GASEOUS H_2-FILLED HELICAL FOFO SNAKE FOR INITIAL 6D IONIZATION COOLING OF MUONS*

Y. Alexahin†, FNAL, Batavia, IL 60510, USA

Abstract

H2 gas-filled channel for 6D ionization cooling of muons is described which consists of periodically inclined solenoids of alternating polarity with 325MHz RF cavities inside them. To provide sufficient longitudinal cooling LiH wedge absorbers are placed at the minima of transverse beta-function between the solenoids. An important feature of such channel (called Helical FOFO snake) is that it can cool simultaneously muons of both signs. Theoretical considerations as well as results of simulations with G4beamline are presented.

INTRODUCTION

Ionization cooling is the only method fast enough for cooling of muons which is essential for realization of muon collider, muon beam-based neutrino factories and other experiments involving muons (see e.g. Ref. [1]). Unfortunately the longitudinal motion is naturally anti-damped in the most suitable momentum range (2-300 MeV/c) since the ionization losses decrease with momentum.

Two basic schemes were proposed in order to achieve longitudinal cooling by forcing muons with higher momentum to take a longer path in the absorber so that they lose more energy. This can be realized by creating dispersion in particle positions (without significant path lengthening) and using wedge absorbers (so called RFOFO channel [2]) or by creating sufficiently large path lengthening with momentum and using a homogeneous absorber (Helical Cooling Channel [3]).

The early versions of the so-called Helical FOFO (HFOFO) snake [4] used a third possibility: locally large path lengthening in slab absorbers due to a large slope of the dispersion function there. This allowed the FOFO snake to cool muons of both signs simultaneously.

Here we present a later version of the HFOFO snake which incorporates wedge absorbers in such a way that simultaneous cooling of μ^- and μ^+ is still possible. This allowed for smaller "snake" amplitude and improved transmission.

BASIC PRINCIPLES

The helical FOFO snake is based on the following principles: alternating solenoid focusing, periodic rotating dipole field and resonant dispersion generation [4].

The focusing magnetic field is created by a sequence of solenoids with alternating polarity and gaps between them (the name FOFO reflects the fact that solenoid focusing does not depend on polarity since it is quadratic in magnetic field). Emittances of the two transverse normal modes** are swapped with each change of polarity so that both modes are cooled.

The transverse magnetic field component necessary for dispersion generation can be created by periodical inclination of solenoids. The idea of the HFOFO snake is to make a rotating dipole field by inclining solenoids in rotating planes $x\cdot\cos(\phi_k) + y\cdot\sin(\phi_k) = 0$, $\phi_k = \pi(1\text{-}2/N_s)(k+1)$, $k = 1, 2,\ldots, N_s$, N_s being the number of solenoids/period.

If N_s=2(2j+1) then μ^- in solenoid $k = k_1$ see exactly the same forces as μ^+ in solenoid $k = k_1 + N_s/2$ since these solenoids have the same inclination but opposite polarity. In the result μ^- and μ^+ orbits have exactly the same form with longitudinal shift by half period ($N_s/2$ solenoids) but are not mirror-symmetric as one might expect. This allows us to find such orientation of wedge absorbers (with periodicity = 2) that they provide longitudinal cooling for both μ– and μ+ at the same time

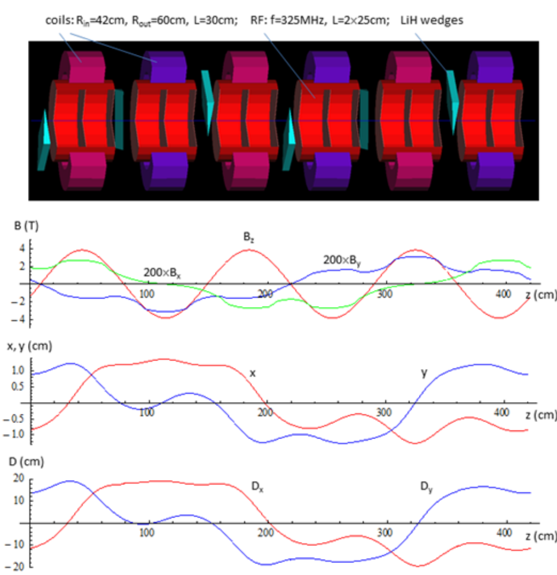

Figure 1: Top to bottom: layout of one HFOFO period, magnetic field for p_0 = 230 MeV/c, μ^+ equilibrium orbit and dispersion.

Large dispersion can be generated if the transverse tune Q⊥ is close to a resonant value. To obtain a positive momentum compaction favorable for longitudinal cooling it must be above the resonant value Q⊥> n + Qs, Qs being the longitudinal mode tune. Despite closeness to a resonance the momentum acceptance of such channel can be sufficiently large owing to higher order chromatic effects.

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† alexahin@fnal.gov

** In an axisymmetric field they are cyclotron (Larmor) and drift modes (see e.g. Ref [5])

LATTICE & OPTICS

Here we present a version of the HFOFO snake filled with high-pressure gaseous hydrogen (GH_2) which allows for higher RF gradient and also works as an absorber. Following the latest HCC design [6] we assume its density to be equal to 20% of liquid hydrogen and take for peak RF gradient E_{max} = 25 MV/m at f_{RF} = 325 MHz. It is assumed that GH_2 will provide sufficient cooling of the cavities windows so that they can be quite thin (0.12 mm of Be in the beginning of the channel).

One period of the channel is schematically shown in Fig. 1 (top). Its length is L_{period}=4.2m. There are N_s = 6 solenoids per period inclined by angle 2.5 mrad in planes rotated by angles $\phi_k = 4\pi/3, 0, 2\pi/3, 4\pi/3, 0, 2\pi/3$; $\phi = 0$ corresponds to pitching in the vertical plane.

The lower two plots in Fig.1 show μ^+ equilibrium orbit and dispersion found for momentum 230 MeV/c. The normal mode tunes and normalized equilibrium emittances are given in Table 1. The tunes were computed from eigenvalues of a one-period transfer matrix; their imaginary part describes oscillation damping due to the regular part of ionization losses.

Table 1: The Normal Mode Tunes and Normalized Equilibrium Emittances in the Beginning of the Channel

Parameter	Mode I	Mode II	Mode III
Tune	1.227+0.010i	1.238+0.004i	0.189+0.005i
Emittance, mm	2.28	6.13	1.93

There is a large difference between the cooling rates and equilibrium emittances of the transverse normal modes (I and II) due to the axial symmetry breaking by the dipole field component. This difference can be reduced with the help of quadrupolar field of constant polarity (but not necessarily of constant gradient) [4]. Such field works for both μ^- and μ^+ despite breaking the translational symmetry between the two beams. However, it excites strong β-beat and is not used here.

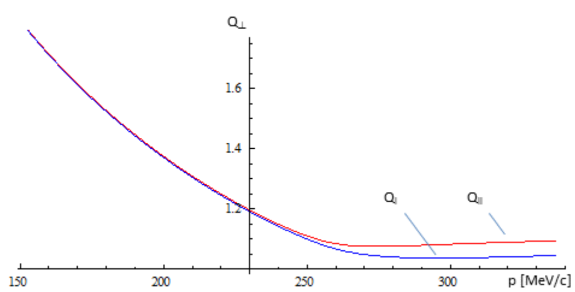

Figure 2: Transverse mode tunes vs. muon momentum.

The momentum acceptance was determined by calculating the tunes for a constant momentum p (Fig. 2). Surprisingly, the transverse tunes are “repelled” from the integer resonance making what we may call static acceptance very large (it is actually larger than shown). However, there is a limitation due to change in the sign of the slippage factor [4] which can be called the dynamic acceptance since it is important only in the presence of RF field. For the considered parameters it limits the available momentum range from above by ≈ 333MeV/c.

Cooling Channel

The considered here HFOFO cooling channel consists of matching-in section, 30 HFOFO periods and matching-out section. The average momentum of the beam core is ~250MeV/c, to pull it farther away from the upper limit and reduce losses the design momentum is lowered along the channel to ~200MeV/c by lowering current in solenoids and adjusting RF phase and LiH absorber wedge angle while keeping the solenoid geometry and RF gradient constant. The total length of the channel is 131m.

Matching to the Front End

The Front End serves for muon production, bunching and phase-energy rotation in multi-frequency RF field [8]. Transversely the muons are focused by solenoidal magnetic field which is constant in the Buncher and Rotator. Matching to the HFOFO oscillating magnetic field is not easy due to a large momentum spread σp/p ≈ 12%. Figure 3 shows the results for fixed momentum. Matching out of the channel is done in a similar way.

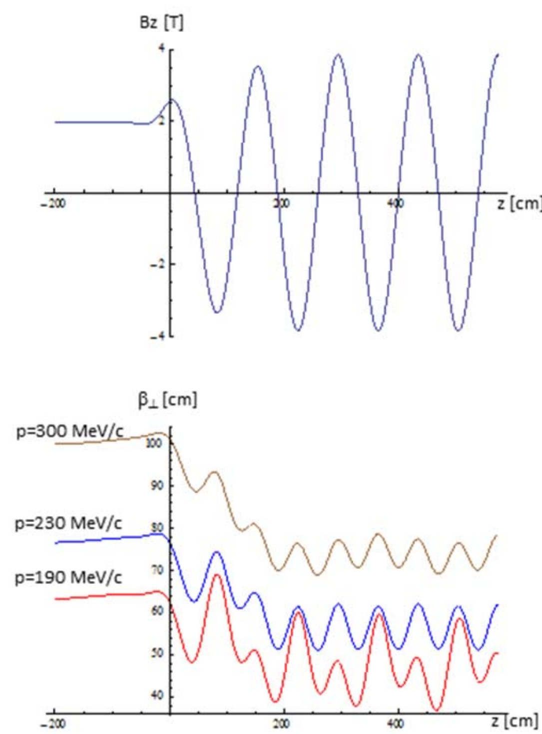

Figure 3. Magnetic field in the transition area (top) and β-function for a few values of momentum (bottom).

G4BL SIMULATIONS

The cooling was simulated using G4beamline code [8] based on Geant4. Figure 4 (top) presents the initial and final μ+ distribution in three phase space planes. All bunches were projected onto the same RF bucket in the right plot. No cuts were applied. Distribution in μ– beam looks similar. Distribution in the total momentum (bottom plot) is a testimony of longitudinal cooling in the channel.

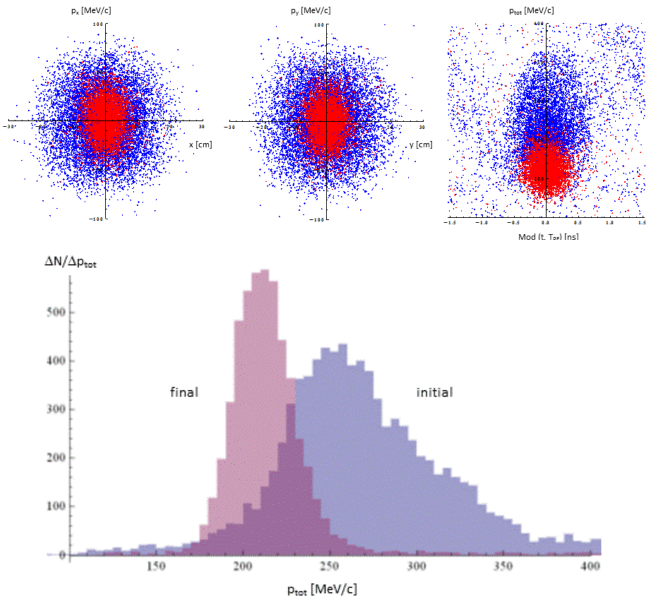

Figure 4: Initial (blue) and final (red) μ^+ distributions.

Computation of beam emittance presents a challenging problem due to long non-Gaussian tails. A multi-dimensional Gaussian fit [9] was used which automatically suppresses halo contribution.

The fit also involves average particle coordinates which we will call the central values to distinguish them from the standard r.m.s. values. Obtained with its help evolution of the central momentum is shown in Fig. 5, while the relative intensity and normalized emittances of the three normal modes is presented in Fig. 6.

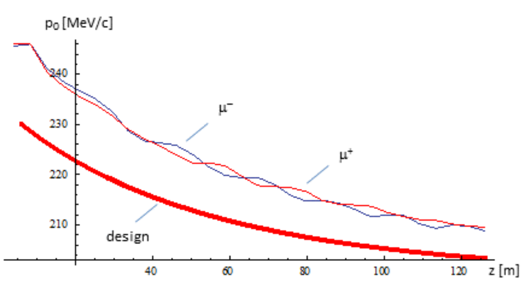

Figure 5: Central momentum of μ^- (blue) and μ^+ (red) beams obtained with Gaussian fit.

Large excess in the central momentum over the design value is explained by increase in the equilibrium momentum for muons with large transverse amplitudes: due to longer path these particles must move faster to be in synchronism with RF.

It can be seen in Fig. 6 that stopping cooling by 10-20 m earlier would actually slightly improve the channel performance due to lower losses.

SUMMARY & OUTLOOK

As it stands now the GH_2-filled HFOFO snake can cool simultaneously muons of both signs with initial normalized emittances for the Gaussian core $\varepsilon_N^{(\mathrm{ini})}$ = 12 mm, 22 mm, 23 mm down to $\varepsilon_N^{(\mathrm{fin})}$ = 1.9 mm, 3.6 mm, 7.6 mm for μ+ and $\varepsilon_N^{(\mathrm{fin})}$ = 1.6 mm, 4.6 mm, 7.2 mm for μ−. If all particles are counted the transmission is only 46% for μ+ and 48% for μ− (decays included), but for the Gaussian core it is 68%. The 6D normalized emittance is reduced from $6{\cdot}10^3$ mm^3 to 51 mm^3.

The 6D emittance can be further reduced by a factor of ~3÷5 by adding a section with scaled down length of all elements and increased solenoid current. A more significant reduction probably can be achieved with higher phase advance per cell providing smaller $\beta_\perp$ at the minima.

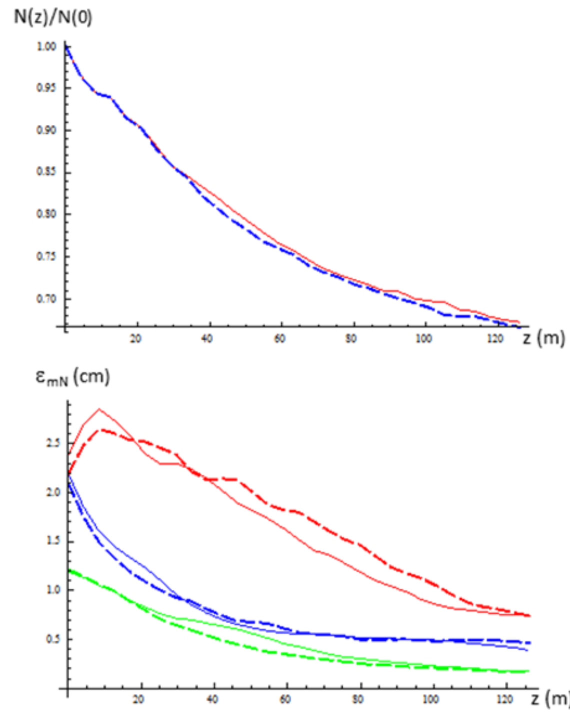

Figure 6: Evolution of the μ+ (solid lines) and μ− (dashed lines) beam intensities (top) and normalized emittances (bottom) obtained with Gaussian fit.

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