

# INTRABEAM SCATTERING IN HIGH BRIGHTNESS ELECTRON LINACS

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## Abstract

Intra-beam scattering (IBS) of a high brightness electron beam in a linac has been studied analytically, and the expectations found to be in reasonable agreement with particle tracking results from the Elegant code. It comes out that, under standard conditions for a linac driving a free electron laser, IBS plays no significant role in the development of microbunching instability. A partial damping of the instability is envisaged, however, when IBS is enhanced either with dedicated magnetic insertions, or in the presence of an electron beam charge density at least 4 times larger than that produced by present photo-injectors.

## INTRODUCTION

The question to which extent intra-beam scattering (IBS) affects the properties of high brightness electron bunches in linacs was posed in [1,2], with attention to the interplay of IBS and microbunching instability (MBI). Following our study in [3], here we aim to provide a quantitative answer and an outlook, by comparing the analysis and particle tracking runs of the ELEGANT code [4], whereas IBS was simulated following prescriptions given in [2,5].

In particular, we wonder whether IBS could play a role when the beam transverse dimension is squeezed with strong focusing (“low-beta”) FODO cells, so to increase the IBS longitudinal growth rate. At first glance, the idea of using IBS to increase the energy spread of an electron bunch traveling in a dedicated FODO channel seems to be attractive for the following reasons: i) IBS heats the beam by avoiding cost, complexities and maintenance of a laser heater (LH) system [6]; ii) the heating level is tuneable with the quadrupoles’ focusing strength; iii) it provides longitudinally uncorrelated energy spread, thus avoiding any side effect associated to the energy modulation induced in a LH at the infrared laser wavelength (*e.g.*, the so-called trickle heating) [7]. We will see however that, to be as effective as a LH, the enhancement of IBS requires a long and densely packed FODO channel. An alternative compact lattice in which the beam recirculates through low-beta FODO channels is investigated. This solution, however, turns out to be not practical because of the coherent synchrotron radiation (CSR) instability that develops through the arcs.

## THEORETICAL BACKGROUND

Ultra-relativistic electron bunches in modern accelerators generally have much smaller velocity spread in the longitudinal direction of motion than in the transverse planes owing to the relativistic contraction by the Lorentz factor  $\gamma$ :  $\sigma_\delta/\gamma \ll \sigma_{x'}, \sigma_{y'}$ , where  $\sigma_\delta$  is the beam rms fractional energy spread and  $\sigma_{x'}, \sigma_{y'}$  the rms angular divergence. If the bunch’s charge density is high

enough and the bunch travels a long path, multiple Coulomb scattering tends to redistribute the beam momenta from the transverse degree of freedom to the longitudinal one. This process is called IBS and its longitudinal growth rate may be comparable to the beam damping time in low emittance electron storage rings. The instantaneous growth rate of the energy spread of a bunched beam circulating in a ring was given in [8,9]. Since there are no synchrotron oscillations in a linac, the formula for a coasting beam should be used here (which results in a growth rate a factor 2 larger than that of a bunched beam) [8]:

$$\frac{1}{\sigma_\delta} \frac{d\sigma_\delta}{dt} \approx \frac{r_e^2 c N}{8\gamma^2 \varepsilon_{n,x} \sigma_x \sigma_z \sigma_\delta^2} \ln \left( \frac{\Delta\gamma_{\max}}{\Delta\gamma_{\min}} \right) \quad (1)$$

Here  $r_e$  is the electron classical radius,  $\beta c \approx c$  the electron velocity,  $N$  the number of electrons in the bunch,  $\varepsilon_{n,x} = \varepsilon_{n,y}$  the rms normalized transverse emittance of a round beam, and  $\sigma_z$  the rms bunch length. The argument of the Coulomb logarithm is the ratio of the maximum and the minimum energy exchange due to a single scattering event, and  $\Delta\gamma_{\max} \propto \gamma^2 \sigma_{x'}$ ,  $\Delta\gamma_{\min} \propto r_e / (\sigma_x \sigma_{x'}) \approx \gamma r_e / \varepsilon_{n,x}$  [1].

Following an argument made in [10], we consider that the IBS energy distribution has a nearly Gaussian core with a long tail. Since we are mostly interested in the energy spread of the Gaussian core, we set the maximum energy transfer to  $\Delta\gamma_{\max} = \gamma \cdot 10^{-5}$  as also done in [1], and find that the logarithm is of the order of 10 for a normalized emittance of  $\sim 1 \mu\text{m}$ . Then, Eq.1 can be integrated and it yields to the final fractional rms energy spread in the presence of IBS cumulated over the distance  $\Delta s$  [3]:

$$\sigma_\delta \approx \sqrt{\sigma_{\delta,0}^2 + \frac{2r_e^2 N}{\gamma^2 \varepsilon_{n,x} \sigma_x \sigma_z} \Delta s} \equiv \sqrt{\sigma_{\delta,0}^2 + \sigma_{\delta,IBS}^2} \quad (2)$$

with  $\sigma_{\delta,0}$  the initial rms fractional incoherent energy spread. Equation 2 is an approximate expression for smooth betatron oscillations, neither energy dispersion nor particle acceleration. If we apply Eq.2 to the low energy part of a linac, we find that an electron beam from a state-of-the-art photo-injector, *e.g.* with beam charge  $Q = 0.5 \text{ nC}$ ,  $\sigma_z = 750 \mu\text{m}$ ,  $\varepsilon_n = 0.6 \mu\text{m rad}$ ,  $\sigma_x = 150 \mu\text{m}$  and  $\gamma = 300$ , collects an absolute rms energy spread  $\sigma_{E,IBS} \approx 3 \text{ keV}$  over  $\Delta s \sim 30 \text{ m}$ . This is comparable to the typical value of  $\sigma_{E,0} \approx 2 \text{ keV}$  out of the photo-injector [11], and still far from the amount of heating required to suppress MBI in an FEL-driver [12,13]. Then, if we assume that the bunch length is magnetically compressed by a factor of, say,  $C \sim 30$ ,  $\sigma_{E,IBS}$  may grow up to  $\sim 100 \text{ keV}$  over hundreds of meters, but its contribution to Landau damping of MBI remains small for two reasons. First, in the linac region immediately following the compressor,  $\sigma_{E,IBS}$  is negligible compared to

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the incoherent energy spread of the compressed beam, which is increased to  $C\sigma_{E,0} \approx 60$  keV by virtue of the preservation of the longitudinal emittance. Second,  $\sigma_{E,IBS}$  grows with  $s$  at a lower rate than the relative energy modulation amplitude of MBI (respectively, square root vs. linear dependence) [5,14]. In conclusion, the impact of  $\sigma_{E,IBS}$  on the development of the MBI is expected to be small. In particular, it is negligible with respect to the effect of a LH *unless* important modifications to the magnetic lattice and/or to the beam parameters are introduced. We finally remark that, with the aforementioned beam parameters, the transverse emittances and the bunch length are substantially unchanged by IBS.

## FODO CHANNEL

Equation 2 says that, for injected beam parameters like those in Tab.1,  $\sigma_{E,IBS} \approx 6$  keV if the rms transverse beam size  $\sigma_{x,y}$  shrinks down to  $25 \mu\text{m}$  (average value) along a beam line 30 m long. With such a system designed for the *maximum* beam heating, *i.e.* minimum betatron function  $\beta$ , a reduction of the total  $\sigma_{E,IBS}$  can be obtained by rearranging the quadrupole strengths so to allow  $\beta$  to expand to higher values. On the opposite, the lower limit of  $\beta$  is set by the optical aberrations excited by strong focusing and by the technical design of the quadrupole magnets. In order to make our system more flexible, compact and easy to build, we set  $\beta = 0.3$  m. This solution ensures a standard technical design of the quadrupole magnets and negligible emittance growth due to optical aberrations.

Table 1: Electron Beam Parameters Out of a State-of-the-Art Photo-Injector and FODO Lattice Parameters

Charge	500	pC
Bunch duration, rms	2.5	ps
Norm. slice emittance, rms	0.6	$\mu\text{m}$
Incoherent energy spread, rms	2.0	keV
Mean energy	150	MeV
FODO length	30	m
Average betatron function in FODO	0.3	m
IBS-induced energy spread, rms (Eq.3)	6.0	keV

$\sigma_{E,IBS}$  cumulated in the FODO channel is evaluated with Eq.2 and shown in Fig.1, in the  $(\beta,Q)$  and the  $(\beta,L)$  space, with  $L$  the FODO channel total length. We assume that the three-dimensional charge density out of the photo-injector remains constant as the injected bunch charge is varied. In other words we assume the following scaling:  $\varepsilon_n [\mu\text{m}] \approx Q[nC]^{1/3}$  and  $\sigma_z [mm] \approx 1.2 \cdot Q[nC]^{1/3}$ , so that  $Q/(\sigma_z \varepsilon_n^2) = \text{const.}$ . In general,  $\sigma_{E,IBS}$  turns out to be quite insensitive to  $Q$  if compared to its dependence on  $s$ ,  $\gamma$  and  $\beta$ , because in our scaling the effect of a higher charge is compensated by a longer bunch duration and a larger transverse emittance.

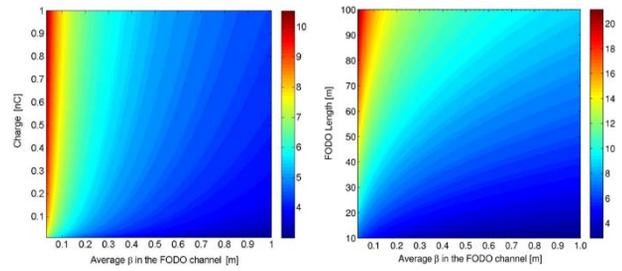


Figure 1: IBS-induced rms energy spread in keV, in the  $(\beta,Q)$  space for  $L = 30$  m (left), and in the  $(\beta,L)$  space for  $Q = 500$  pC. Both plots are for a beam energy of 150 MeV. The beam transverse emittances and the bunch duration are scaled with  $Q$  as explained in the text. Notice that the colour scale is different in the two plots. Copyright of American Physical Society [3].

We benchmarked the analytical estimation of  $\sigma_{\delta,IBS}$  for the beam parameters in Tab.1, with particle tracking runs of the ELEGANT code. ELEGANT implements Bjorken and Mtingwa's formulas [15] for calculating the emittance growth rate in all directions of motion. To take into account non-Gaussian distributed beams, ELEGANT allows beam slice analysis: within each slice, particles are assumed to be Gaussian-distributed in the transverse phase space and in energy, and uniformly distributed in  $z$ . The incoherent energy spread induced by IBS along the FODO channel is shown in Fig.2. Its final rms value, averaged over the bunch slices, is 4.5 keV for the sliced beam (not shown) and 6.0 keV for the unsliced one. Such a discrepancy is due to the non-uniform heating of the sliced beam because of the lower charge density at the bunch edges. The simulations confirm that the bunch length remains substantially unchanged in the presence of IBS (not shown).

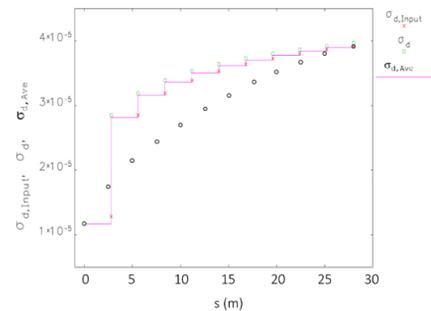


Figure 2: Electron beam slice rms fractional energy spread along the FODO channel in the presence of IBS, for the unsliced beam (see parameters in Tab.1). In the legend, “ $\sigma_{d,Input}$ ” is the energy spread at the entrance of each “IBS module” depicted in ELEGANT; “ $\sigma_d$ ” is the energy spread at the exit of each IBS module and “ $\sigma_{d,Ave}$ ” is the rms fractional energy spread, averaged over all bunch slices. The rms fractional energy spread estimated with Eq.2 is also shown (circles). Copyright of American Physical Society [3].

By scaling the simulation result with Eq.2, we estimate a FODO channel as long as  $\sim 100$  m to achieve  $\sigma_{E,IBS} \sim 10$  keV. At this point, the scheme would start having a large impact on the machine design and cost. Alternatively, while keeping the 30 m long FODO channel, a beam charge density  $\sim 4$  times higher than in Tab.1 should be provided, which seems to be out of the horizon of present facilities. We can therefore conclude that a relatively *compact single-pass* low-beta FODO channel could only about double the incoherent energy spread of typical high brightness electron beams produced by nowadays photo-injectors. This is not sufficient for best performance of x-ray FELs, although it might be suitable, *e.g.*, for longer wavelength FELs driven by shorter linacs, lower peak current and/or requiring weaker magnetic compression than in FERMI and LCLS, *i.e.*, having a lower MBI gain.

## RECIRCULATION

As an alternative to the single-pass FODO channel, we investigated a recirculating IBS beam line (RIBS) to cumulate a larger  $\sigma_{E,IBS}$  and to minimize the impact on the total linac length. The bunch is injected into, and extracted from, the RIBS by fast kicker magnets. After M-turns into the RIBS, the beam has passed through a low-beta FODO channel  $2M+1$  times. A sketch of the RIBS at 150 MeV with realistic sizes is shown in Fig.3. The two arcs are basically a copy of the design by Douglas *et al.* [16]. In our design, the arcs are achromatic and quasi-isochronous ( $R_{56} = 2 \times 10^{-4}$  m,  $T_{566} = 4 \times 10^{-3}$ ) and connected to the FODO channels by matching sections made of additional quadrupole magnets. An ultra-relativistic bunch takes approximately 360 ns to make one turn in the RIBS. Kickers with rise and fall time pulse duration of a few tens of nanosecond are therefore adequate for our purposes.

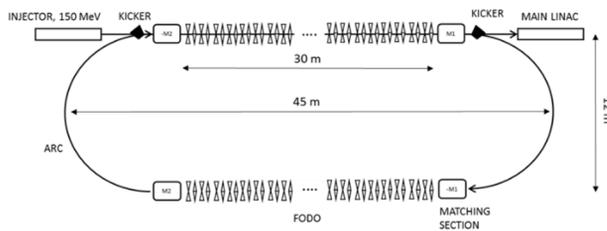


Figure 3: Sketch (not to scale) of the recirculating IBS beam line. Copyright of American Physical Society [3].

A 150 MeV, 250 pC beam at the entrance of RIBS was generated by including the relativistic velocity spread, the geometric longitudinal wakefield and the RF curvature in an upstream 12 m long S-band injector. Other beam parameters are:  $\varepsilon_n = 0.4 \mu\text{m rad}$ ,  $\sigma_z = 375 \mu\text{m}$  and  $\sigma_{E,0} = 2$  keV. The total rms energy spread is 0.1%. This beam is expected to generate  $\sigma_{E,IBS} \approx 10$  keV in half a turn (see Eq.2). In principle, the number of turns in RIBS should be a compromise between the amount of desired  $\sigma_{\delta,IBS}$ , which is proportional to the square root of the length of the traversed FODO channel, and the tolerable degradation of the beam six-dimensional emittance due to chromatic aberrations and CSR instability. After one turn, the

incoherent energy spread has grown to 10 keV rms, but largely at the expense of the deeply modulated longitudinal phase space, as shown in Fig.4. We conclude that the longitudinal CSR instability prevents beam recirculation. In addition, the CSR-induced energy loss modulates the beam correlated energy spread through the arc. This amplifies the variation of the bunch length at the dipole magnets (since  $R_{56}$  oscillates in the range  $\pm 30$  mm, see [10]) and partially invalidates the optics scheme for emittance preservation in the presence of CSR, which requires the same bunch length at the dipoles [17,18].

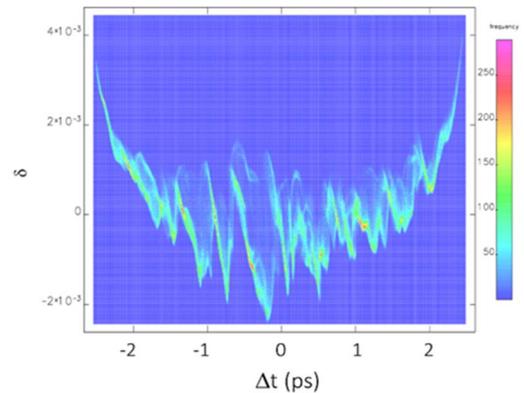


Figure 4: Electron beam longitudinal phase space after one turn in RIBS. Copyright of American Physical Society [3].

## CONCLUSION

The impact of IBS on the six-dimensional emittance of high brightness electron beams like those driving x-ray FELs, has been studied. The analytical estimation based on the Piwinski's formalism is in rough agreement with the particle tracking results obtained with the ELEGANT code. They confirm that IBS is relevant neither to the FEL energy-normalized bandwidth in the ultra-violet – x-ray wavelength range, nor to the gain of the MBI in the main linac. A low-beta FODO channel has been investigated to increase the longitudinal growth rate of IBS at the linac injection. This solution is far from being as efficient as a LH system: the channel requires tens of quadrupole magnets over tens of meters to generate an incoherent energy spread in the range 5–10 keV rms, for beam charges in the range 100–500 pC. As an alternative, a recirculating beam line was explored to cumulate IBS-induced energy spread in a relatively compact lattice. Unfortunately, the CSR instability in the arcs, driven by the high charge density and the low beam rigidity, deeply modulates the beam longitudinal phase space after only one turn. In conclusion, a relatively compact single-pass low-beta FODO channel at the linac injection could almost double the incoherent energy spread of high brightness beams with charge in the range 100–500pC. A beam heating above the 10 keV rms level is envisaged at the end of the FODO channel for charge densities at least  $\sim 4$  times higher than generated by state-of-the-art photo-injectors.

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## REFERENCES

- [1] Z. Huang, Technical Note LCLS-TN-02-8 (2002).
- [2] A. Xiao and M. Borland, in *Proc. of the 23<sup>rd</sup> Part. Accel. Conf.*, Vancouver, BC, Canada, 2009, paper TH5PFP038.
- [3] S. Di Mitri, *Phys. Rev. Special Topics – Accel. Beams* 17, 074401, 2014.
- [4] M. Borland, APS LS-287, 2000.
- [5] A. Xiao, in *Proc. of the 24<sup>th</sup> Linear Accel. Conf.*, Victoria, BC, Canada, 2008, paper MOP093.
- [6] E. L. Saldin, E. A. Schneidmiller and M.V. Yurkov, *Nucl. Instrum. Methods Phys. Res., Sect. A* 528, 355, 2004.
- [7] Z. Huang *et al.*, *Phys. Rev. Special Topics – Accel. Beams* 13, 020703, 2010.
- [8] A. Piwinski, “Intrabeam Scattering” in *CERN Accelerator School*, 1991, p. 226.
- [9] A. Piwinski, in *Proc. of the 1983 SSC Workshop*, Ann Arbor, MI, USA, 1983, p. 59.
- [10] T. Raubenheimer, *Part. Accel.* 45, 1994, p. 111.
- [11] M. Hüning and H. Schlarb, in *Proc. of the 2003 Part. Accel. Conf.*, Portland, OR, USA. IEEE Catalog Number 03CH37423C, ISBN 0-7803-7739-7, 2003, paper WUPAB017.
- [12] Z. Huang, *et al.*, *Phys. Rev. Special Topics – Accel. Beams* 7, 074401, 2004.
- [13] S. Spampinati, *et al.*, *Phys. Rev. Special Topics – Accel. Beams* 17, 120705, 2004.
- [14] E. L. Saldin, E. A. Schneidmiller and M. V. Yurkov, *Nucl. Instrum. Phys. Res. A* 490, 2002, p. 1.
- [15] J. D. Bjorken and S. K. Mtingwa, *Part. Accel.* 13 (1983) 118.
- [16] D. R. Douglas *et al.*, JLAB-ACP-14-1751, 2014, arXiv:1403.2318.
- [17] D. Douglas, JLAB-TN-98-012, 1998.
- [18] S. Di Mitri, M. Cornacchia, S. Spampinati, *Phys. Rev. Letters* 110, 014801, 2013.