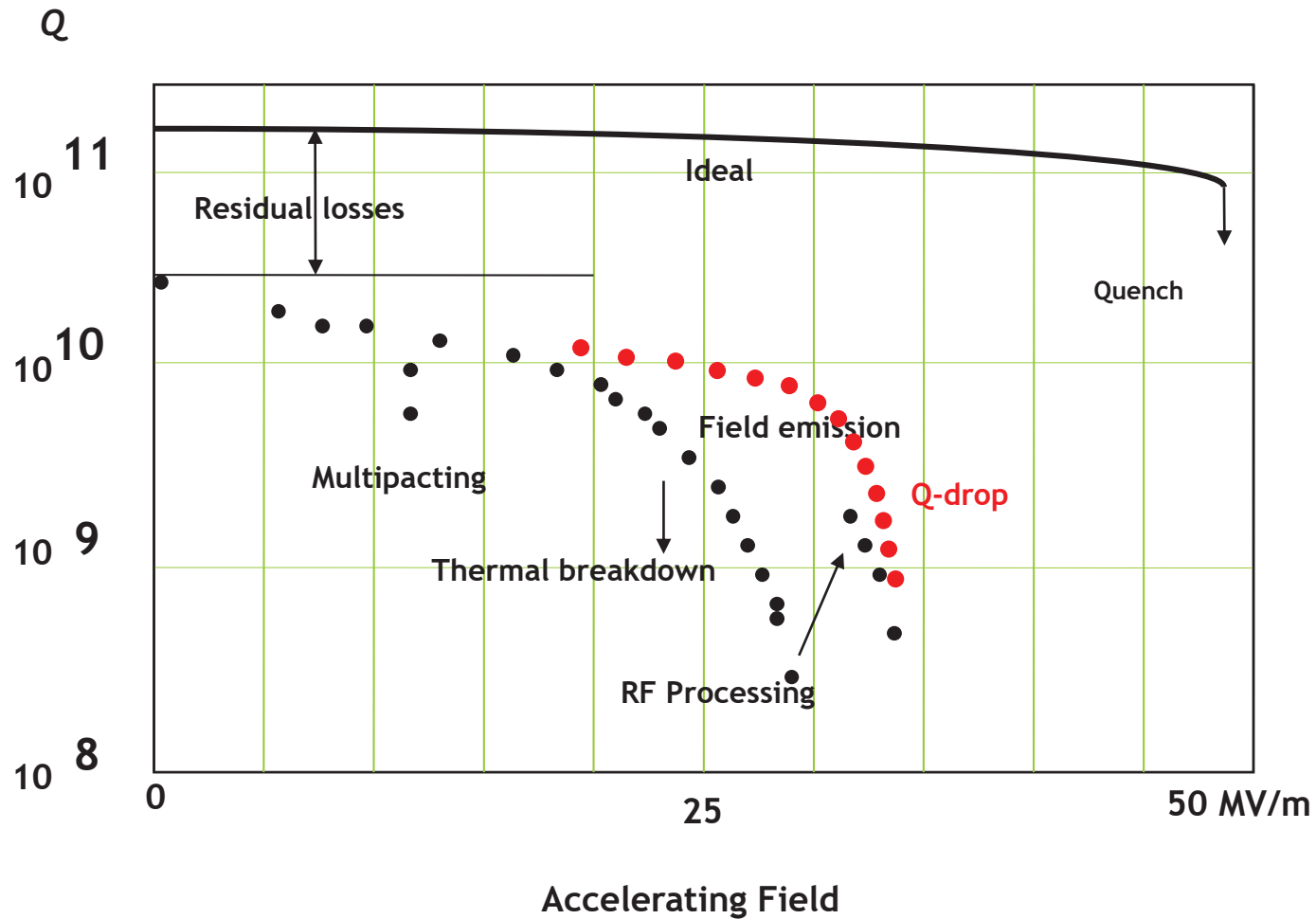
# High Field Q-Drop

- Decrease of Q at high field not associated with x-rays
- Still an area of investigation
- Many models
  - Magnetic field related
  - Electric field related
- Strong indication that it is related to the concentration of oxygen at the surface
- Reduced or eliminated by mild baking around 120C

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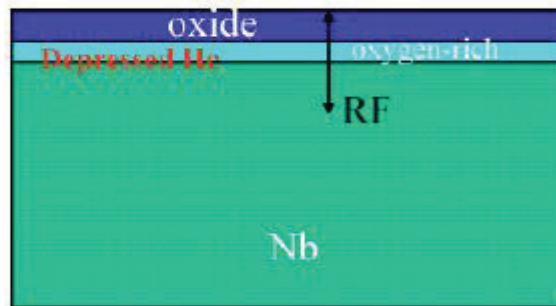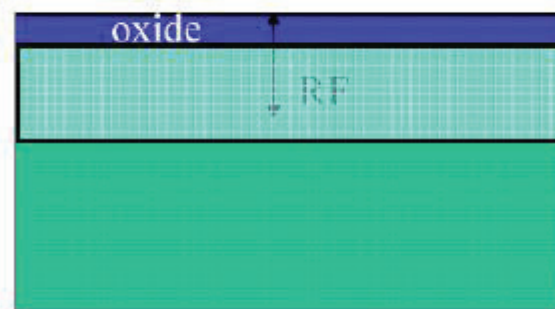
# Q-drop and Baking
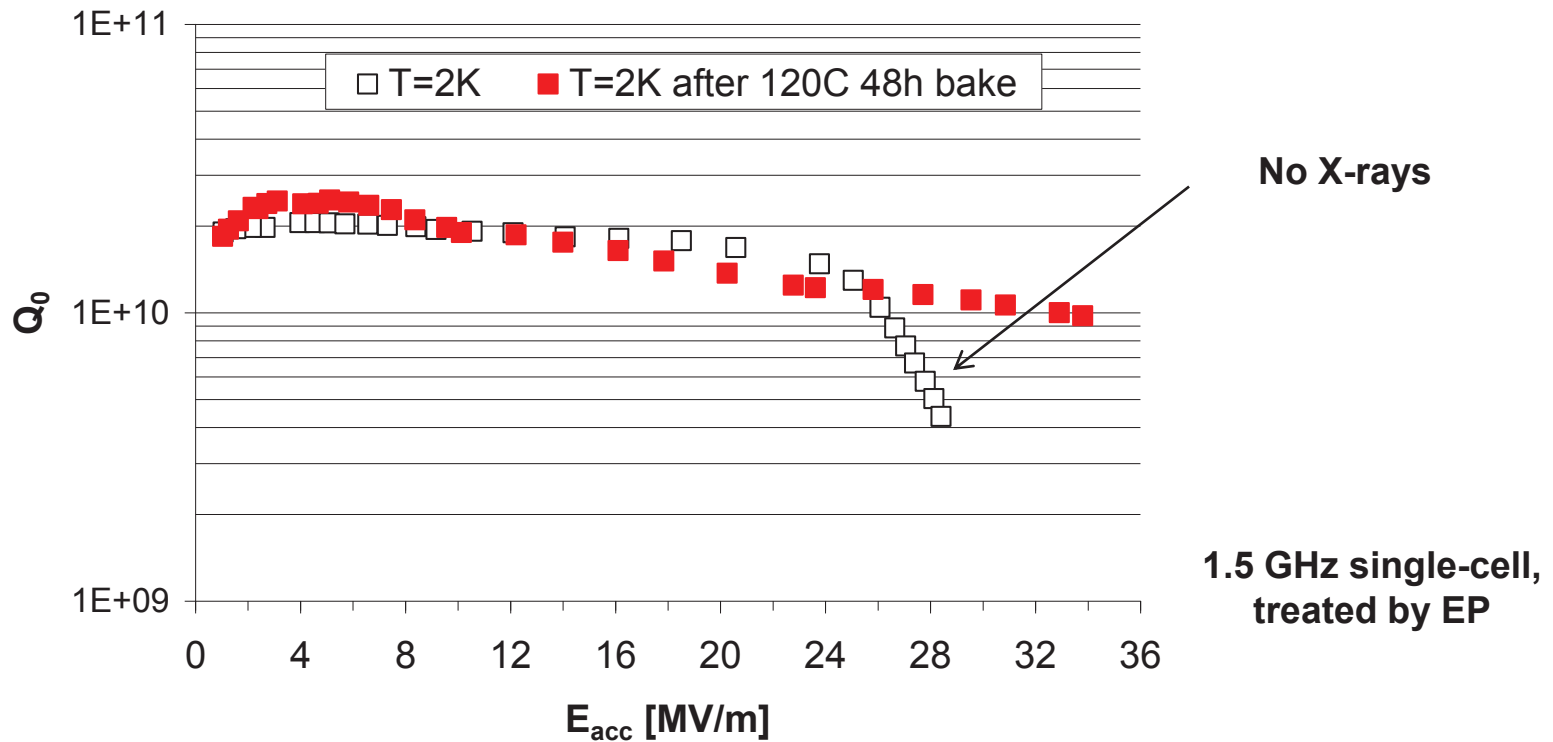


- **The origin of the Q-drop is still unclear. Occurs for all Nb material/treatment combinations**
  - **The Q-drop recovers after UHV bake at 120 °C/48h for certain material/treatment combinations**

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# Models of Q-drop & Baking

- Magnetic field enhancement
- Oxide losses
- Oxygen pollution
- Magnetic vortices

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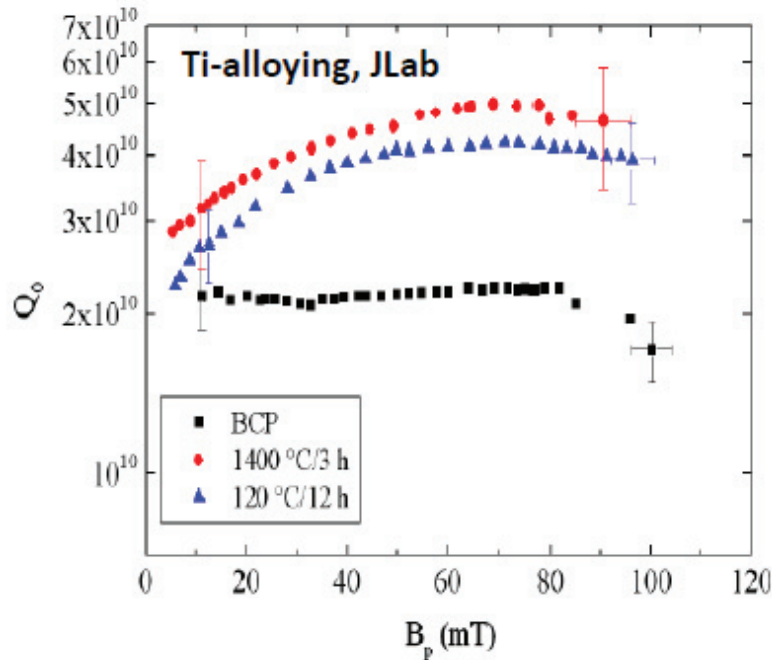
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# Fluxons as Source of Hot-Spots

- Motion of magnetic vortices, pinned in Nb during cool-down across $T_c$, cause localized heating

- Periodic motion of vortices pushed in & out of the Nb surface by strong RF field also cause localized heating
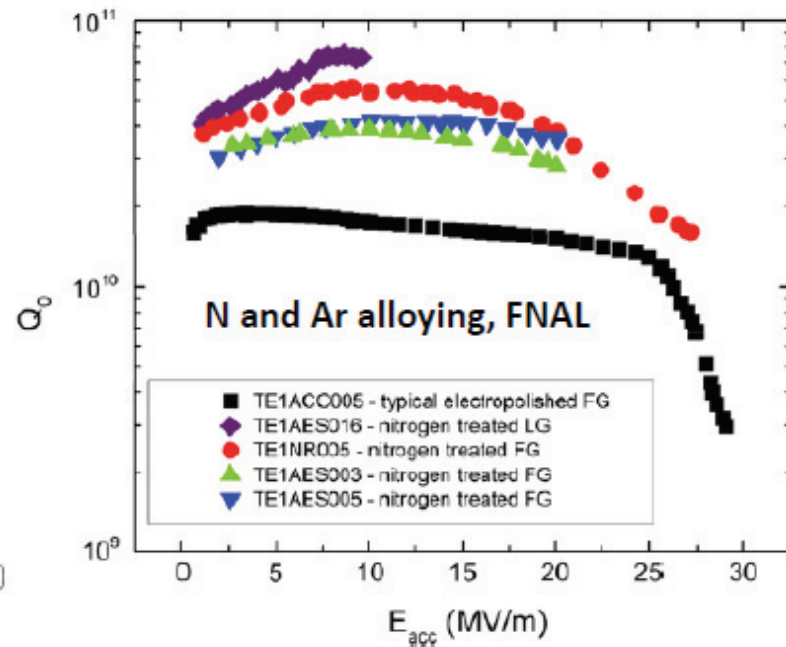
The small, local heating due to vortex motion is amplified by $R_{BCS}$, causing cm-size hot-spots

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# Recent Developments: Impurity Doping

P. Dhakal et al, Phys Rev. ST-AB 16, 042001 (2013)

A. Grasselino et al, SUST 26, 102001 (2013)

Ti-alloying, JLab

N and Ar alloying, FNAL

- BCP
- 1400 °C/3 h
- 120 °C/12 h

- TE1ACC005 - typical electropolished FG
- TE1AES016 - nitrogen treated LG
- TE1NR005 - nitrogen treated FG
- TE1AES003 - nitrogen treated FG
- TE1AES005 - nitrogen treated FG

- Dirty layer due to diffusion of N or Ti into a few μm thick layer ( >> λ = 40 nm) at the surface

- <u>Decrease</u> of $R_s(B)$ up to **B ≈ 0.5B$_c$: microwave suppression of surface resistance**

# Recent Developments: $Nb_3Sn$

## Later Coatings
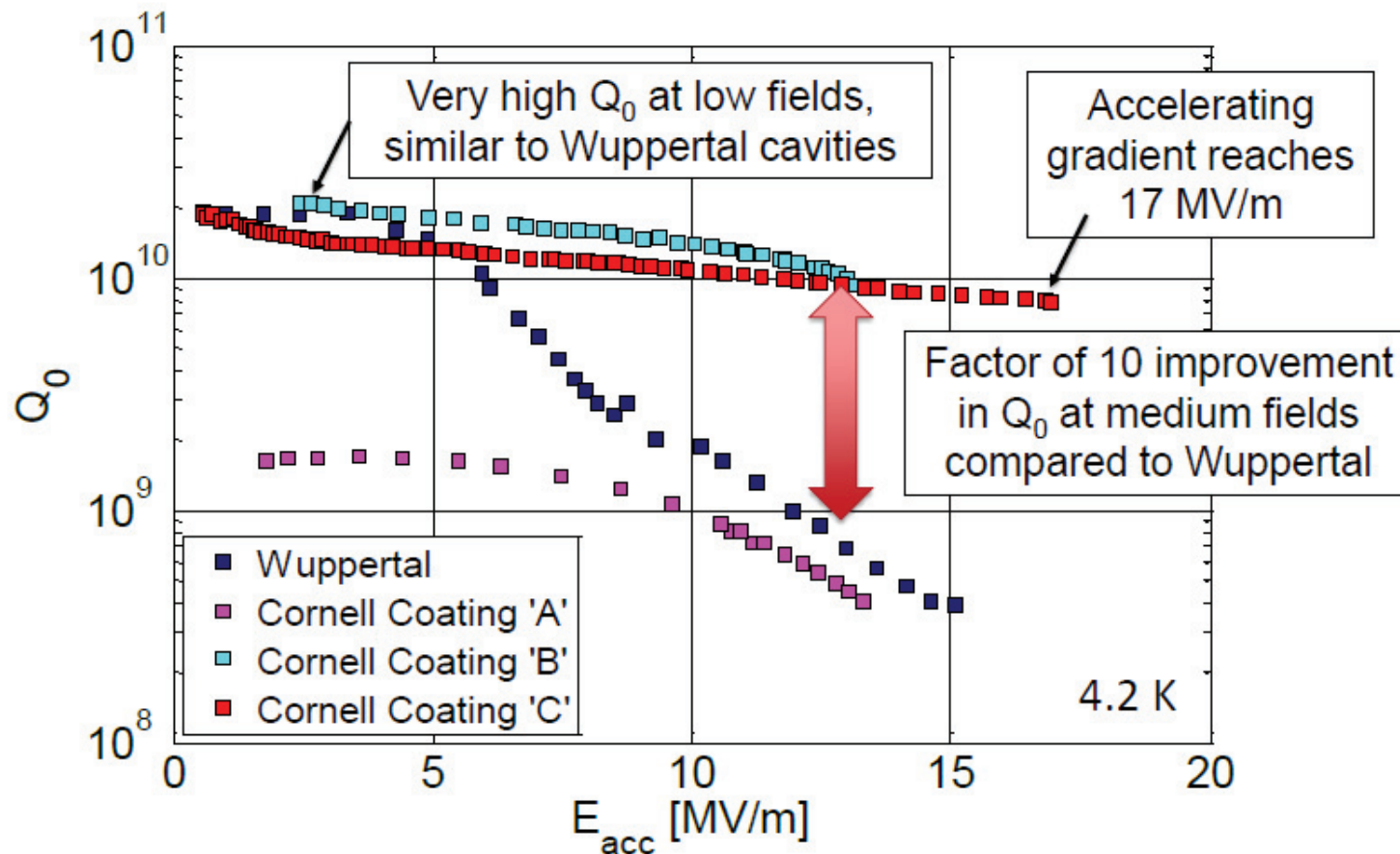
# Parting words

SRF in now 50 years old

Much progress has been made

Many accelerators are operating successfully

But…our full understanding of the rf properties of superconductors and the fundamental limitations of srf cavities is still incomplete and new techniques are still being developed

Still many opportunities to make a difference

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