

MODULATOR SIMULATIONS FOR COHERENT ELECTRON COOLING

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Abstract

Highly resolved numerical simulations of the modulator, the first section of the proposed coherent electron cooling (CEC) device, have been performed using the code SPACE. The beam parameters for simulations are relevant to the Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL). Numerical convergence has been studied using various numbers of macro-particles and mesh refinements of computational domain. A good agreement of theory and simulations has been obtained for the case of stationary and moving ions in uniform electron clouds with realistic distribution of thermal velocities. The main result of the paper is the prediction of modulation processes for ions with reference and off-reference coordinates in realistic Gaussian electron bunches with quadrupole field.

INTRODUCTION

Coherent electron cooling (CEC) [1,2] is a novel technique for rapidly cooling high-energy, high-intensity hadron beams. CEC consists of three sections: a modulator, where the ion imprints a density wake on the electron distribution, an amplifier, where the density wake is amplified, and a kicker, where the amplified wake interacts with the ion, resulting in dynamical friction for the ion. Figure 1 illustrates a general schematic of CEC.

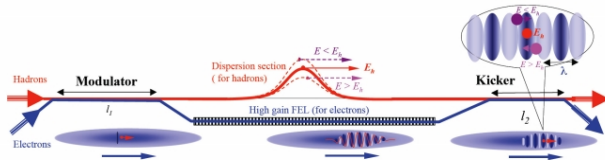


Figure 1: Schematic of coherent electron cooling concept.

Table 1: Parameters of Electron and Ion Beams

	Electron	Ion, Au ⁺⁷⁹
Beam energy	$\gamma=42.9$	$\gamma=42.9$
Peak current	100 A	
Bunch intensity	10 nC	1e+9
Bunch length	10 ps (full)	2 ns (r.m.s)
R.M.S. emittance	5π mm mrad	2π mm mrad
R.M.S. energy spread	1e-3	
Beta function	4 m	

Beam parameters for numerical simulations are relevant to Relativistic Heavy Ion Collider (RHIC) at the

Brookhaven National Laboratory (BNL), which are listed in Table 1.

METHODS AND TOOLS

SPACE, a parallel, relativistic, 3D electromagnetic Particle-in-Cell (PIC) code [3] has been used for the study of plasma dynamics in a dense gas filled RF cavities [4], and the study of mitigation effect by beam induced plasma [5]. SPACE contains modules for solving equations in electrostatic approximation. The module was used in present work as the modulation problem is electrostatic in co-moving frame. This module employs two different approaches. One is traditional PIC method for Poisson-Vlasov equation, which is effective for uniform beam and periodic boundary condition. The other is adaptive Particle-in-Cloud (AP-Cloud) method [6] that replaces traditional PIC mesh with an adaptively chosen set of computational particles, which is beneficial for non-uniform beams, geometrically irregular computational domains and mixed type boundary conditions.

Mechanism for modulation process is Coulomb interactions between ions and surrounding electrons. For relativistic beam energy, the relative density modulation of electrons due to their interaction with ions is orders of magnitudes smaller than unity. Therefore it is viable to treat each ion individually and use superposition principle to obtain net responses of electrons to all ions in the beam. Analytical solution of electrons' response to a moving ion exists for electrons with uniform spatial distribution [7]. For spatially non-uniform electrons, numerical approaches are employed either by solving the Vlasov equation [8], or by direct macro-particle simulation [9]. One of the difficulties in a macro-particle simulation is the fact that the modulation signal is too weak compared to the shot noise resulting from the discreteness of macro-particles. To extract modulation signal from shot noise, we perform two simulations with identical initial electron distribution. One simulation operates only with electron beam while the other simulation contains the electron beam and an ion. With the assumption that the Coulomb force from an ion only slightly changes the trajectories of the electrons over modulator, the influences of the ion can be obtained by taking difference in the final electron distributions of the two simulations. Similar approach has been successfully applied to simulate the FEL amplification process in the presence of shot noise [10].

NUMERICAL CONVERGENCE

Various numbers of macro-particles and refinements of computational domain are used to study the numerical convergence. Typical results are shown in Fig. 2.

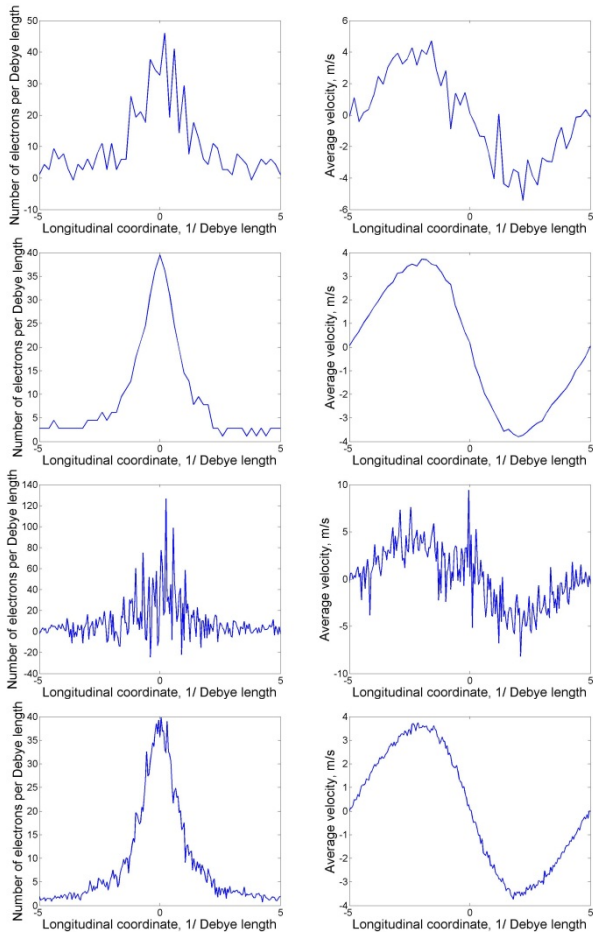


Figure 2: Convergence study of longitudinal density (left column) and velocity (right column) modulation with various mesh refinements (5 grids per Debye Length in first and second row, 20 grids per Debye Length in third and fourth row) and numbers of particles (3e+5 macro-particles in first and third row, 3e+7 macro-particles in second and fourth row).

Using coarse mesh and small number of macro-particles (first row) gives density and velocity modulation close to correct value. Increasing number of macro-particles and using the same mesh (second row) improves the smoothness of curves. Using fine mesh and small number of macro-particles (third row) induces largest noise in Fig. 2. Finally, refinement of 20 grids per Debye Length and 3e+7 macro-particles (fourth row) improves the resolution of gradients and reduces the overall simulation error, but induces some noise. Due to parallel scalability of our code, we used such a resolution and resolve real number of electrons in all simulations presented in this work. An additional smoothing technique is applied to reduce the noise in the plots.

VERIFICATION

In this section, we compare theory and simulation for the case of a stationary ion and a moving in uniform electron clouds. Kappa-2 probability density function is used to model thermal velocity distribution. The 3D form of kappa-2 probability density function is given in [7],

4: Hadron Accelerators

A11 - Beam Cooling

and the 1D and 2D forms are given in [11]. Theoretical values for density and velocity modulation, respectively, are given in [11] and [12]. While the theory assumes uniform distribution in infinite space for electrons, we used periodic boundary conditions in longitudinal and transverse and uniform electron beam filling the entire domain, to make fair comparison with theory. Comparison in Fig. 3 shows a good agreement of theory and simulations using a stationary ion and a moving ion.

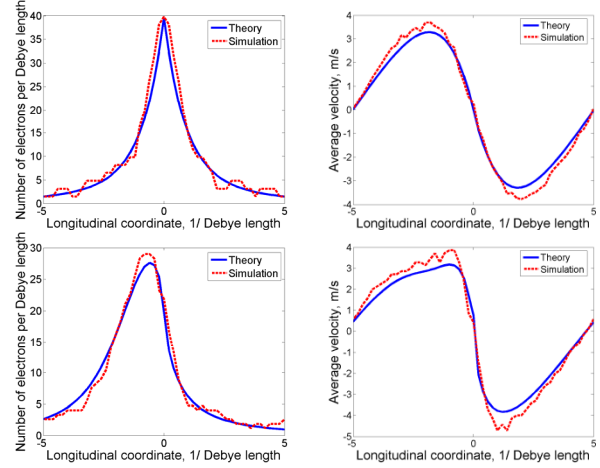


Figure 3: Comparison of theory and numerical simulations of density (left) and velocity (right) modulation by stationary (top) and moving (bottom) ion with respect to uniform electron cloud.

PREDICTION

In this section, we present simulation results of the modulation processes for an ion in Gaussian electron beams with a quadrupoles focusing field. We have used the quadrupoles focusing field with hard edge, given in Eq. (1). Quadrupoles focusing field with fringe has also been used, and our results show that effect of fringe field on modulation processes is negligibly small.

$$\begin{cases} B_y = K \cdot x \\ B_x = K \cdot y \end{cases} \quad (1)$$

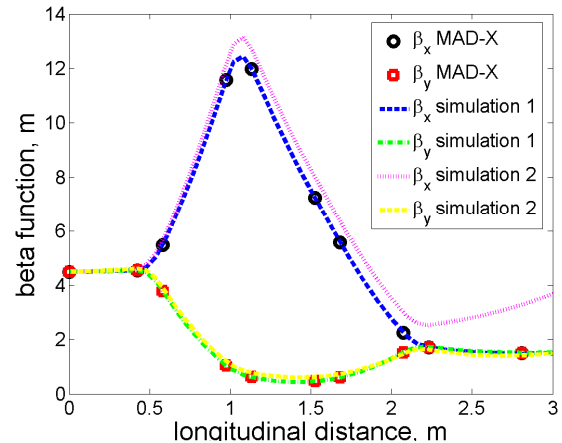


Figure 4: Comparison of MAD-X and SPACE of transverse β function changes in quadrupoles field.

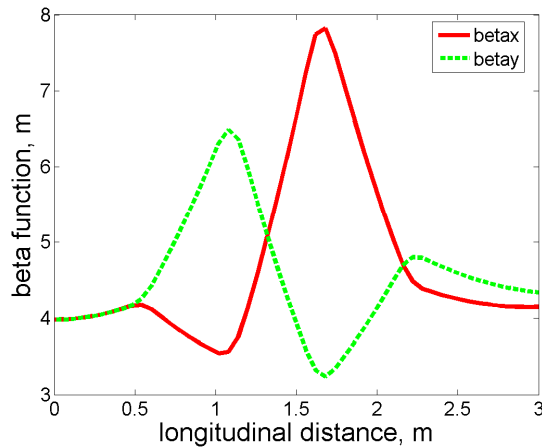


Figure 5: Transverse β function changes due to quadrupoles field used in modulator simulation.

Figure 4 shows the transverse β function changes of the electron beam due to quadrupoles magnetic field. We used results from code MAD-X as benchmark, without considering space charge effect. Simulation 1 used the electron beam given in Table 1 with reduced charge of electrons to make fair comparison with MAD-X, and agreed with MAD-X. Simulations 2 used regular charge of electrons, and differed from MAD-X, as space charge effect was included.

The quadrupoles magnetic field used in Fig. 4 is artificially set to make the transverse β_x and β_y match at the end of quadrupoles, for the purpose of comparison. Quadrupoles magnetic field for modulator simulation uses more realistic setting, and is relevant to RHIC at BNL. Figure 5 shows the transverse β function changes in such quadrupoles magnetic field.

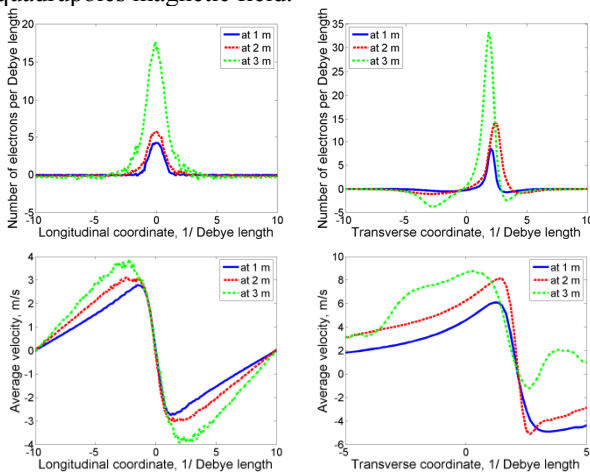


Figure 6: Density (top) and velocity (bottom) modulation in longitudinal (left) and transverse (right) by ion one σ off the center of the Gaussian electron beam in quadrupoles field.

Modulation results for ion in the center of the Gaussian electron beam are similar with the case using uniform electron beam. We studied modulation process with ion located at various off-center positions. Figure 6 gives modulation results for ion one σ off the center of the

Gaussian electron beam, where σ is RMS value for the Gaussian distribution of electron beam. Figure 7 is the 2D plot of density modulation for ion one σ off the center of the Gaussian electron beam in quadrupoles magnetic field.

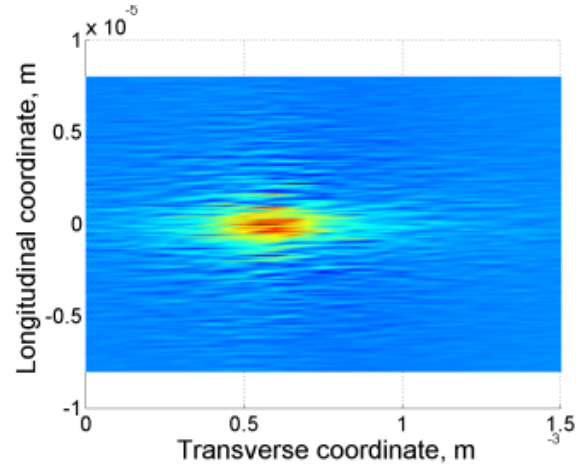


Figure 7: 2D density modulation plot by ion one σ off the center of the Gaussian electron beam in quadrupoles field.

CONCLUSION

Numerical convergence study shows that our code is self-consistent and helps us optimize settings for numerical simulations. Comparisons with theory in modulation effect and quadrupoles magnetic field effect show that our simulation results are reliable. Our prediction for more realistic case, in which ion co-propagate with Gaussian electron beam in quadrupoles magnetic field, is a help for the design of CEC in RHIC at BNL.

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