HIGHER ORDER MODES ANALYSIS OF FERMILAB'S RECYCLER CAVITY*

M. Awida#, J. Dey, T. Khabiboulline, V. Lebedev, and R. Madrak
Fermilab, Batavia, IL 60510, USA

Abstract

Two recycler cavities are being employed in Fermilab's Recycler Ring for the purpose of slip stacking proton bunches, where 6 batches of 8 GeV protons coming from the Booster are stacked on top of 6 circulating batches. Slip stacking requires two RF cavities operating near 52.809 MHz with frequency slip of 1.26 kHz. In this paper, we report on the analysis of higher order modes in the Recycler cavity, and present their values for R/Q and shunt impedances. Knowing the frequencies and properties of higher order modes is particularly critical for understanding beam stability margins.

INTRODUCTION

Two RF cavities are utilized in Fermilab's Recycler Ring to achieve the slip stacking of proton bunches [1, 2]. Each cavity is a quarter wave resonator, as shown in Figure 1, but they are tuned to slightly different frequencies. One is operating near 52.809 MHz, while the other is detuned by -1.26 kHz. A relatively small ferrite loaded transmission line is used to electronically tune the cavity to the target resonance value. Several ports exist on the cavity. They are used for sensing the RF power with field probes and for the initial mechanical tuning of resonance frequency.

The Recycler cavities are made of copper and are designed to sustain a gap voltage of 150 kV with a maximum power of 150 kW. The designed value of shunt impedance for the fundamental operating mode is 75 kΩ, while R/Q is 13 Ω.

RF MODEL

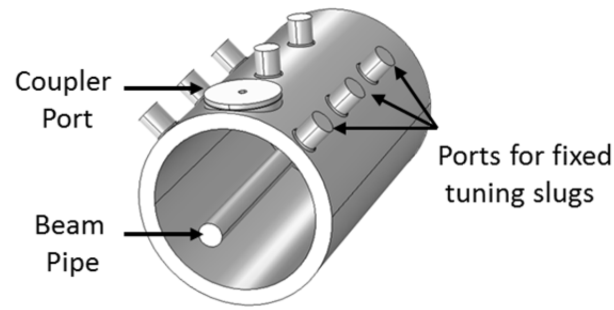
A full 3D model for the Recycler cavity was created to analyse the cavity's higher order modes (HOM). Figure 2 illustrates the electric field of the fundamental mode resonating at 51.518 MHz. The field is scaled to produce a 150 kV gap voltage. Assuming a conductivity of 5.998e7 S/m for copper, the corresponding dissipated power is 111 kW.

The scope of this work is to compute the R/Q for the monopole modes, and the transverse R/Q_{tr} for the dipoles and quadrupoles, up to 500 MHz. In this perspective the computed quantities are defined as follows:

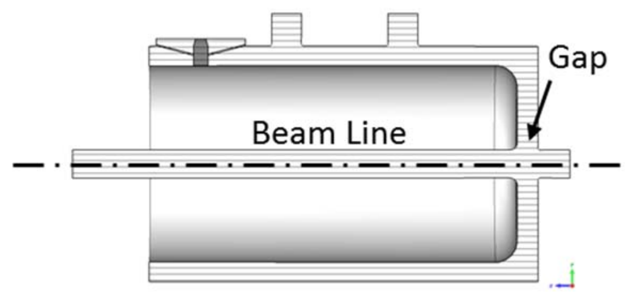
$$R/Q = \frac{|V|^2}{2\omega U}$$

$$R/Q_{tr} = \frac{|V_{kick}|^2}{2\omega U}.k = \frac{|V_{kick}|^2}{2U.c}$$

where ω is the angular frequency, U is the cavity's stored energy, k is the propagation constant, V is the longitudinal gap voltage, and V_{kick} is the transverse kick voltage.



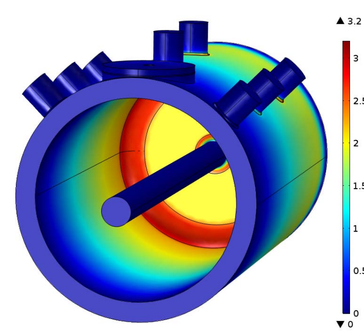
(a)



(b)

Figure 1: Fermilab's Recycler cavity. (a) 3D model. (b) Cross sectional view.

(a)



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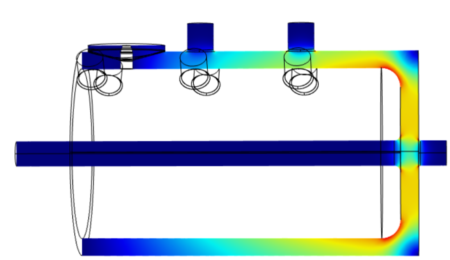

Figure 2: Surface electric field in MV/m of the fundamental mode resonating at 51.518 MHz inside the Recycler cavity. (a) 3D view. (b) Cross sectional view.

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#mhassan@fnal.gov

The gap voltage is calculated along the length of the cavity L as

$$V = \int_0^L E_z e^{j\frac{k}{\beta}z}\, dz$$

Meanwhile, the transverse kick voltage can be computed using the transverse electric and magnetic fields [3] as

$$V_{kick_x} = \Delta Px.c = \int_0^L \left(\frac{Ex}{\beta} - jZ_0\, Hy\right) e^{j\frac{k}{\beta}z}\, dz$$

$$V_{kick_y} = \Delta Py.c = \int_0^L \left(\frac{Ey}{\beta} + jZ_0\, Hx\right) e^{j\frac{k}{\beta}z}\, dz$$

Or from the longitudinal electric field off axis, using Panofsky's theorem [4] as

$$V_{kick_x} = \Delta Px.c = \frac{j}{k}\int_0^L \frac{(Ez^{+\Delta x} - Ez^{-\Delta x})}{2\Delta x} e^{j\frac{k}{\beta}z}\, dz$$

$$V_{kick_y} = \Delta Py.c = \frac{j}{k}\int_0^L \frac{(Ez^{+\Delta y} - Ez^{-\Delta y})}{2\Delta y} e^{j\frac{k}{\beta}z}\, dz$$

HOM ANALYSIS

The HOM analysis was carried out using Comsol Multiphysics electromagnetic module [5]. A perfect magnetic conductor boundary condition was assumed on the coupler port. Figure 3(a) demonstrates the computed resonance frequencies and actual measured ones. Simulation and measurements are mostly in good agreement. Some modes were observed in simulation but not in measurements. Most likely these modes have very weak coupling to the excitation antenna used in measurements. The computed quality factors for the resonant modes are shown in Figure 3(b). The quality factor of the fundamental mode is 6860 and it increases with frequency for the HOMs reaching 23273 for the highest computed mode at 562 MHz.

Figures 3(c) and (d) present the calculated longitudinal shunt impedances and R/Q for the HOMs. The computed value of shunt impedance for the fundamental mode is 101.5 kΩ, while R/Q is 14.8 Ω. The largest shunt impedance of 134.3 kΩ is exhibited by the monopole mode at 243 MHz, for which R/Q is 8.4 Ω.

On the other hand, Figure 4 compares the longitudinal and transverse shunt impedances for HOMs. Both horizontal and vertical transverse shunt impedances are computed using the definitions presented earlier. The largest horizontal shunt impedance of 976 kΩ/m is exhibited by the horizontal dipole mode at 390.5 MHz, with its transverse R/Q_{tr} of 48.3 Ω/m, while the largest vertical shunt impedance of 1161 kΩ/m is exhibited by the vertical dipole mode at 390 MHz, with its transverse R/Q_{tr} of 57.9 Ω/m.

Table 1 summarizes the results of the higher order mode study presenting the quality factors and shunt impedances, both longitudinal and transverse, for all resonating modes up to 560 MHz. Based on the computed longitudinal and transverse shunt impedances, resonating modes were divided into monopoles, horizontal dipoles, vertical dipoles, and quadrupoles, as shown in the table. The presented data should be useful for further beam instability studies. It is worth noting here that some of the vertical dipole modes have non-negligible accelerating shunt impedance, which is most likely due to the vertical asymmetry of the cavity's geometry.

Table 2 compares the measured and simulated values of shunt impedance for the first six monopole modes. Measurements were done using the stretched wire method [1]. Measured and simulated values agree to within 20%.

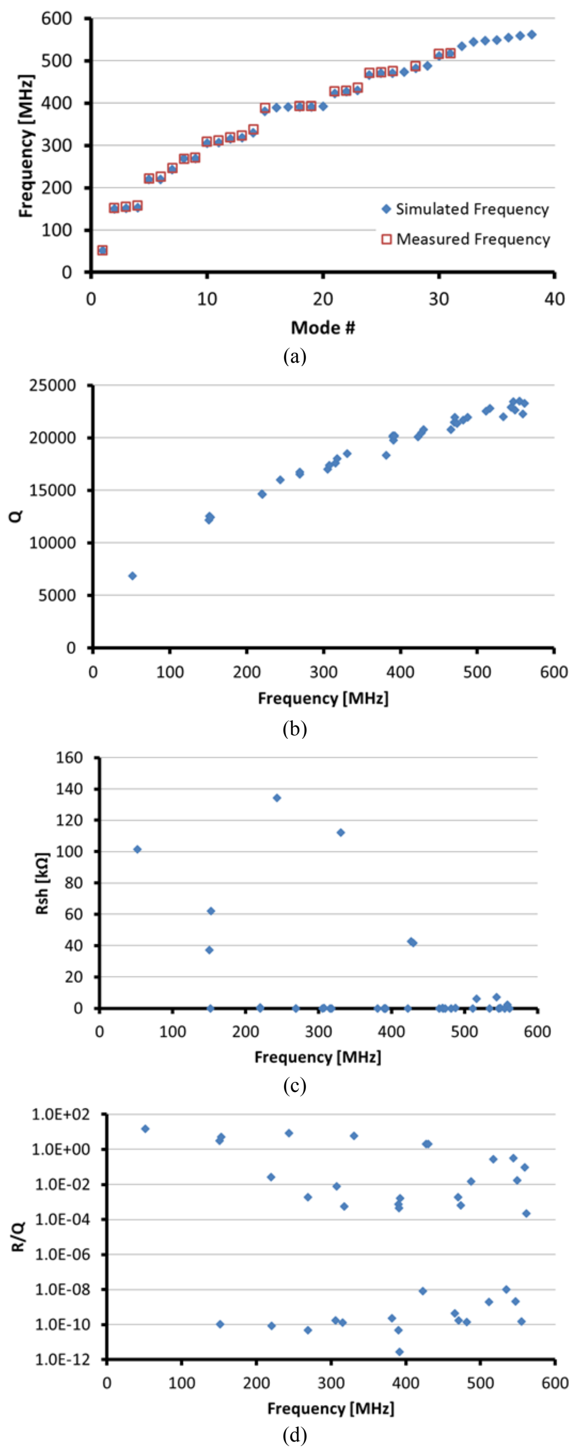

Figure 3: Recycler cavity HOM properties (a) Computed versus measured resonance frequencies. (b) Quality factors. (c) Longitudinal shunt impedance. (d) R/Q.

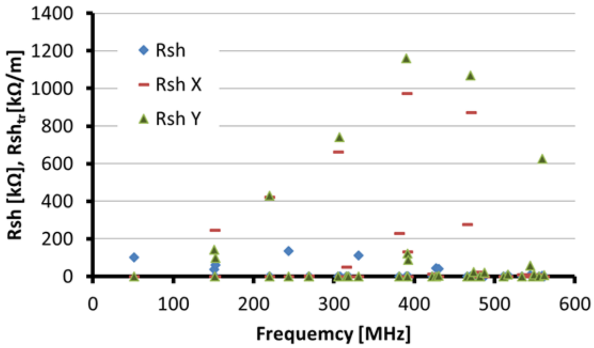

Figure 4: Simulated longitudinal and transverse shunt impedances of the HOMs for the Recycler cavity.

Table 1: Summary of the HOM Analysis

Frequency [MHz]	Q	Rsh [kΩ]	Rsh_x [kΩ/m]	Rsh_y [kΩ/m]
51.52	6859.63	101.52	0.00	0.01
150.69	12186.64	37.13	0.00	142.35
151.65	12555.77	0.00	246.79	0.00
152.64	12453.90	61.92	0.00	96.96
219.74	14690.94	0.39	0.00	431.22
220.03	14638.01	0.00	423.82	0.00
243.24	16015.87	134.30	0.00	0.83
268.87	16557.29	0.00	1.65	0.00
269.01	16762.17	0.03	0.00	0.26
305.58	17018.75	0.00	664.57	0.00
307.31	17392.76	0.13	0.00	738.22
315.51	17593.88	0.00	51.40	0.00
317.75	18025.21	0.01	0.00	0.36
330.34	18474.77	112.20	0.00	1.51
381.25	18323.34	0.00	229.63	0.00
390.00	20069.22	0.02	0.00	1161.07
390.47	20197.90	0.00	975.75	0.00
391.14	19759.72	0.01	0.00	121.27
391.27	20130.38	0.00	131.73	0.00
392.24	20212.95	0.03	0.00	86.82
422.69	20073.47	0.00	12.92	0.00
426.94	20465.46	42.78	0.00	1.10
429.96	20764.18	41.56	0.00	4.51
465.61	20760.83	0.00	278.79	0.00
469.93	21463.32	0.04	0.00	1068.03
470.72	21953.18	0.00	871.90	0.00
473.64	21388.77	0.01	0.00	24.54
481.75	21678.94	0.00	23.24	0.00
487.61	21973.33	0.32	0.00	19.90
511.23	22543.82	0.00	1.45	0.00
516.80	22791.68	6.21	0.00	9.71
534.25	22029.94	0.00	9.48	0.00
544.08	22892.55	7.27	0.00	56.65
547.24	23434.70	0.00	10.35	0.00
549.24	22623.98	0.37	0.00	8.59
555.10	23492.46	0.00	11.05	0.00
558.99	22275.30	2.13	0.00	625.56
561.65	23272.70	0.01	0.00	8.31
Monopole Mode	Horizontal Mode	Vertical Mode	Quadrupole Mode	

Table 2: Comparison of Measured vs. Simulated Shunt Impedances for the First Six Monopoles

f_m	f_s	Q_m	Q_s	Rsh_m	Rsh_s
52.89	51.52	6130	6860	84.46	101.52
155.23	152.64	10501	12454	63.64	61.92
250.38	243.24	13337	16016	123.86	134.30
337.22	330.34	15478	18475	104.28	112.20
429.02	426.94	17458	20465	42.98	42.78

Finally, Figure 5 illustrates the electric field distribution of some modes of interest; the monopole mode at 243 MHz, the two quadrupole modes at 269 MHz, and the dipole modes at 390 MHz, and 470 MHz.

CONCLUSION

Higher order modes can cause beam instabilities. Analysis of higher order modes of the Recycler cavity yielded the shunt impedances for both longitudinal and transverse modes. The 3rd monopole mode resonating at 246 MHz exhibits the largest longitudinal shunt impedance, and the vertical dipole resonating at 390 MHz gives the largest vertical impedance, while the horizontal dipole at 470 MHz has the largest horizontal impedance. Measured and simulated longitudinal shunt impedances are in reasonable agreement.

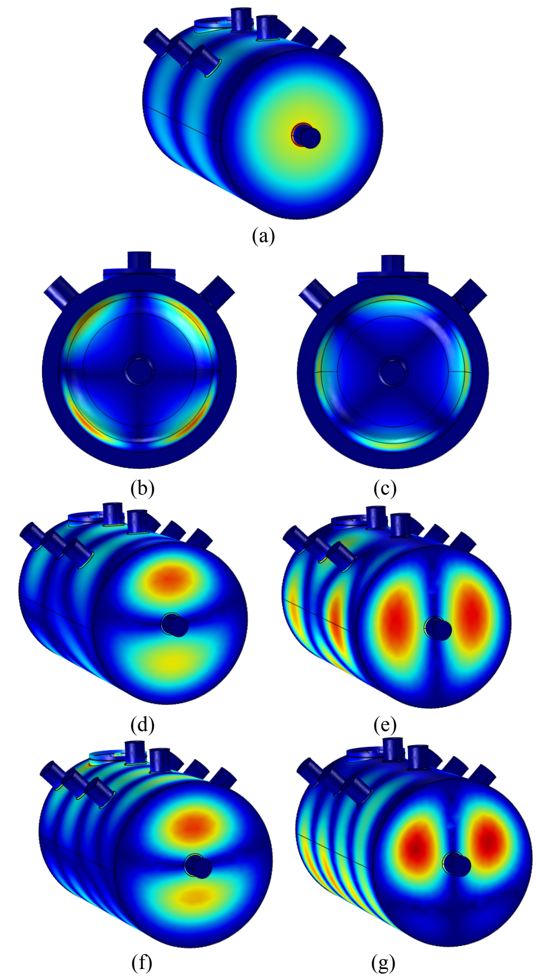

Figure 5: Surface electric field of (a) Monopole at 243 MHz, (b), (c) Quadrupole modes at 269 MHz, (d), (e) Dipoles at 390 MHz, (f), and (g) Dipoles at 470 MHz.

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