Dynamics of Intense Beam in Quadrupole-Duodecapole Lattice Near Sixth Order Resonance

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Abstract

The presence of duodecapole components in quadrupole focusing field results in excitation of sixth-order single-particle resonance if the phase advance of the particles transverse oscillation is close to 60°. This phenomenon results in intensification of beam losses. We present analytical and numerical treatment of particle dynamics in the vicinity of sixth-order resonance. The topology of resonance in phase space is analyzed. Beam emittance growth due to crossing of resonance islands is determined. Halo formation of intense beams in presence of resonance conditions is examined.

Duodecapole Component

Magnetic vector potential of lens with quadrupole symmetry

$$A_z = -[\frac{G_2}{2} r^2 \cos 2\theta + \frac{G_6}{6} r^6 \cos 6\theta + ...]$$

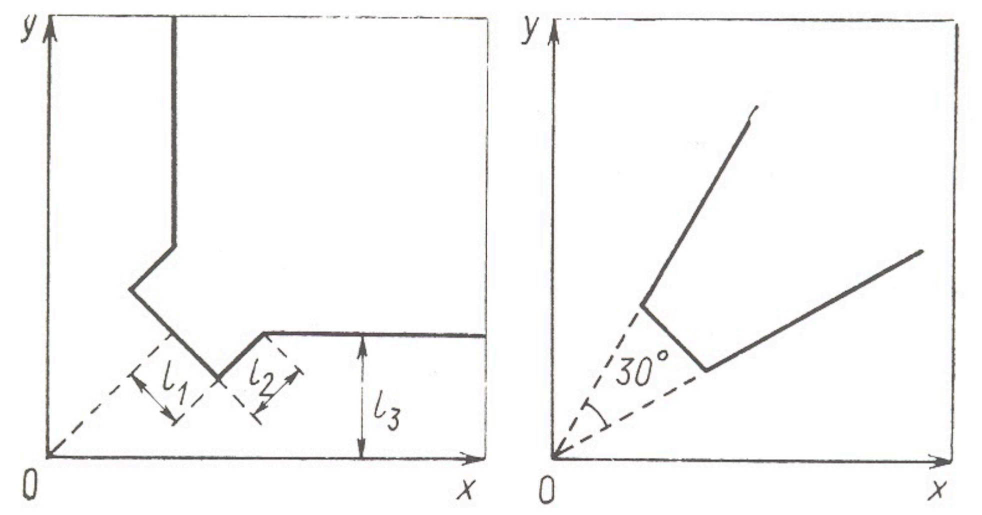
Vertical component of magnetic field along abscissa

$$B_y(x,0) = G_2 x + G_6 x^5 + ...$$

Particles traveling through quadrupole receive kick which contains both linear and non-linear parts

$$\Delta \frac{dx}{dz} = \frac{qD}{mc\beta\gamma}(G_2 x + G_6 x^5 + ...)$$

Minimization of G_6



Minimization of duodecapole component $l_1 = 0.4852$, $l_2 = 0.5741$, $l_3 = 0.77$ (I.M. Kapchinsky, "Theory of Linear Resonance Accelerators", Harwood, 1985)

Hamiltonian of Sixth Order Resonance

Phase advance per FODO period:

$$\mu_o = \frac{S}{2D}\sqrt{1 - \frac{4}{3}\frac{D}{S}}\frac{qG_2 D^2}{mc\beta\gamma}$$

Matrix of focusing period

$$\begin{pmatrix} x_{n+1} \\ p_{n+1} \end{pmatrix} = \begin{pmatrix} \cos\mu_o & \sin\mu_o \\ -\sin\mu_o & \cos\mu_o \end{pmatrix} \begin{pmatrix} x_n \\ p_n + \Delta p_n \end{pmatrix}$$

p is the modified momentum and *Δp* is the non-linear quick due to presence of duodecapole component

$$p = \frac{S}{\mu_o}\frac{dx}{dz} \qquad \Delta p = \delta_5 x^5$$

Strength of non-linear duodecapole kick

$$\delta_5 = 2\frac{qG_6 DS}{mc\beta\gamma\ \mu_o}$$

Transformation to action-angle variables

$$x = \sqrt{2J}\cos\psi$$

$$p = -\sqrt{2J}\sin\psi$$

Normalized emittance of the beam

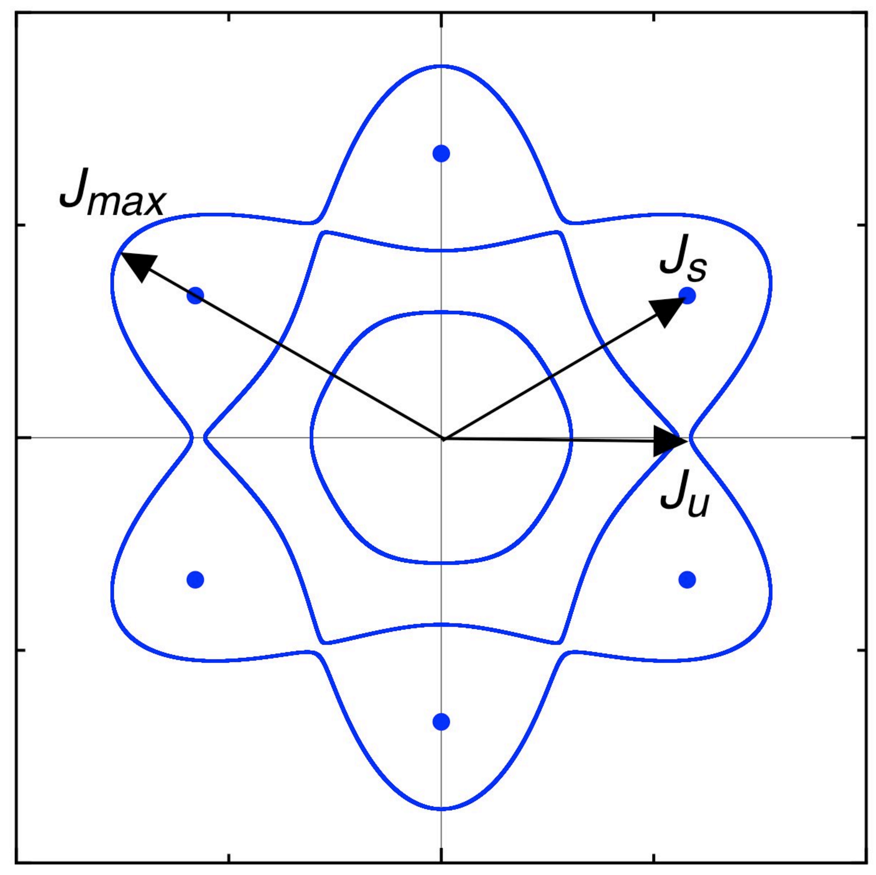
$$\varepsilon = 2J\beta\gamma\frac{\mu_o}{S}$$

Deviation from "resonance" angle 60°

$$\vartheta = \mu_o - \pi/3$$

Hamiltonian describing slow motion near sixth order resonance

$$\boxed{H(J,\psi) = J\vartheta - \frac{\delta_5}{4}J^3 - \frac{\delta_5}{24}J^3\cos 6\psi}$$



Topology of 6th order resonance.

Fixed Points and Island Size

Fixed points (stable and unstable) are determined by equations:

$$\frac{dJ}{dn} = -\frac{\partial H}{\partial\psi} = -\frac{\delta_5}{4}J^3\sin 6\psi = 0$$

$$\frac{d\psi}{dn} = \frac{\partial H}{\partial J} = \vartheta - \frac{3}{4}\delta_5 J^2[1 + \frac{\cos 6\psi}{6}] = 0$$

First equation has a solution sin 6ψ = 0 or cos 6ψ = +/-1.

Unstable points: cos6ψ = 1
Stable points: cos6ψ = -1

Action at unstable point

$$J_u = \sqrt{\frac{8}{7}\frac{\vartheta}{\delta_5}} \qquad \psi_u = \frac{\pi}{3}k$$

Action at stable point

$$J_s = \sqrt{\frac{8}{5}\frac{\vartheta}{\delta_5}} \qquad \psi_s = \frac{\pi}{6} + \frac{\pi}{3}k$$

Maximum Action value

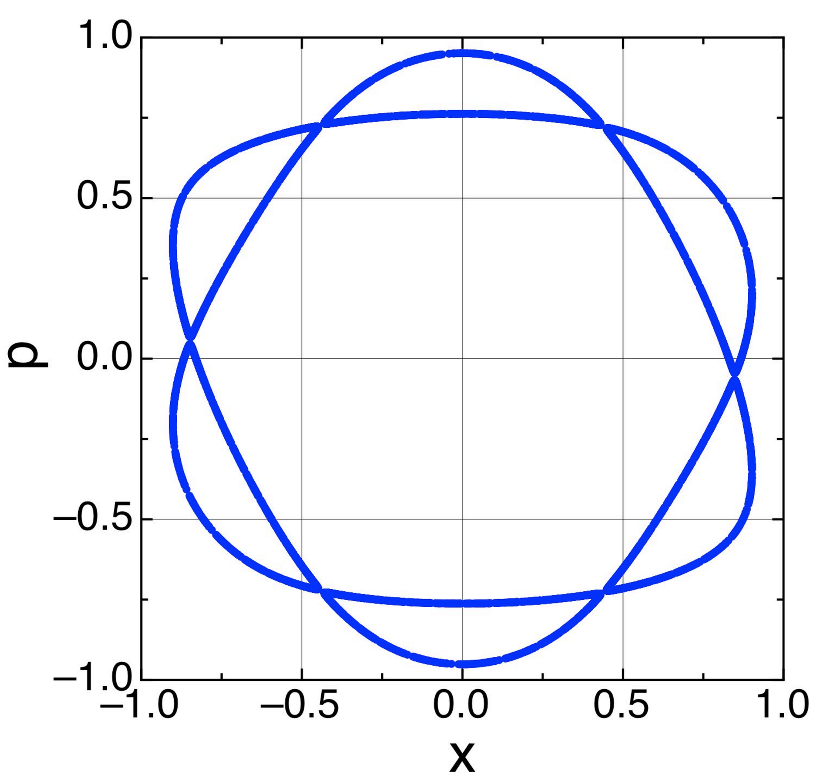
$$J_{max} = 1.54\sqrt{\frac{8}{7}\frac{\vartheta}{\delta_5}}$$

Beam emittance limited by unstable points

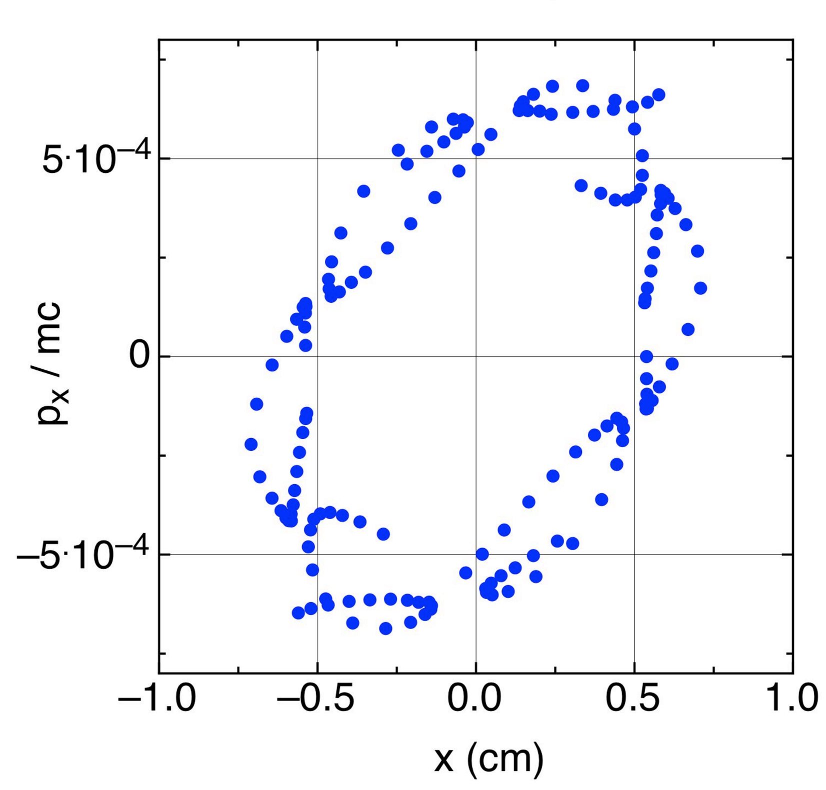
$$\varepsilon_u = \sqrt{\frac{32}{7}\frac{\vartheta}{\delta_5}}\beta\gamma\frac{\mu_o}{S}$$

Single Particle Dynamics in FODO Lattice

Matrix Method

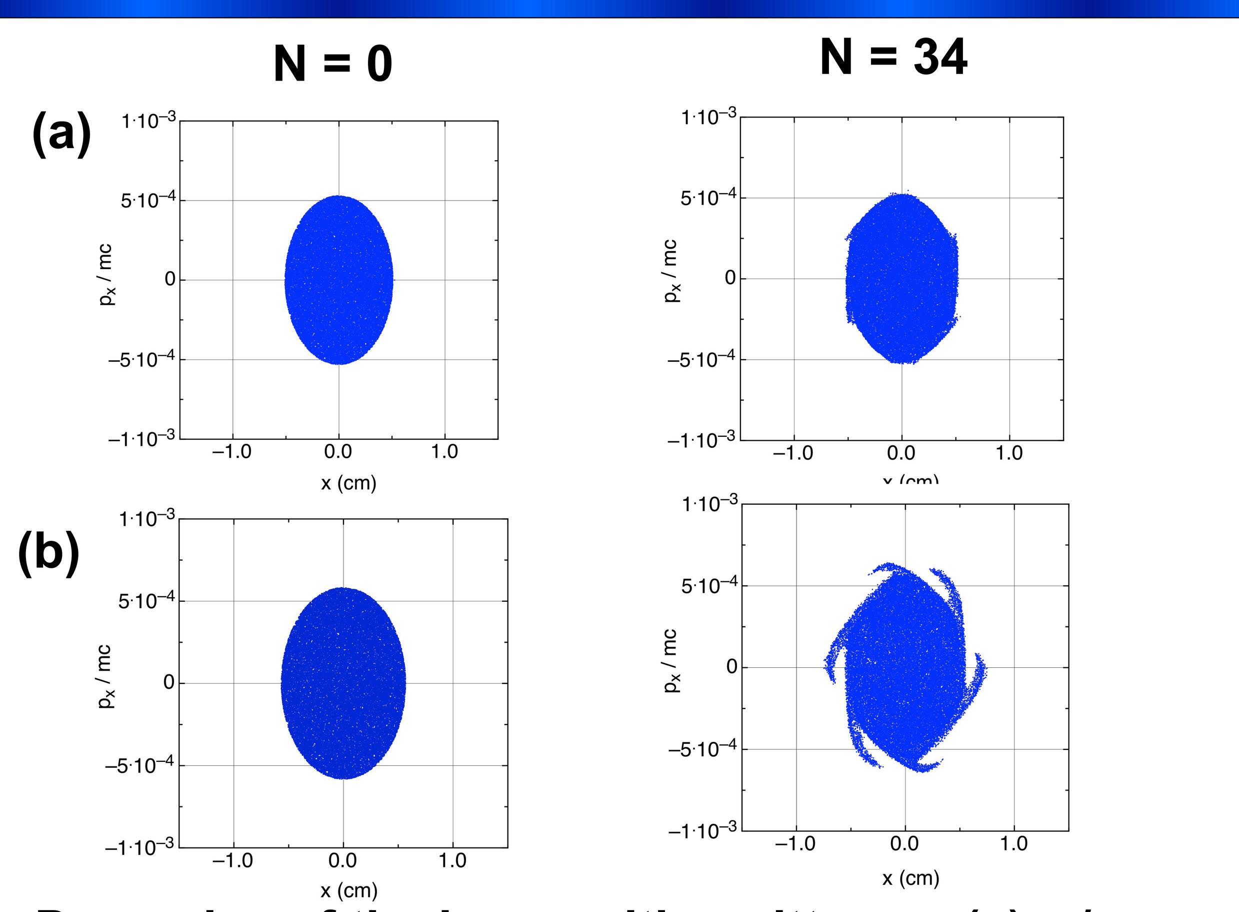


Direct Integration



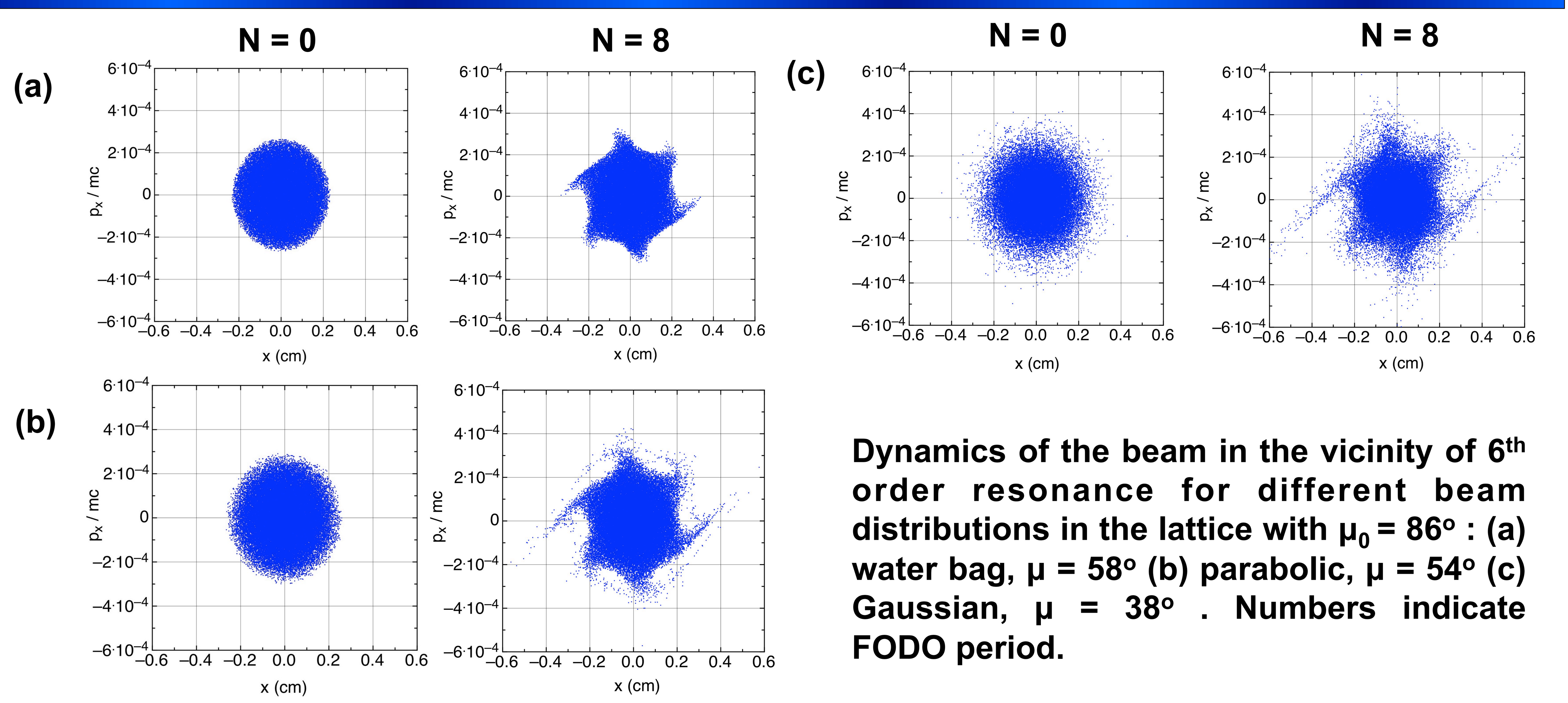
Dynamics of single particle in FODO focusing channel with D/S=1/3 and phase advance μ_o= 62.6°.

Distortion of Beam Emittance Near 6th Order Resonance



Dynamics of the beam with emittance (a) $\varepsilon / \varepsilon_u$= 0.5 (b) $\varepsilon / \varepsilon_u$ = 0.6. Numbers indicate FODO period.

Effect of Space Charge on Beam Dynamics Near Resonance



Dynamics of the beam in the vicinity of 6th order resonance for different beam distributions in the lattice with μ_0 = 86° : (a) water bag, μ = 58° (b) parabolic, μ = 54° (c) Gaussian, μ = 38° . Numbers indicate FODO period.