Microbunching instability (MBI) has been a challenging issue in high-brightness electron beam transport for modern accelerators. Our Vlasov analysis of MBI is based on single-pass configuration. For multi-pass recirculation or a long beamline, the intuitive argument of quantifying MBI by successive multiplication of individual MBI gains was found to underestimate the effect. More thorough analyses based on concatenation of gain matrices aimed to combine both density and energy modulations for a general beamline. Yet, quantification still focuses on characterizing longitudinal phase space; microbunching structures residing in $(t,z)$ or $(t',z')$ was observed in particle tracking simulations. Inclusion of such cross-plane microbunching structures in Vlasov analysis shall be a crucial step to systematically characterize MBI for a beamline complex in terms of concatenating individual beamline segments. We derived a semi-analytical formulation to include the microbunching structures in longitudinal and transverse phase spaces. Using these generalized formulas, we studied an example lattice and found the microbunching gains calculated from multiplication of concatenated gain matrices can be considered as upper limit to the ultimate effect.

**References**

[3] C.-Y. Tsai and R. Li, IPAC’16 (TUPOR020)

**V. Summary**

In this paper we derived a set of governing equations for microbunching in different dimensions, including density, energy, and transverse-longitudinal modulations, and apply to an example of recirculating machine. The Vlasov solutions and tracking simulations agree qualitatively with each other. Although the Vlasov results from concatenated sections do not match well with those obtained directly from very beginning (start-to-end), $G_{MBI} = \sum_{n=0}^{\infty} G_n$ gives upper limit for the modulation spectra. In addition, the extended formulations can give us further insights on how upstream beamline sections can accumulate density, energy modulation, and/or transverse-longitudinal microbunching, when the full-ring lattice is not provided.