ON THE POSSIBILITY OF USING NONLINEAR ELEMENTS FOR LANDAU DAMPING IN HIGH-INTENSITY BEAMS *

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Abstract

Direct space-charge force shifts incoherent tunes downwards from the coherent ones breaking the Landau mechanism of coherent oscillations damping at high beam intensity. To restore it nonlinear elements can be employed which move back tunes of large amplitude particles. In the present report we consider the possibility of creating a “nonlinear integrable optics" insertion in the Fermilab Recycler to host either octupoles or hollow electron lens for this purpose. For comparison we also consider the classic scheme with distributed octupole families. It is shown that for the Proton Improvement Plan II (PIP II) parameters the required nonlinear tune shift can be created without destroying the dynamic aperture.

INTRODUCTION

The Fermilab Recycler [1] is a 3.3 km long 8.9 GeV proton storage ring originally constructed to accumulate antiprotons for Tevatron Run II and based on permanent magnets. For suppling megawatt beams for the Long-Baseline Neutrino Facility, PIP II foresees the replacement of the 400 MeV Linac by a new 800 MeV superconducting one, while the old rings Booster, Main Injector and Recycler, are expected to undergo only minor improvements.

With PIP II parameters [2], the Recycler is expected to store at least a factor 1.5 more protons wrt current operation increasing the incoherent space charge tune shift from -0.06 to -0.09 at 1σ. Currently coupled bunch instabilities in the Recycler are controlled by dampers. However their use during slip stacking is problematic and new dampers are now under consideration.

In general, there are two contributions to Landau damping for bunched beams: the intrinsic one due to the space charge tune spread, and the external one due to machine nonlinear elements. For the PIP II Recycler with its very high space charge parameter, $\Delta Q_{sc}/Q_s$ [3], the intrinsic damping is too weak to be useful and external nonlinear lenses are needed to provide stabilization. However if the required non-linearities are large they may compromise the beam Dynamical Aperture (DA).

For the Recycler it has been evaluated that a tune shift of $\simeq 1.4\times10^{-3}$ at 1σ is needed in presence of a damper and $\simeq 5.6\times10^{-3}$ without it [4] .

We have studied the possibility of inserting octupole magnets into the existing Recycler ring as well as the possibility of modifying the lattice for a “nonlinear integrable optics" [5] insertion to host either octupoles or a hollow electron lens.

DISTRIBUTED OCTUPOLES

The detuning due to an octupole of integrated strength $O_3 = \frac{\partial^3 B_y}{\partial x^3}\ell/B\rho$ is

$$\Delta Q_{x,y} = \frac{O_3}{16\pi}(\beta_{x,y}^2 J_{x,y} - 2\beta_x\beta_y J_{y,x})$$

$J_{x,y}$ being the action ($2J_{x,y} = a_{x,y}$, with $a_{x,y}$ Courant-Snyder invariant). To minimize the 2nd order chromaticity and the cross-term detuning which may limit the DA the octupoles should be located where D_x=0 and $\beta_z^2 \gg \beta_x\beta_y$ (z=x,y).

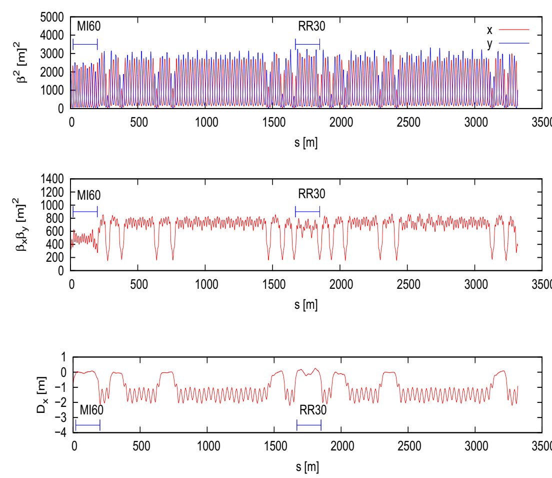

Figure 1: Recycler optics.

For minimizing the direct impact of the octupoles on DA a scheme where the phase advance between consecutive octupoles satisfies the condition $\Delta\mu_z = k\pi/N_{oct}$ (z=x,y) with k=1,2,...$N_{oct}-1$ and $\Delta\mu_z \neq \pi/2$, has been considered. The only resonance excited in this case by normal octupoles is the 4th order $2Q_x$-$2Q_y$=*integer* one. Two families of 5 octupoles each were located in the straight section MI60 (see Fig. 1). The 9 MI60 tuning quadrupoles and the quadrupoles QD531, QF532, QF610 and QD611, assumed to be tunable, were used to move $\Delta\mu_{x,y}/2\pi$ between octupoles from 0.26 to 0.3 keeping the optics unperturbed outside. The RR30 straight section tuning quadrupoles were used for optimizing the betatron tunes and 4 quadrupoles were added for re-matching the optics. With octupole settings for a tune shifts of about 0.005 at 1σ, the on-energy DA computed with 1000 turns tracking and ignoring synchrotron motion is only about 3σ for 2.5×10^{-6} m rms normalized emittance,

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to be compared to 10σ over 1×10^5 turns for the lattice w/o octupoles.

NONLINEAR INTEGRABLE OPTICS INSERTION

Recycler Lattice Change

The non linear integrable optics insertion requires that β_x=β_y along the insertion. The straight section RR30 was modified by increasing to 22.5 m the distance, ℓ_{ins}, between the Q305A and Q305B quadrupoles. With α^*_x=α^*_y=0 in the Middle Point, MP, and asking for a $\Delta\mu_x$=$\Delta\mu_y$=0.2 across the insertion it is β^*_x=β^*_y=15.5 m. The RR30 tuning quadrupoles and 5 quadrupole pairs left and right of MP were used for matching the insertion and setting the total tunes to Q_x=25.29 and Q_y=24.28. The insertion optics is shown in Fig. 2.

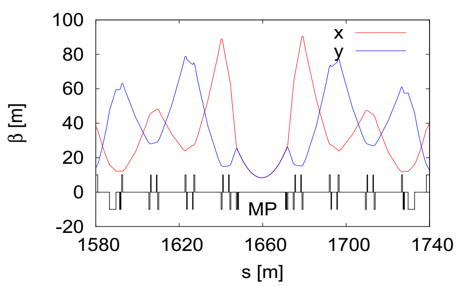

Figure 2: Insertion optics.

Octupole Lens Insertion

The octupole field must be shaped so that $K^{(3)}\ell(s) = k_0\beta^{-3}(s)$, $K^{(3)} \equiv \partial^3 B_y/\partial x^3/B\rho$. The integrated strength is then

$$O_3 = \frac{2k_0}{\beta^{*2}}\,\frac{1}{8}\Big[\frac{a(5+3a^2)}{(1+a^2)^2} + 3\tan^{-1} a\Big]$$

with $a \equiv \ell_{ins}/2\beta^* = \tan(\Delta\mu/2)$. Fig. 3 shows O_3/k_0 vs. insertion phase advance for three ℓ_{ins} values.

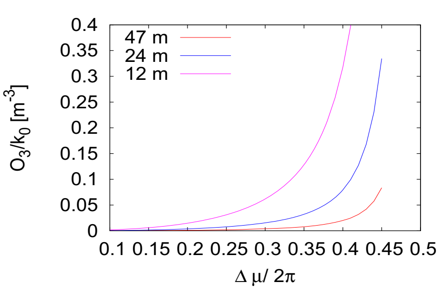

Figure 3: O_3/k_0 vs. insertion phase advance for ℓ_{ins}=47, 24 and 12 m.

Owing to the shape of the octupole field the detuning may be written in terms of the phase advance across the insertion

$$\Delta Q_{x,y} = \frac{k_0}{16\pi}(J_{x,y} - 2J_{y,x})\Delta\mu$$

The integrated octupole field increases with $\Delta\mu$, however the effect on the detuning coefficients increases (linearly) too.

The octupole field has been modeled by introducing 41 thin octupoles with strength varying as $k_0\beta^{-3}(s)$ and k_0 has been determined so to get 1.4×10^{-3} tune shift at x_0=σ_x and y_0=0 (and x_0=0 and y_0=σ_y) as computed by MAD-8 STATIC. The values are in reasonable agreement with analytical calculations as well as with the tune shift evaluated by a Fourier analysis of the Turn-by-Turn (TBT) position. Fig. 4 shows the resulting on-energy DA evaluated by tracking over 1000 turns, w/o space charge and ignoring synchrotron motion.

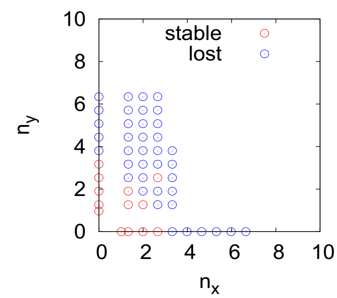

Figure 4: On-energy DA with octupole lens insertion for a tune shift of 1.4×10^{-3} at 1σ. The particles have been tracked over 1000 turns with MAD8-LIE4.

By increasing the phase advance across the insertion from 0.2 to 0.3 (2π units) or by adding a 12-pole component as described in [6] the DA does not improve. The reason for these disappointing results is going to be investigated.

Hollow Electron Lens Insertion

Electron lenses have been used for beam-beam compensation and they are under study for a variety of accelerator applications such as long range beam-beam compensation and beam collimation. To be effective the lens electrons must move in the direction opposite to the primary beam.

We can think of a hollow lens as the superposition of two beam lenses with different charge sign. If N^+/N^-=$(\sigma_+/\sigma_-)^2$, with σ_+ and σ_- being the radii of the positively and negatively charged round lenses respectively and N^+ and N^- being the number of positively and negatively charged particles, the linear tune shift vanishes and the the horizontal tune shift as function of the betatron amplitudes a_x and a_y is given by

$$\Delta Q_x(a_x,a_y) = 4\xi_0^- \int_0^{1/4} dt\Big\{I_0(a_y^2t/\sigma_-^2)\Big[I_0(a_x^2t/\sigma_-^2) - I_1(a_x^2t/\sigma_-^2)\Big]\exp\Big[-\frac{a_x^2+a_y^2}{\sigma_-^2}t\Big]$$
$$-I_0(a_y^2t/\sigma_+^2)\Big[I_0(a_x^2t/\sigma_+^2) - I_1(a_x^2t/\sigma_+^2)\Big]\exp\Big[-\frac{a_x^2+a_y^2}{\sigma_+^2}t\Big]\Big\}$$

where I_0 and I_1 are the modified Bessel functions of the first kind and $\xi_0^- > 0$ is the linear tune shift due to the negatively

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charged lens alone. The expression for ΔQ_y is obtained by exchanging the subscripts x and y. Fig. 5 shows the tune shift (in units of ξ_0^-), vs. particle amplitude (in units of σ_-), for $r \equiv \sigma_+/\sigma_-$=0.85. Unlike a pure octupole, the tune shift decreases above $2\sigma_-$. Developing the potential up to the

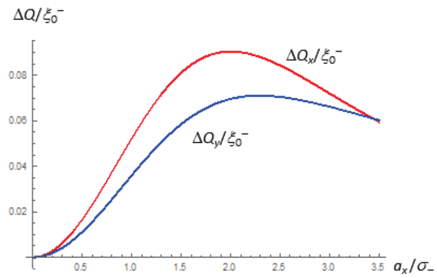

Figure 5: Total hollow lens nonlinear tune shift for r=0.85.

fourth order [7], we get the octupole component related tune shift

$$\Delta Q^O_{x,y} = \frac{F}{2\pi}\frac{r_p}{\gamma_p}\frac{N^+(1-r^2)}{16\sigma_+^4}\left[3\beta^2_{x,y}J_{x,y} + 2\beta_x\beta_y J_{y,x}\right]$$

with $F \equiv (1+\beta_p\beta_e)/[\beta_p(\beta_p+\beta_e)]$, which shows that the tune shift is positive for $r < 1$, namely for an actual electron hollow lens, and that unlike usual octupoles, the terms in J_x and J_y add.

For the non linear integrable optics it must be $\sigma_- = \lambda\beta(s)^{3/4}$ (with $\beta \equiv \beta_x$=β_y), λ being a proportionality factor. The proportionality factor has been fixed so to get the maximum tune shift at about 1σ. For β_{MP}=15.5 m and ϵ^N_{rms}=2.5×10^{-6} m it is λ=1.3×10^{-4} m$^{1/4}$.

The effect of the hollow lens has been modeled in MAD8 by superimposing 41 positively and 41 negatively charged beam-beam elements; the ratio r has been fixed in order to get the desired tune shift with a reasonable electron current. With r=0.85 and λ=1.3×10^{-4} m$^{1/4}$, to get ΔQ_x=0.03 at x_0=σ_x and y_0=0 (and ΔQ_y=0.03 for x_0=0 and y_0=σ_y) it must be N^-=N^+/r^2=4.76×10^{10}. The actual total number of electrons, n_e, is obtained by scaling the value of $N^- - N^+$ used in the MAD-8 model with the actual F factor. With 10 KeV electrons it is β_e=0.195 and n_e=41×0.996×($N^- - N^+$)=54.1×10^{10} leading to a current of 0.23 A.

As MAD-8 STATIC (as well as MAD-X SODD) does not take into account beam-beam elements, the effect of the hollow beam on the tune has been evaluated by a Fourier analysis of the TBT position. With the chosen parameters the DA evaluated by tracking with MAD8-LIE4 over 1×10^5 turns (the number of turns protons will actually perform is about 7×10^4), is at least 6σ, i.e. larger than the physical aperture. Synchrotron motion was ignored and space charge not included. It is worth noting that by choosing λ=2.6×10^{-4} m$^{1/4}$ so to match the electron and the proton sizes at the MP and setting N^-=N^+/r^2 so to get the same tune shift of 0.03 at 1σ the motion is no more a simple oscillation.

The on-energy DA without and with hollow lens in presence of space charge corresponding to a tune shift of -0.06, is shown in Fig. 6 . Particles are tracked over 7×10^4 turns by using MAD-X thin-lens tracking. Synchrotron motion is ignored.

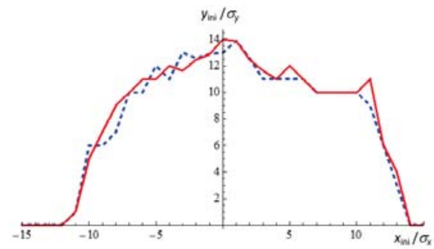

Figure 6: DA without (blue dashed) and with (solid red) hollow lens (N^-=4.76×10^{10}). Particles are tracked over 7×10^4 turns with MAD-X thin-lens tracking including space charge. The DA is about 10σ.

SUMMARY

Several options for restoring Landau damping in the Fermilab Recycler for the PIP II era have been investigated. Distributed octupoles drastically reduce the DA. Thus a special insertion has been considered in the RR30 straight section for hosting either a sequence of octupoles or a hollow electron lens according to the recipe of nonlinear integrable optics. While the first one performs worse than the distributed octupoles scheme, the latter has little impact on the DA even for a tune shift of 0.03, a factor 5 larger then the estimated value needed in the absence of dampers.

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