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OPTIMIZATION OF COMPTON SOURCE PERFORMANCE THROUGH ELECTRON BEAM SHAPING*

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Abstract

We investigate a novel scheme for significantly increasing the brightness of x-ray light sources based on inverse Compton scattering (ICS) - scattering laser pulses off relativistic electron beams. The brightness of these sources is limited by the electron beam quality since electrons traveling at different angles, and/or having different energies, produce photons with different energies. Therefore, the spectral brightness of the source is defined by the 6d electron phase space shape and size, as well as laser beam parameters. The peak brightness of the ICS source can be maximized then if the electron phase space is transformed in a way so that all electrons scatter off the x-ray photons of same frequency in the same direction. We describe the x-ray photon beam quality through the Wigner function (6d photon phase space distribution) and derive it for the ICS source when the electron and laser rms matrices are arbitrary. We find the optimal uncorrelated electron beam phase space distribution resulting in the highest brightness of the ICS source for the simple on axis case as an example.

INTRODUCTION

Currently there is a strong national need for high quality light sources at hard X-rays [1]. The quality of a light source is characterized by its brightness, or photon density in phase space. Larger brightness corresponds to radiation with a higher degree of coherency and thus permits higher resolution in imaging experiments. Relativistic electron beams are routinely used to generate radiation above optical frequencies. Over the past 50 years light sources based on synchrotron radiation have improved their average brightness by over 15 orders of magnitude [2, 3]. The 3rd and 4th generation light sources produce radiation via magnetic devices known as undulators – arrays of alternating permanent magnets which wiggle the electron trajectory. The wavelength of radiation generated by relativistic beams in these devices is on the order of $\lambda_{x-ray} \sim \lambda_u/(2\gamma^2)$. The undulator wavelength λ_u is limited to $\sim 1\ cm$ for practical devices. As a result, generating X-rays with wavelengths < 1 nm requires multi-GeV electron beams. Since the accelerator size and cost scales with energy, the facilities are correspondingly large and expensive.

A large-amplitude electromagnetic wave can also serve to undulate the trajectory of an electron beam. The electron trajectory in the wave field deviates from the straight line motion in a manner similar to that of an undulator, resulting in radiation. Alternatively, this process can be viewed as inverse Compton scattering (ICS) in which photons increase their energy after scattering off the relativistic electrons [4,5]. Regardless of viewpoint, the end result is the potential to generate hard X-rays using optical wavelength light and 0.1 GeV-range electron beams. X-ray ICS sources have been proposed and demonstrated over the past decade [6–8]. It produces photons with energies of tens of keV. Estimates for the peak brightness of these sources are close to each other and are on the order of $10^{20} - 10^{22}$ ph/mm^2/mrad^2/s/0.1% BW, more than 10 orders of magnitude below the estimate for hard x-ray free electron lasers (FELs). As a result, ICS sources based on current technology cannot compete with FELs [9]. Significant increases in brightness are needed to make ICS sources more attractive.

The spectrum of a single-particle ICS radiation strongly depends on its energy γ and angle at which it travels in respect to the axis $x' = dx/dz$ (plots (a) and (b) in Fig. 1). As a result, angular divergence and energy spread of the electron beam increase the bandwidth $\Delta w/w$ of the backscattered radiation as $(\gamma\Delta x')^2$ and $2\Delta\gamma/\gamma$, respectively [10, 11]. For example, both effects result in the same 1% ICS bandwidth for the following beam parameters: 50 MeV beam having 250 keV energy spread, 10 µm normalized emittance, and 100 µm rms spot size. However, the beam phase space may be conditioned by redistributing electrons in the phase space so that high energy electrons travel at large angles and low energy electrons at small angles. In this case all the electrons emit photons of the same energy in some direction (plot (c) in Fig. 1) and ICS peak brightness may reach the limit of a single-electron radiation. Collimation of emitted radiation at this angle would result in a significant increase in ICS source quality compared to the normal, unconditioned case.

In this manuscript we demonstrate the brightness of an ICS source as a function of the 6-dimensional electron distribution in the phase space. In addition, we show how it behaves for the simple uncorrelated electron beam with an observer located on z-axis.

6D WIGNER FUNCTION

First, we introduce the 6D Wigner function as an autocorrelation function of the radiated electric field to characterize the distribution of emitted photons in the phase space, based on the Quantum Mechanical Wigner function

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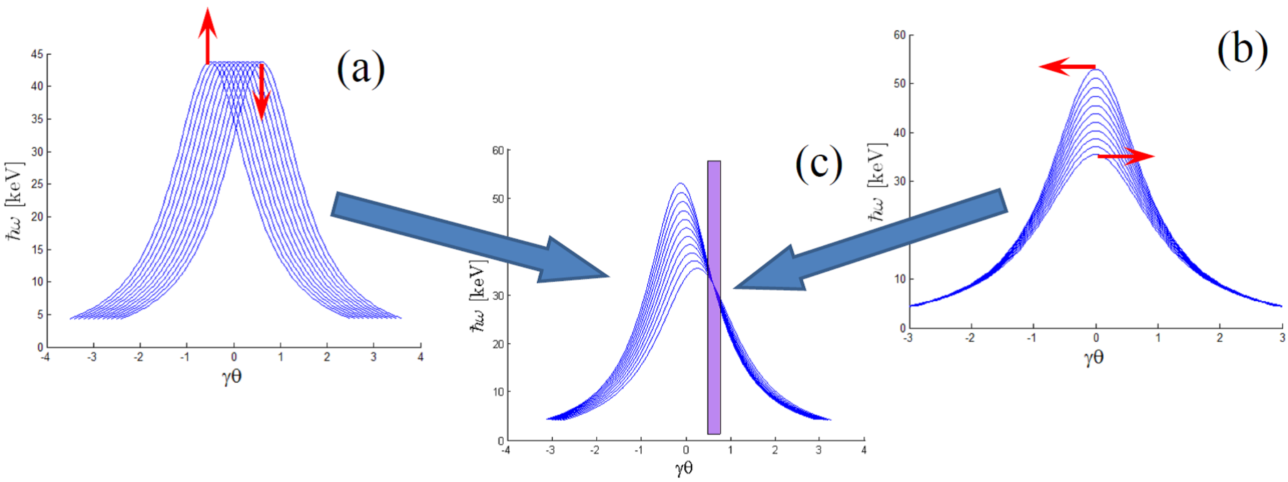

Figure 1: ICS spectra emitted by different electrons due to (a) finite angular divergence, (b) finite energy spread, and (c) imposed $x' - \gamma$ correlation in the beam.

introduced by Eugene Wigner in 1932 [12]:

$$W_{6d}(\vec{r}_{3d}, \vec{k}_{w3d}; t) \sim \int_{-\infty}^{\infty} E_{rad}(\vec{r}_{3d} + \frac{\vec{\xi}_{3d}}{2}; t) E^{*}_{rad}(\vec{r}_{3d} - \frac{\vec{\xi}_{3d}}{2}; t) e^{i\vec{k}_w \vec{\xi}_{3d}} d\vec{\xi}_{3d} \quad (1)$$

This Wigner function is an extension of description introduced by K.J.Kim for describing transverse Brightness of light sources [13]. The description of radiation with full 6D Wigner function is necessary since we are interested in increasing the brightness of a source by narrowing its bandwidth

CONVOLUTION THEOREM

In the general case, one would have to find total radiated electric field by different sources, then derive Wigner function using Eq. (1) . However, under the approximation that all electrons radiate incoherently the total Wigner function of scattered radiation can be found as the convolution of the single-electron Wigner function derived before with the electron distribution function in 6D phase space:

$$W_{beam} \sim \int_{-\infty}^{+\infty} W_{1e}(\vec{\zeta}_e, \vec{\zeta}_{ph}) f(\vec{\zeta}_e) d^6 \vec{\zeta}_e \quad (2)$$

SINGLE ELECTRON WIGNER FUNCTION

Relativistic electron, traveling with the velocity $\vec{\beta}_0 c$, scatters of the small amplitude ($a_0 << 1$) incoming wave with the wave-vector ($\vec{k}$) and in the Far Field approximation emits radiation:

$$E_{rad} = \alpha_f E_0 e^{-i\vec{q}(\vec{k})\vec{r} + iq(\vec{k})t} \quad (3)$$

where dimensionless coefficient is defined as $\alpha_f = \frac{1}{2\gamma_0^5}\frac{e^2}{Rmc^2}$. The wave-vector of the emitted radiation is: $\vec{q}(\vec{k}) = q(\vec{k})\vec{n} = \frac{1-\vec{\beta}_0\vec{k}}{1-\vec{\beta}_0\vec{n}} k\ \vec{n}$, and R is the distance from the electron to the observer.

In the case of the homogeneous time-limited (Gaussian profile) plane wave approximation for the incoming field ($\sigma_x >> \sigma_z$) we find the single electron Wigner function:

$$W_{1e} \sim \delta\left(\frac{n_x}{n_z}k_{w_z} - k_{w_x}\right)\delta\left(\frac{n_y}{n_z}k_{w_z} - k_{w_y}\right) e^{\frac{-(r-ct)^2}{\sigma_{rad}}} e^{\frac{-(k_{w_z}/n_z - k_{rad})^2}{\sigma_{k_{rad}}}} \quad (4)$$

where $k_{rad} = \frac{k_{z_{las}}(1+\beta_z)}{1-\vec{\beta}\vec{n}}$, $\sigma_{rad} = \frac{\sigma_z(1-\vec{\beta}\vec{n})}{(1+\beta_z)}$, $\sigma_{k_{rad}} = \frac{1}{\sigma_{rad}}$. These coefficients have to be corrected by multiplication to the numerically evaluated coefficients in the case of a strong focusing.

WIGNER FUNCTION OF RADIATION EMITTED BY ELECTRON BEAM

We can rewrite the expression (2), in case of a Gaussian distribution of the electron beam described its 6D Σ-matrix as:

$$W_{beam} = \sqrt{\pi}^{-6}\sqrt{Det[\Sigma^{-1}]}\int_{-\infty}^{+\infty} W_{1e}(\vec{\zeta}_e, \vec{\zeta}_{ph}) e^{-\vec{\zeta}_e^{\,T}\Sigma^{-1}\vec{\zeta}_e} d^6\vec{\zeta}_e \quad (5)$$

Moreover, the expression in (4) can be presented in the similar to the electron distribution form by representing each delta-function as a limit of Gaussian function with its standard deviation (δ_i) approaches to "*zero*":

$$W_{beam} = W_0 \frac{1}{\sqrt{\pi}\delta_x}\frac{1}{\sqrt{\pi}\delta_y} e^{-(\vec{\zeta}_{ph}^{(i)} - \vec{\zeta}_e)^T M_0 (\vec{\zeta}_{ph}^{(i)} - \vec{\zeta}_e)} \quad (6)$$

where $\vec{\zeta}_{ph}^{(i)} = Drift^{-1}\vec{\zeta}_{ph}^{(f)}$ is the 6D vector of a photon in the phase space at the moment it was emitted, M_0 - is the characteristic matrix for the single electron radiation Wigner function, describing emitted photons distribution similar to Σ^{-1}-matrix describing the electron distribution in the phase space. This expression looks similar to the 4D case studied by K. J. Kim [13], but here $\vec{\zeta}_{ph} - \vec{\zeta}_e$ cannot be expressed in symplectic coordinates due to the specific feature of the ICS process: $w_{rad} \sim \gamma^2$. If photon ($\vec{\zeta}^T_{ph} = \{\Delta x, \frac{\Delta k_{w_x}}{k_{w_{z0}}}, \Delta y, \frac{\Delta k_{w_y}}{k_{w_{z0}}}, \Delta z, \frac{\Delta k_{w_z}}{k_{w_{z0}}}\}$) and electron ($\vec{\zeta}_e^{\,T(s)} = \{x_e, \Delta\beta_x, y_e, \Delta\beta_y, z_e, \Delta\beta_z\}$) coordinates expressed in the position-momentum symplectic representation, then the transformation from the symplectic coordinates for an electron to ones which were used in (8): $\vec{\zeta}_e = \Omega\vec{\zeta}_e^{\,(s)}$ is described by the matrix Ω, where all elements are similar to identity matrix but Ω_{66} is substituted with $2\gamma_0^2$.

We find the Wigner function of radiation scattered by the electron beam as a 6-dimensional form:

$$W_{beam} = W_0 \cdot lim_{\delta_x,\delta_y \longrightarrow 0}\left\{\frac{\sqrt{Det[\Sigma^{-1}]}}{\delta_x\delta_y\sqrt{Det[M]}} e^{-(\vec{\zeta}^{(i)}_{ph})^T(M_0 - M_0 M^{-1} M_0)\vec{\zeta}^{(i)}_{ph}}\right\} \quad (7)$$

where $M = M_0 + \Sigma^{-1}$. The limit should exist for any $\vec{\zeta}_{ph}$, which means the expression before the exponent should converge itself, and defines the peak brightness of the source:

$$B_{peak} \sim \frac{1}{\sqrt{Det[Q_0 M_0]\Sigma + Q_0}}, \quad (8)$$

where $Q_0 = lim_{\delta_x,\delta_y \longrightarrow 0}\{Q\}$ and $Q_0 M_0 = lim_{\delta_x,\delta_y \longrightarrow 0}\{Q \cdot M_0\}$, have no singular elements, and $Det[Q] = (\delta_x\delta_y)^2$.

PEAK BRIGHTNESS FOR THE UNDULATOR CASE

For the simple "undulator" case, when the electron beam is traveling along the z-axis in the moment of interaction with the incoming field and the observer is located on-axis we obtain:

$$B_{peak} \sim \left[x^2_{rms} y^2_{rms}\left(\frac{z^2_{rms}}{\sigma^2_{rad}} + k^2_{rad}\epsilon^2_z\right)\right]^{-1} \quad (9)$$

If the emittances ϵ_x, ϵ_y and ϵ_z are fixed to maximize the peak brightness one would have to squeeze the beam in transverse directions by increasing the divergences, which can be realized via set of quadrupoles and drifts to compress the beam simultaneously in both x- and y- directions. Taking into account $k_{rad} \simeq 4\gamma_0^2 k_{z_{las}}$, $\sigma_{rad} \simeq \frac{\sigma_z}{4\gamma_0^2}$ and quasi-monochromatic approximation for the incoming radiation $k_{rad}\sigma_z >> 1$, we can have longitudinal emittance dominated regime for the second term, if $k_{rad}\sigma_z\frac{\Delta\gamma}{\gamma} >> 1$ (no chirp approximation). Electron beam duration dominated regime would be observed under the opposite condition. In this case, increasing the peak beak brightness can be realized via using a bunch compressor.

CONCLUSION

We have demonstrated that the Brightness of an ICS source can be represented as a Wigner function in the 6D photon phase space and found it as a 6D Gaussian form of an arbitrary electron phase space distribution described by 6D $\Sigma - matrix$. We found that the convolution theorem proved in 4D for the on-axis observer fails in 6D for an arbitrary direction to the observer in symplectic coordinates for a photon and an electron simultaneously, but can be expressed in non-symplectic coordinates for, at least, one of them. We have derived how an electron beam should be shaped in the simple case of on-axis observer and no correlations allowed between transverse and longitudinal electron phase space to reach the maximum peak brightness. We will present the optimal electron phase space distribution for the general off-axis case in the peer reviewed paper.

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