Simulated Measurements of Beam Cooling in Muon Ionization Cooling Experiment

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Abstract

- The **international Muon Ionization Cooling Experiment (MICE)** aims to demonstrate ionization beam cooling:
  - Muon beam is passed through an absorbing material to reduce its phase-space volume (emittance).
- Why cooled muon beams:
  - **Neutrino Factory**: for intense and pure neutrino beams.
  - **Muon Colliders**: for compact lepton colliders with energies of up to several TeV.
- The figure of merit for cooling: root-mean-square (RMS) emittance reduction.
- Alternative figures of merit for cooling: changes in phase-space density and volume using **Kernel Density Estimation (KDE)** technique.

Introduction

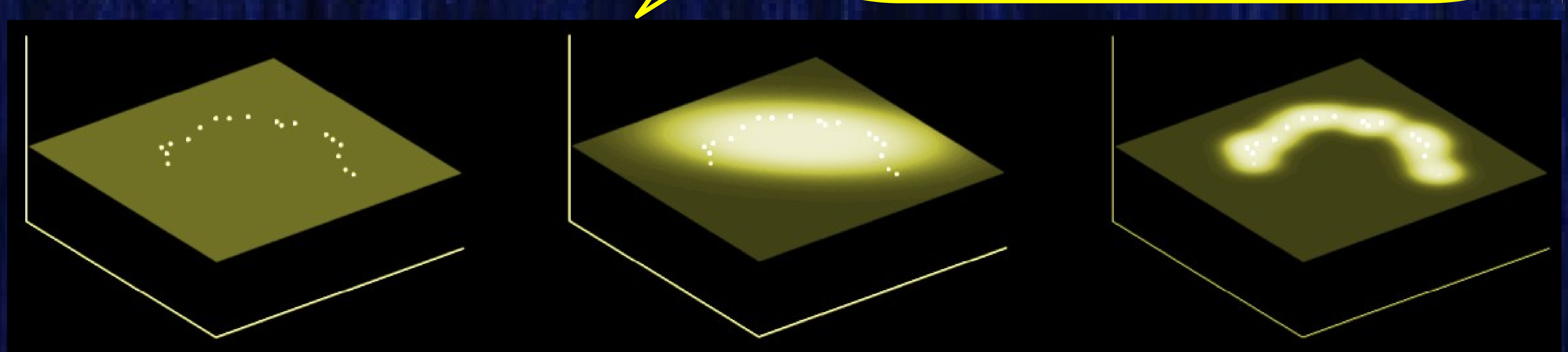
- How MICE demonstrates beam cooling:
  - Ensure muon beam purity using PID detectors (time-of-flight, Cherenkov, electron muon ranger).
  - Reconstruct muon transverse coordinates $X_i=(x_i, p_{xi}, y_i, p_{yi})$ using the trackers.
  - Compute RMS emittance from transverse coordinates.

BUT a different measure of cooling is needed because of the sensitivity of the RMS emittance to non-linear effects.

Kernel Density Estimation in MICE

- Kernel Density Estimation (KDE) technique:
  - Well known in image processing.
  - No assumptions are made about the distribution.

- High probability density regions shown in lighter shades.
- Actual distribution shown on the left.
- Compared to Gaussian density (middle), KDE better reveals the actual distribution.



M. Rousson, et. al., "Efficient Kernel Density Estimation of Shape and Intensity Priors for Level Set Segmentation", (MICCAI) (2005)

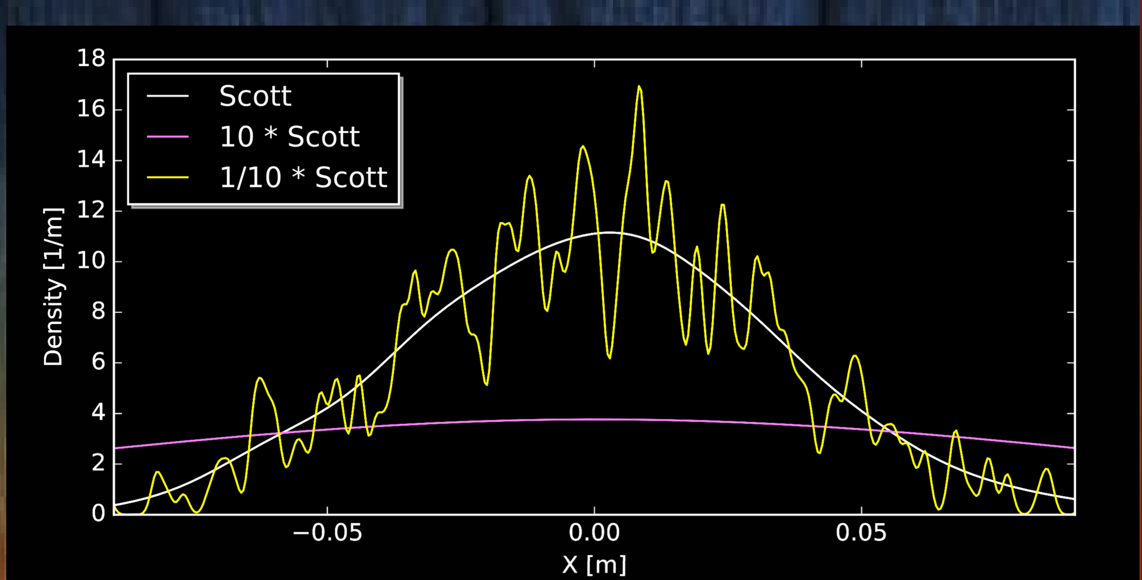
- How MICE demonstrates beam cooling using **KDE**:
- Center a four dimensional Gaussian kernel function (weighting function shaped as multi-dimensional ellipse of variance $h = h_f\Sigma$) at each muon.
- Estimate the density at an arbitrary point $x = (x, p_x, y, p_y)$ by summing the contributions from all muons.

$$\hat{f}(\vec{x}) = \frac{|\Sigma|^{-1/2}}{nh_f^d\sqrt{(2\pi)^d}}\sum_{i=1}^{n}\exp\left[-\frac{(\vec{x}-\vec{X}_i)^T\Sigma^{-1}(\vec{x}-\vec{X}_i)}{2h_f^2}\right]$$

- h_f and h are the bandwidth factor and parameter. Σ is the covariance matrix of the muon coordinates.
- h has a strong effect on the estimated density. Scott's rule of thumb was used here, $h = \Sigma n^{\frac{-1}{d+4}}$

Bandwidth Factor Effect

- Estimated density vs. x position plot for 500 muons.
- Scott's rule of thumb bandwidth parameter multiplied by a large factor oversmoothes the density.
- A smaller factor leads to a noisier density.



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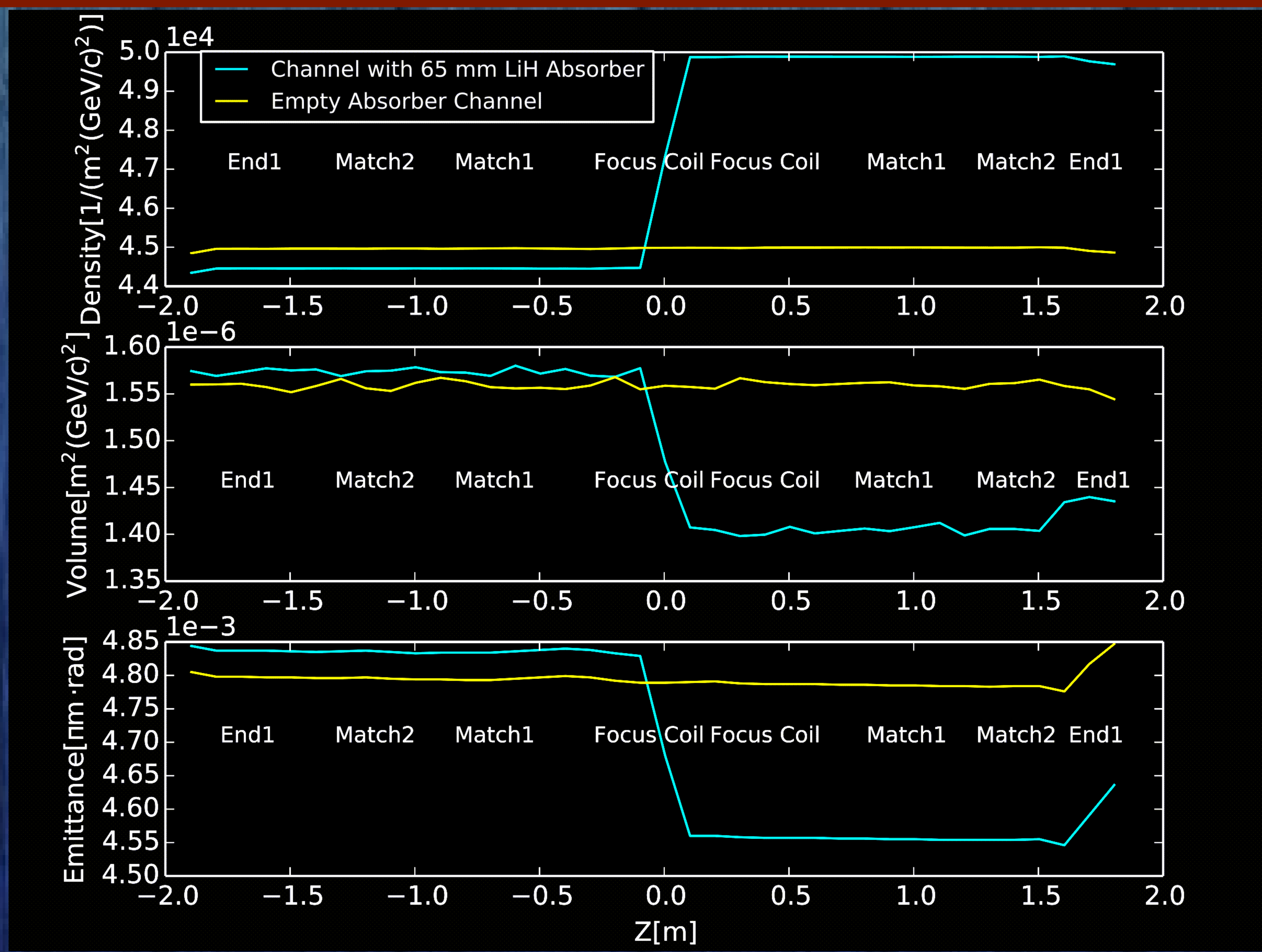
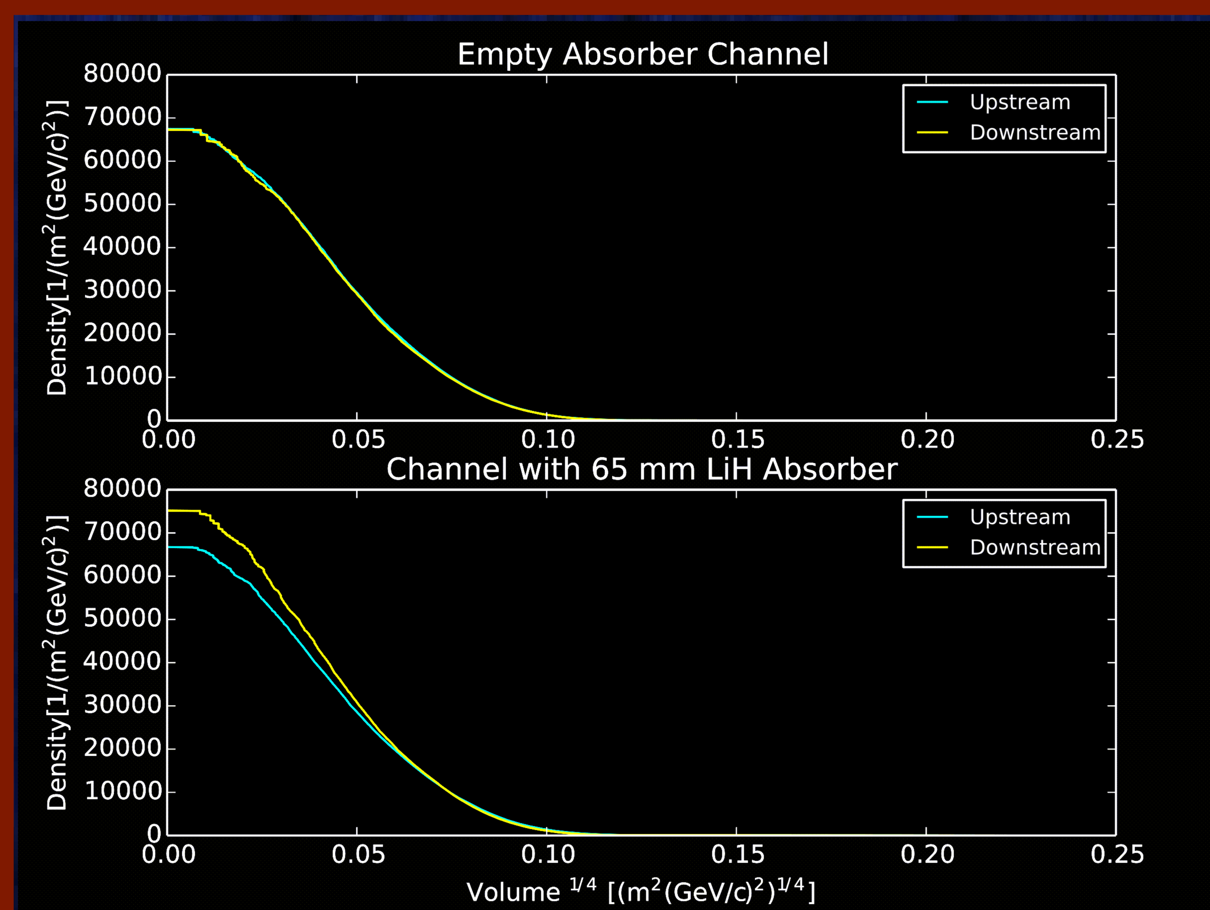
Simulation Results



- The preliminary density, volume and emittance evolution plots in the MICE Step IV channel:
- The yellow curves represent a channel with no absorber.
- The blue curves represent a channel with a 65 mm LiH absorber.
- The evolution curve remains constant for an empty channel except at $z=1.5$ m due to the turned off downstream Match 1 and Match 2 coils.

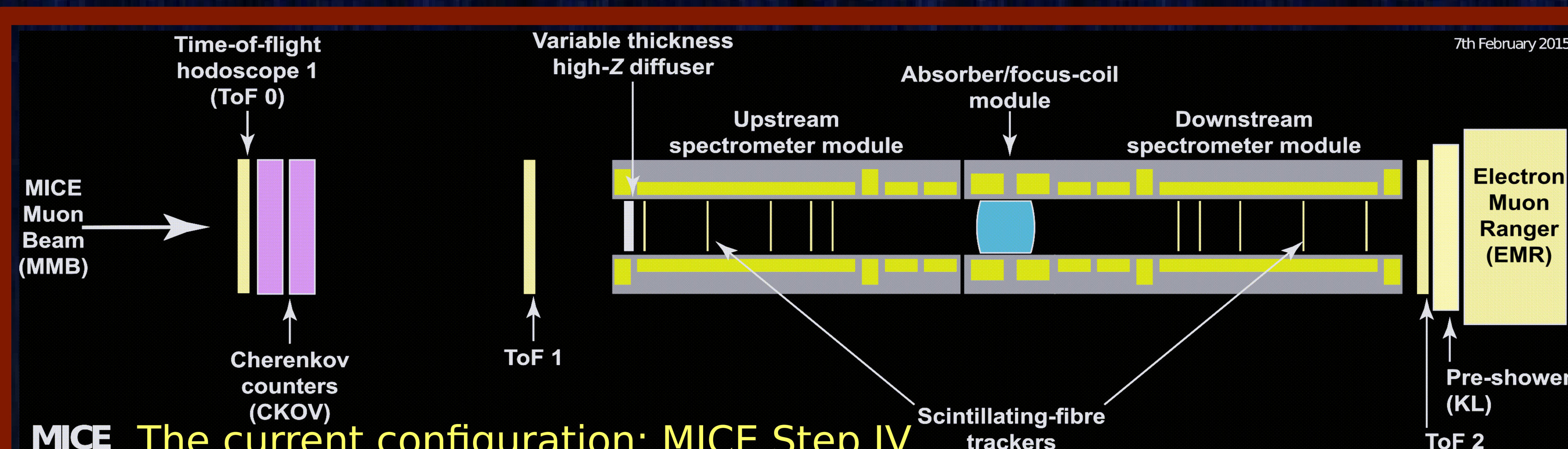


- Preliminary density vs. volume$^{1/4}$ plots:
- In the channel with 65 mm LiH absorber, the density at smaller radius increases as the beam passes through the absorber.
- In an empty channel, no change in density is observed.

Conclusion

- Studied a MICE Step IV lattice with Match 2 and the in-operable Match 1 coil fields set to zero in the downstream Spectrometer Solenoids.
- Demonstrated cooling through phase-space density increase and phase-space volume decrease using KDE.

MICE



The current configuration: MICE Step IV

Before and after MICE photos: the cooling channel (left, 2015) enclosed by the partial return yoke (PRY) (right, 2016).