ANALYTICAL MODELING OF ELECTRON BACK-BOMBARDMENT INDUCED CURRENT INCREASE IN UN-GATED THERMIONIC CATHODE RF GUNS

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Abstract

In this paper we derive analytical expressions for the output current of an un-gated thermionic cathode RF gun in the presence of back-bombardment heating. We provide a brief overview of back-bombardment theory and discuss comparisons between the analytical back-bombardment predictions and simulation models. We then derive an expression for the output current as a function of the RF repetition rate and discuss relationships between back-bombardment, field-enhancement, and output current. We discuss in detail the relevant approximations and then provide predictions about how the output current should vary as a function of repetition rate for some given system configurations.

INTRODUCTION

Thermionic cathodes are widely known as a robust source. When used in un-gated RF guns they result in a simple electron injector that is robust and suitable for a wide range of applications. However, one primary drawback of this system is that the gun will accelerate electrons whenever the field on the cathode is negative. As a result, some electrons that are emitted late relative to the RF period will not gain enough energy to exit the cathode cell. These electrons will be decelerated when the field changes sign and eventually accelerated back towards the cathode surface. These so-called back-bombarded electrons deposit their energy on the cathode surface in the form of heat. This back-bombardment heating impedes the ability to regulate the output current of the gun.

Recent work developed scaling laws relating back-bombardment power to different gun design parameters [1,2,3]. These scaling laws can be used for initial gun design and trade-space optimization prior to a detailed simulation-based design. While instructive, these models do not provide any estimates for how the output current will be affected by gun designs with different back-bombardment power levels. In order to address this, we combined the analytic models for back-bombardment power with the Richardson-Dushman equation to study the effect of gun design parameters on the output current. For a fixed geometry, there are three parameters which directly affect the back-bombardment power levels: the initial output of the cathode, the field on the cathode, and the RF duty factor.

In this paper we will address how changing the RF duty factor and the peak field on the cathode will affect the output current of the gun in the presence of back-bombardment heating. While this work is concerned with estimating the change in output current due to back-bombardment heating for a general class of thermionic guns, the Advanced Photon Source (APS) injector test stand gun will be used as an example to provide justification for the necessary approximations. For a different gun, the approximations made here would need to be re-evaluated. We will begin with a brief overview of the existing back-bombardment power models, and discuss their range of validity. Then we derive the relationship between repetition rate and output current in the presence of back-bombardment heating, followed by a discussion of how field enhancement in these guns changes the back-bombardment effect.

OVERVIEW OF BACK-BOMBARDMENT THEORY

The back-bombardment power for a particular gun design is defined by Equation 1 [1].

$$P_{\mathrm{ave}} = \frac{3IE_0c^2}{4v_{\mathrm{eff}}f\alpha^2}TKD_{\mathrm{F}} \tag{1}$$

In Equation 1, I is the average beam current, E_0 is the peak field gradient, c is the speed of light in vacuum, f is the RF frequency of the gun, α is the normalized gap length of the cathode cell, defined by $\alpha = c/(fL_{\mathrm{gap}})$, T is the transit time factor, v_{eff} is the effective velocity (Equation 2), K is the normalized field strength (Equation 3), and D_{F} is the duty factor of the RF system.

$$v_{\mathrm{eff}} = c\sqrt{1-\left(1+\frac{qE_0\lambda}{2m_0c^2\alpha}\right)^{-2}} \tag{2}$$

$$K = \frac{\int_0^{L_{\mathrm{gap}}} E(z)dz}{E_0\lambda/\alpha} \tag{3}$$

For the APS Injector test stand, the geometry parameters α, T, and K, are given by Table 1. Equation 1 has been demonstrated to match simulation and measurement of back-bombardment power to order of magnitude accuracy for

Table 1: Geometry Parameters for the APS Gun

Parameter	Symbol	Value
Transit Time Factor	T	0.73
Normalized Gap Voltage	K	0.80
Alpha	α	3.36

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a wide range of gun designs [1,2,3]. We can evaluate the effectiveness of this model or a particular gun design by examining the ψ parameter (Equation 4) [3].

$$\psi = \frac{\alpha f}{E_0} \tag{4}$$

For gun designs with $\psi < 200$, Equation 1 will generally make more reliable predictions of the back-bombardment power [3]. For the APS injector test stand ψ will vary for different peak fields. For a peak field in the cathode cell ranging from 30 MV/m to 50 MV/m, ψ will vary from 320 to 210. As the ψ is close to the cutoff between reliable and unreliable predictions for the APS gun care must be taken when interpreting results that use Equation 1. Therefore the APS gun was simulated and compared with Equation 1 in order to ensure appropriate application of Equation 1 to this particular design. The comparison showed a peak difference between simulation and theory of a factor of two over the previously specified range of peak fields in the gap.

ANALYSIS OF REPETITION RATE DEPENDENT EFFECTS

The output current of a thermionic cathode is given by the Richardson-Dushman equation with the Schottky correction for field enhancement, Equation 5. Here T_s is the surface temperature, W is the work function, E_0 is the surface electric field applied to the cathode, and A_c is the cathode area.

$$I = A_c A_0 T_s^2 \exp\left(-\frac{W - \sqrt{\frac{q^3 E_0}{4\pi\epsilon_0}}}{k_b T_s}\right) \tag{5}$$

To evaluate the impact of back-bombardment power on output current, we begin by approximating the relationship between cathode temperature and the heater power. In the absence of back-bombardment heating, the cathode temperature can be expressed as a linear dependance of the heater power.

$$T_s = \alpha_c P_h + T_0 \tag{6}$$

This linear coefficient α_c and offset temperature T_0 are specific to the particular machine in question and would be experimentally determined. The heater power/temperature relationship in the APS gun was measured to be $\alpha_c = 19.23$ K/W and $T_0 = 707$ K. We include the impact of back-bombardment power on temperature by assuming that it acts in the same fashion as the cathode heater and therefore can be added to the linear term in Equation 6,

$$T_s = \alpha_c(P_h + P_{bb}) + T_0. \tag{7}$$

Substituting Equation 7 into Equation 5 gives the output current as a function of temperature in the presence of back-bombardment heating, Equation 8.

$$\begin{aligned} I =& A_c A_0 \left(\alpha_c(P_h + P_{bb}) + T_0\right)^2 \\ & \exp\left(-\frac{W - \sqrt{\frac{q^3 E_0}{4\pi\epsilon_0}}}{k_b(\alpha_c(P_h + P_{bb}) + T_0)}\right) \end{aligned} \tag{8}$$

Using Equation 8, one can derive the relationship between the output current and any of the gun design parameters using the methodology presented in this paper. In order to aid in understanding the relationship between output current and RF duty factor, we make the following substitutions: $A = A_c A_0$, $\Delta W = \sqrt{E_0 q^3/4\pi\epsilon_0}$, and $P_{bb} = G_F D_F I(D_F)$. Where G_F is referred to as the geometry factor and accounts for the terms in Equation 1 that do not depend on the RF duty factor. Note that because the back-bombardment heating depends on the output current we include the output current as some function of the duty factor, $I(D_F)$. Making these substitutions and differentiating Equation 8 with respect to the duty factor results in Equation 9.

$$\begin{aligned} \frac{dI(D_F)}{dD_F} =& \left\{2\alpha_c A G_F \left(D_F \frac{dI(D_F)}{dD_F} + I(D_F)\right)\right. \\ & \left.\left[\alpha_c P_h + \alpha_c G_F D_F I(D_F) + T_0 + \left(\frac{W - \Delta W}{2k_b}\right)\right]\right\} \\ & \exp\left[\frac{-W + \Delta W}{k_b(\alpha_c P_h + \alpha_c G_F D_F I(D_F) + T_0)}\right] \end{aligned} \tag{9}$$

Equation 9 describes the general relationship between the change in duty factor and the change in current due to the presence of back-bombardment heating. As written, Equation 9 is not analytically solvable, however it is separable. By defining the functions $f(I)$ and $g(I)$ as Equations 10 and 11 respectively we can simplify Equation 9 giving Equation 12.

$$f(I) = \left(\alpha_c P_h + \alpha_c G_F D_F I(D_F) + T_0 + \left(\frac{W - \Delta W}{2k_b}\right)\right) \tag{10}$$

$$g(I) = \frac{-W + \Delta W}{k_b(\alpha_c P_h + \alpha_c G_F D_F I(D_F) + T_0)} \tag{11}$$

$$\frac{dI(D_F)}{dD_F} = \frac{2A\alpha_c G_F f(I) e^{g(I)} I(D_F)}{1 - 2A\alpha_c G_F f(I) e^{g(I)} D_F} \tag{12}$$

Equation 12 has no analytical solution. However, we will show next that the current dependance in both $f(I)$ and $g(I)$ is not the dominating factor and therefore both of these functions can be approximated as constants. For the APS gun, the normal range of the terms in Equation 11 is given in Table 2.

Table 2: Approximate Range of Terms in $f(I)$

Term	Range
$\alpha_c P_h$	384.6 ↔ 673.05
$\alpha_c G_F D_F I(D_F)$	0.277 ↔ 24.2
$\frac{W-\Delta W}{2k_b}$	9804 ↔ 10687
T_0	707.46 K

Table 2 shows that work function divided by twice the Boltzmann constant is the dominant factor in $f(I)$. Additionally, because the output current of the gun is between 50-300 mA, the current dependent term is quite small. Therefore we can approximate Equation 10 as the constant K_1.

$$K_1 = \left(\alpha_c P_h + T_0 + \left(\frac{W - \Delta W}{2k_b}\right)\right) \tag{13}$$

The results in Table 2 also allow us to neglect the current dependent term in Equation 11 and therefore approximating $g(I)$ as the constant K_2.

$$K_2 = \frac{-W + \Delta W}{k_b(\alpha_c P_h + T_0)} \tag{14}$$

In essence, the simplification resulting in Equation 13 and 14 is neglecting the second order effect caused by recursion, i.e. increasing the duty factor will increase the back-bombardment which increases the current, but the increase in current will have a second order increase in the back-bombardment power, which in principle will increase the current further. Because we are neglecting this effect, Equation 12 can be integrated resulting in Equation 15.

$$I(D_F) = \frac{I_0}{1 - K_r D_F} \tag{15}$$

Here K_r is a constant that includes the geometry, heater power, and field enhancement effects, thus defining the state of the gun (Equation 16), and I_0 is the output current of the cathode without any back-bombardment heating.

$$\begin{aligned} K_r =& 2A_c A_0 \alpha_c G_F \left(\alpha_c P_h + T_0 + \left(\frac{W - \Delta W}{2k_b}\right)\right) \\ &* \exp\left(\frac{-W + \Delta W}{k_b(\alpha_c P_h + T_0)}\right) \end{aligned} \tag{16}$$

Equation 15 shows that for a given gun state, the output current should always increase with RF duty factor. In a practical gun, the current would increase until the output was either space charge limited, the interlocks tripped, or the gun was damaged. For example, recent studies designed to examine the effect of back-bombardment on output current showed that for dispenser cathodes the surface coating can be degraded by the back-bombarding current causing reduced performance of the gun [4].

FIELD ENHANCEMENT EFFECTS ON BACK-BOMBARDMENT

Next we want to address the impact of field enhancement on back-bombardment and its subsequent effect on the output current. In Equation 1 gives a linear dependance of back-bombardment on electric field. This relationship is derived from the energy of the particles striking the cathode. However, due to field enhancement, there is also in increase in current with field which results in a nonlinear dependance of back-bombardment on peak field. Using Equation 5 we can calculate the output current as a function of the field on the cathode, Equation 17.

$$I = I_0 \exp\left[k\sqrt{E_0}\right] \tag{17}$$

In Equation 17, $k = \sqrt{(q^3/(4\pi\epsilon_0))}/(k_b T)$, where q is the fundamental unit of charge, $k_b$is the Boltzmann constant, $T$is the cathode temperature, ϵ_0 is the permittivity of free space, and I_0 is the output current in the absence of field enhancement. Substituting Equation 17 into Equation 1 gives a corrected back-bombardment power relationship that takes into account this enhancement.

$$P_{\text{ave}} = \frac{3I_0 E_0 c^2}{4v_{\text{eff}} f \alpha^2} T K D_F \exp\left[k\sqrt{E_0}\right] \tag{18}$$

When analyzing the relationship between output current and peak field in the presence of back-bombardment, the change in field enhancement must be taken into account. Then following a similar analysis as was done for RF duty factor, one could analyze the impact of output current on the peak field in the presence of back-bombardment heating.

CONCLUSIONS

We have shown that for thermionic cathode RF guns we can calculate the output current as a function of repetition rate in the presence of back-bombardment heating. Additionally we have discussed the effect of changes in the field enhancement on back-bombardment and how that might impact the output current. This paper lays the groundwork for additional theoretical analysis, simulation studies, and experiments to better understand and quantify how back-bombardment will impact the output current of the gun.

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REFERENCES

[1] J. P. Edelen *et al.*, *PRAB*, vol. 18, p. 043402, April 2015.

[2] J. P. Edelen *et al.*, “An Improved Analytic Model of Electron Back-Bombardment in Thermionic-Cathode RF Guns,“ in Proc. of IPAC 2015, Richmond, VA, USA, May 2015, paper TUPMA018, p. 1872.

[3] J. P. Edelen, “Theory and mitigation of electron back-bombardment in thermionic cathode radio frequency guns,“ Ph.D. Thesis, Colorado State University, 2015.

[4] J. P. Edelen, Y. Sun, J. R. Harris, J. W. Lewellen, “Observation of repetition-rate dependent emission from an un-gated thermionic cathode RF gun,“ presented at NAPAC 2016, Chicago, IL, USA, October 2016, paper WEPOB26, this conference.