## Accelerator physics

## A brief introduction Z.Martí

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- Introduction
- Accelerator technology
- Beam dynamics



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- Accelerator technology
- Beam dynamics



Why accelerators

# Why would someone make so much efforts to break something?...





Why accelerators

# Why would someone make so much efforts to break something?...



To see what there is inside!



- The more energy we put in the particles the bigger the detail!  $\uparrow \qquad \lambda = \frac{h}{-}$
- As de Broglie (1923) would say: the more momentum (p), the smaller the associated wavelength (λ): the deeper in the laws of nature we can reach!



Why accelerators



• Particle physics.



- Particle physics.
- Medicine.



- Particle physics.
- Medicine.
- Biology.



- Particle physics.
- Medicine.
- Biology.
- Material science.



- Particle physics.
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- Chemistry.



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Roentgen's Lab (1895):





The first xray picture: Roentgen's wife hand!



## How to build your accelerator

#### We could use gravitational fields!





## We could use gravitational fields! Using the Earth: 11,2 km/s << c=300000 km/s





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## How to build your accelerator

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#### Ok, gravity is weak, what about magnetic fields?

$$\vec{F} = q\vec{v} \times \vec{B}$$

Only acts upon charged particles!



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It is not very useful to accelerate since it does not increase the particle energy:

$$\int \vec{F} \vec{ds} = q \int \left( \frac{\vec{ds}}{dt} \times \vec{B} \right) \vec{ds} = 0$$



# Ok, magnetic fields do not accelerate particles, what about **electric fields**?

$$\vec{F} = q\vec{E}$$

Only acts upon charged particles!



# Ok, magnetic fields do not accelerate particles, what about **electric fields**?

$$\vec{F} = q\vec{E}$$

Only acts upon charged particles!

It does a good job, but has a some limitations...



$$W = \oint \vec{F} \, \vec{ds} = q \oint \vec{E} \, \vec{ds} = 0$$



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Which is equivalent to Faraday's induction law.


#### A varying electric can easily accelerate:





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In summary, mother nature offers us:

- 1. Gravitational field: is too weak.
- 2. Magnetic field: is strong but it is only good for bending, it does not increase the particle energy.
- 3. Electric field: can produce higher energy gains if it varies with time.



So, basically everything we do with accelerators is explained by Maxwell laws:

Coulomb's (Gauss') law
$$q = - \oint \epsilon \vec{E} \, \vec{da}$$

Gauss' law:

$$\oint \vec{B} \vec{da} = 0$$

Faraday's law:

$$\oint \vec{\vec{E}} \, \vec{ds} = -\iint \frac{\partial \vec{\vec{B}}}{\partial t} \, \vec{da}$$

Ampere's law:  $\oint \frac{\vec{B}}{\mu} \vec{ds} = \iint \frac{\partial \epsilon \vec{E}}{\partial t} \vec{da} + \iint \vec{J} \vec{da}$ 



#### Fact: the speed of light is irrelative!

 $= mc^2$ .

 The measured elapsed times and the distances are different in different inertial reference systems (time dilatation and length contraction).



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 Particles have a maximum propagation velocity: the speed of light (c).

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#### *E* 0 =*mm c* 2 *cc c* 2 *2 c* 2 *. Fact*: the speed of light is irrelative!

As Albert Einstein showed to the world in 1905:

- Particles have a maximum propagation velocity: the speed of light (c).
- Particles have a minimum energy ("rest energy"):  $E \ 0 \ 0 \ 0 = mc^2$ .
- The measured elapsed times and the distances are different in different inertial reference systems (time dilatation and length contraction).



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The particle's momentum p and the kinematic energy E are expressed as follows:

$$E = \gamma m c^2 \left(\frac{1}{2} m v^2\right)$$
$$p = \gamma m v \quad (mv)$$



## If the truth be told, nowadays, many accelerators do almost no acceleration:





#### The muon experiment

(forget about acc. for 1 sec.):



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*Rossi, B. and Hall (1941) in Mt.Washington in New Hampshire* 





The muon experiment (forget about acc. for 1 sec.):

 $T_0 = 2.2 \mu s$   $\mu \overline{\phantom{v}}_{W} \overline{\phantom{v}}_{e}$  .):

Given such lifetime, measuring muon's energy and flux at the two labs, whithout time dilatation should be 22, but it is 1.4 in agreement with relativity.

Rossi, B. and Hall (1941) in Mt.Washington in New Hampshire













Lab Frame







They don't hold together, but in the lab frame, due to time dilatation they separate slowly! (same as muons lifetime seems longer)



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- After some acceleration, particles do not get faster any more, which is good, because we can transfer more energy without changing the accelerator geometry!
- 2. Yes, we can accelerate a bunch of particles with the same charge if we accelerate them rapidly!



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However, the maximum electric field achievable is limited and a series of accelerating structures are disposed one after the other...

This leads to the concept of linear accelerator (Linac) in contrast to circular accelerator.



#### Linear acceleration $E_0 \gg E (v \ll c)$ :



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#### Circular acceleration $E_0 \ll E (v \approx c)$ :





#### Typically the two schemes are combined :



# There are around 90 synchrotron radiation facilities in the world using this configuration.



# First of all, how do we close the circle? Magnetic field.

$$\vec{F} = q\vec{v} \times \vec{B} \qquad q \qquad \vec{F}$$



 $\vec{R}$ 

First of all, how do we close the circle? Magnetic field.  $\vec{F} = q\vec{v} \times \vec{B}$  q  $\vec{F} = \vec{v}$ 

But wait a minute, could we use electric field?



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Yes, but...





#### If, $v \approx c$ (3x10<sup>8</sup>m/s) then, a magnetic field of 1T (modest field) equals in strength to a 3MV/cm electric field (high tech field)!

#### *c*, electric fields will

always be more effective.

In circular accelerators most guiding fields are magnetic while in the first Linac stages guiding field is electric.



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	ALBA	LHC
Num. Mag.	32	1232
θ[°]	11.25	0.3



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$$F_{Mag.} = \mathbf{q}\boldsymbol{v}\boldsymbol{B}$$

$$F_{Centr.} = \frac{\gamma m v^2}{\rho} \begin{cases} \rho = \frac{p}{q\boldsymbol{B}} \end{cases}$$

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Most circular accelerator work in the ultra relativistic regime (p =  $\gamma m v \approx \gamma m c \approx E/c$ ):  $\rho = \frac{E}{acR}$ 



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General Electric synchrotron accelerator



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At higher energies, in 1946 they found out some light was coming out when particles passed through the guiding magnetic field.



General Electric synchrotron accelerator





Inertial rest Frame

$$\overbrace{}^{q} \overbrace{}^{\vec{E}}$$

Coulomb's law:

$$\frac{q}{\epsilon_0} = - \oint \vec{E} \, \vec{da}$$



Inertial rest Frame Non Inertial rest Frame





Coulomb's law:



Faraday's law (vacuum)

$$\oint \vec{E} \vec{ds} = -\iint \frac{\partial \vec{B}}{\partial t} \vec{da}$$

Ampere's law (vacuum)

$$\oint \vec{B} \vec{ds} = \mu_0 \epsilon_0 \iint \frac{\partial \vec{E}}{\partial t} \vec{da}$$



Inertial rest Frame





Non Inertial rest Frame



Lab Frame

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Synchrotron light is not different to the antenna radiation. However:

- 1. Particles move with a velocity respect to the observer, which adds a relativistic Doppler effect: a factor  $\gamma$ .
- 2. At relativistic speeds, time dilatation also adds another factor  $\gamma$ .

As particles travel in the accelerator they find magnetic elements at a frequency around GHz, then, X rays are produced!



### Focusing the beam

# Both in case of colliders or synchrotron radiation facilities, the smaller the beam size, the better!



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ct:

$$\vec{F} = -k \overrightarrow{\Delta x}$$

We need to cancel the field at the beam center!



A simple: way opposing two magnets!





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Or more efficient (4 poles working):





For a proper focusing we need to place the poles 45 deg (called quadrupole):





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Unfortunately we focus in one plane and defocus in the other:

$$F_x = k\Delta x$$

$$F_y = -k\Delta y$$



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• Magnets.



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- Magnets.
- Accelerating systems.



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- Magnets.
- Accelerating systems.
- Vacuum systems.



### Magnet technology

#### Regarding magnetic field usage (bend and focus), up to three different technologies are used:

Permanent magnets:

Normal conducting electromagnets:

Superconducting electromagnets:









### Magnet technology

#### Ampere's law (intra medium)

$$\oint \frac{\vec{B}}{\mu} \vec{ds} = \iint \frac{\partial \epsilon \vec{E}}{\partial t} \vec{da} + \iint \vec{J} \vec{da}$$



### Magnet technology

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#### Magnet technology

#### Ampere's law (intra medium)





#### Magnet technology

#### High permeability ( $\mu \gg \mu_0$ ) materials allow to:

Substance	Permeability
Air, dry	Slightly more than 1
Aluminum	Slightly more than 1
Bismuth	Slightly less than 1
Cobalt	60 to 70
Ferrite	100 to 3000
Free space (vacuum)	1 (exactly)
Iron	60 to 8000
Nickel	50 to 60
Permalloy	3000 to 30,000
Silver	Slightly less than 1
Steel	300 to 600
Specialized alloys	100,000 to 1,000,000

 Concentrate flux lines in the gap:

$$\frac{B_0}{g} \simeq \frac{I\mu_0}{g}$$

2. Generate equipotential surfaces:

$$B_{0,\parallel}\simeq 0$$



Both permanent magnets and electromagnets make use of the high permeability:



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 Dipoles: constant magnetic field, equipotential lines are straight lines.





Both permanent magnets and electromagnets make use of the high permeability:

- Dipoles: constant magnetic field, equipotential lines are straight lines.
- 2. Quadrupoles: linear magnetic field, equipotential lines are hyperbolas.







# Both permanent magnets and electromagnets are used in the iron dominated regime:





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For magnetic fields above 2-3T, the field is dominated by the coil. At such large currents, heat dissipation starts to be an issue...



# Both permanent magnets and electromagnets are used in the iron dominated regime:





#### Magnet technology

Coil dominated magnets can produce also pure dipole fields: 2r



**Coil dominated** magnets can produce also pure dipole fields: 2rВ And adding a displaced cable with opposite current...



Magnet technology

#### A pure dipole field can be obtained:





Magnet technology

#### A pure dipole field can be obtained:



# Similarly, overlapping concentric ellipses gives a pure quadrupolar field:





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Iron dominated:

- 1. Finite permeability.
- 2. Finite size.

Coil dominated (SC):

- 1. Finite malleability.
- 2. Finite size.
- 3. Finite  $T_c/B_c$ .
- 4. Quenching.



### Now you can probably better recognize the three different technologies of these quadrupoles:









**2. RF sources (amplifiers)**: Klystrons, Magnetrons, Tetrodes, IOTs or Solid state amplifiers...



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**4.** Accumulate: RF (resonant) cavities, which can be normal or super conducting.



Major breakthroughs in the development of RF accelerating happened at the end of World War II, when RF power sources in MW ranges at frequencies of MHz were developed...

All because they wanted more powerful Radars...













Vacuum tubes (Klystrons, Magnetrons, Tetrodes, IOTs) are still the most used RF power source in accelerators:



Bunching mechanism at MHz







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**Heated High** 



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voltage cathode Bunching mechanism at MHz

However, SSA are modular, don't need high voltage and are more stable and endurable... Just wait for the market to grow!

**Heated High** 















\*The **grid** must be very close to cathode, (1/4 mm) otherwise electrons do not have time to cross it!





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#### RF Cavities are **conductor geometries** designed so that specific frequencies (modes) can be accumulated without losses (resonate).





# **Unavoidably**, high order modes (HOM) can resonate as well. This can be avoided by feeding only the main mode and damping the other modes.




#### Damping HOMs can be achieved by enabling those modes to escape the cavity and being absorbed (for example with ferrites<sup>\*</sup>).



\*ferrites: high permeability and high resistivity.



The Cavity can be normal conducting (Cu) or superconducting. Normal conducting have limited current densities, hence, the geometry has to enhance the field, also getting closer to the beam:





The frequency adjustment can be done thought a hole in the cavity, modifying the cavity geometry. Since the beam pipe is small, specific HOM dampers are needed:



#### RF technology

#### ALBA (500MHz) RF cavity:





#### RF technology

In superconducting cavities, electric field can be much higher. However surface should be smooth to avoid quenches (multipacting). The separation to the beam can be bigger allowing HOM to escape through the vacuum chamber.





In this case, the frequency adjustment can be done by mechanically deformation of the cavity. The resaon is again avoiding sharp shapes to prevent quenching.





#### RF technology

#### **CESR** (500MHz) supeconducting RF cavity:

#### **RF power input**



#### Beam pipe



Ok, we can accelerate, bend and focus charged particle beams. But, charged particles interact **very strongly** with air (or any gas) since air is full of charged particles!





# Electrons/Protons at GeV energies stop after few/several km of air (lifetime is microseconds).

#### bar) (UHV).

#### At UHV the beam lifetime is around **hours**.



#### $0 \ 10 \ -12 \ -12 \ 10 \ -12 \ bar)$ (UHV).

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#### To keep UHV the vacuum chamber has to ensure, apart from good sealing, resistance to radiation, conductivity...





To keep UHV the vacuum chamber has to ensure, apart from good sealing, resistance to radiation, conductivity... which is needed to ensure the image beam continuity and avoid charge build-up: -q





# Vacuum chambers are then metallic (steel, Al or Cu). For some particular applications chambers can also be ceramic.



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Vacuum systems is a large topic (as the others), but we will stop here...



#### The end of the first part...

#### Thanks! **Questions**?





- Introduction
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Quadrupoles act like lenses for the charged particle beams: accelerators layouts (also called lattice) are similar to a camera focusing systems:



The question is, how do we choose the lattice?



#### To know how the beam is focused, we need to know the individual particle trajectories. First, the proper **reference system** should be used:





## We will restrict ourselves to the paraxial approximation:

 $x \ll \rho$  and  $y \ll \rho$ 

This is generally the case since the beam movement is of the order of mm, and the accelerator bending radius around tens of m:

	ALBA	LHC
$ ho \ [m]$	7	2812
θ[°]	11.25	0.3



The longitudinal position *s* with respect to the reference particle is chosen as independent variable (not the time *t*).



Notice that the trajectory angle is now the derivate respect to the independent variable:



Well aligned Quadrupole magnets have a linear magnetic field:

$$B_x = -gy \quad B_y = gx$$

If it focuses in one plane it defocuses in the other plane  $k = \frac{g}{B\rho}$ :





In this reference system, bending magnets are similar to simple drift sections except they add some extra focusing in the horizontal plane:





#### **Equations of motion**

#### Then the equations of motion are:

$$\frac{d^2x}{ds^2} + \left(k(s) + \frac{1}{\rho^2(s)}\right)x = 0$$
$$\frac{d^2y}{ds^2} - k(s)y = 0$$

In circular accelerators (of circumference *L*) these quantities are periodic:

k(s + L) = k(s) $\rho(s + L) = \rho(s)$ 



Equations of motion

# Such type of equation had been solved in 1886 by **G.W.Hill** when studying the motion of the moons perigee.





#### Equations of motion

Hill's equations are also called pseudo-harmonic oscillation, indeed they resemble the harmonic oscillator:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

Solution: 
$$x(t) = Acos(\omega t - \mu_0)$$

Amplitude (constant)

X



The solution are the *betatron* oscillations:



- *J*: action [m]: depends on initial conditions.
- $\mu_0$ : Initial phase [rad]:depends on initial conditions.
- $\beta(s)$ : beta function [m]: it is periodic:  $\beta(s + L) = \beta(s)$
- *μ*: betatron phase [rad]: does NOT grow linearly:

$$\mu(s) = \int_{0}^{s} \frac{ds}{\beta(s)}$$



## The beta function depends on the lattice focusing elements distribution (lattice).

Trajectories of different particles with different **initial conditions** have the same envelope given by the **beta function**.













The tracks correspond to different turns of the same particle, but could also be different particles starting with an initial phases changed by  $\mu(L)$ 







## Ok, we know how a single particle varies its position.

However, circular accelerators are characterized by the emittance which is a measure of the beam focalization and collimation.

#### Emittance lives in the position-angle space.



Every particle trajectory can be parameterized by  $(J,\mu_0)$ . Equivalently we can use initial position and angle  $(x_0, x'_0)$ . The angle can be obtained by simple derivation:

$$x'(s) = \sqrt{\frac{J}{\beta(s)}} \sin(\mu(s) - \mu_0) - \alpha(s)\sqrt{J\beta(s)}\cos(\mu(s) - \mu_0)$$

Where:

$$\alpha(s) = -\frac{\beta'(s)}{2}$$



#### Positions and angles

So, position and angle:

$$x(s) = \sqrt{J\beta(s)}\cos(\mu(s) - \mu_0)$$
$$x'(s) = \sqrt{\frac{J}{\beta(s)}}\sin(\mu(s) - \mu_0) - \alpha(s)\sqrt{J\beta(s)}\cos(\mu(s) - \mu_0)$$

We can isolate the phase dependence:

$$\frac{x^2(s)}{\boldsymbol{J}\beta(s)} = \cos^2(\mu(s) - \boldsymbol{\mu_0}) \qquad \qquad \gamma(s) \equiv \frac{1 + \alpha^2(s)}{\beta(s)}$$

Substituting:

$$J = \beta(s)x'^{2}(s) + 2\alpha(s)x(s)x'(s) + \gamma(s)x^{2}(s)$$


Positions and angles

position and angle describe an ellipse:

$$J = \beta(s)x^{\prime 2}(s) + 2\alpha(s)x(s)x^{\prime}(s) + \gamma(s)x^{2}(s)$$





# Positions and angles





# **Beam optics**





# Beam optics





The total phase variation over  $2\pi$  in one turn is called the tune (aka number of oscillations):

 $\boldsymbol{Q} = \boldsymbol{\mu}(L)/2\pi$ 



Since  $\beta(s)$  is periodic:  $Q = (\mu(L + s) - \mu(s))/(2\pi)$ , then the tune is **constant**.



At a given position in the ring only the non integer part of the tune is observable:



At a given position in the ring, the picture would be de same if the tune was Q + 1 or Q + 2...



#### The tune times the revolution frequency $(f_0)$ is the resonant frequency of the system...



Exciting the beam at such frequency leads easily to a beam loss.



# Also, the accelerator is unstable if the tune in integer:



Any small dipole error would produce an orbit amplitude growth turn after turn:



Similarly, the accelerator is unstable if the tune is half-integer :



Any small quadrupole error would produce an orbit amplitude growth turn after turn:



# Trajectory Example with working point $(Q_x, Q_y) = (4.16, 2.00)$ and small dipole error:





# Trajectory Example with working point $(Q_x, Q_y) = (4.50, 1.69)$ and small quadrupolar error:





# More in general, circular accelerators must avoid resonant lines: $nQ_x + mQ_y = q$

Dipole resonantLines:  $Q_x = i$  $Q_y = j$ 

Quadrupole resonant lines  $Q_x = i + \frac{1}{2}$   $Q_y = j + \frac{1}{2}$   $Q_x - Q_y = i - j$  $Q_x + Q_y = i + j + 1$ 





The tune *Q* is **constant** and all particles in the beam have more or less the same tune. All particles have different action *J*: there is a distribution of *J* values:



The RMS of the action distribution is called Emittance  $\varepsilon = RMS(J) = \sigma_I$ .



# The **beam size** $\sigma_x$ varies along the ring: $\sigma_x = RMS(x) = \sqrt{\varepsilon_x \beta(s)}$

#### at the same time:

$$\sigma'_{x} = RMS(x') = \sqrt{\frac{\varepsilon_{x}}{\beta(s)}}$$



 $\sigma' x x x \sigma' x$  at the same time:

The **beam size**  $\sigma_x$  varies along the ring:

$$\sigma' x = RMS(x') = \sqrt{\frac{\varepsilon_x}{\beta(s)}}$$
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$$\sigma'_x = RMS(x') = \sqrt{\frac{\varepsilon_x}{\beta(s)}}$$



Also, emittance is a key parameter for beamlines because they want:

- **Resolution**: determined mostly by  $\sigma_x$ .
- Flux of photons: determined mostly by  $\sigma'_x$ .

The emittance contains both:

$$\varepsilon_x = \sigma_x \sigma'_x$$



Generally speaking, beamlines focalize at the sample ( $\sigma_x$ ), and for a given optical aperture they want as much photons as possible ( $\sigma'_x$ ):



#### Still, beam size is the first requirement.



#### Emittance values for some Light Sources:

	Elettra	ALBA	Diamond	ESRF	APS	Spring-8
Energy (GeV)	2	3	3	6	7	8
Circumference (m)	259	269	562	845	1104	1436
Lattice	DBA	DBA	DBA	DBA	DBA	DBA
Emittance (nmrad)	7.4	4.4	2.7	4	3.1	3.4

Emittance of few *nmrad* means roughly 100 $\mu$ m× 100 $\mu$ rad  $\sigma_x \times \sigma'_x$ .



#### In modern light sourced (picture from SLS), beam sizes are of the order of a hair thickness:





# Lattice design

## Quadrupoles must be placed and powered so that they produce **small beam sizes** where desired.



# Lattice design

Quadrupoles must be placed and powered so that they produce small beam sizes where desired.



Other practical details to take into account in Lattice design:

1. Phase advance between elements.



- 1. Phase advance between elements.
- 2. injection scheme.



- 1. Phase advance between elements.
- 2. injection scheme.
- 3. Straight sections space (diagnostic, RF, IDs...).



- 1. Phase advance between elements.
- 2. injection scheme.
- 3. Straight sections space (diagnostic, RF, IDs...).
- 4. Maximum beam size.



# Lattice design

#### Design *p* repetitive structures (cells) is simpler and cheaper, and also, allows to avoid resonances:

$$nQ_x + mQ_y = q$$



Design *p* repetitive structures (cells) is simpler and cheaper, and also, allows to avoid resonances:

$$npQ_x + mpQ_y = q$$

The most simple cell (the unit block to build a lattice) consists of alternating FOcusing and De-fOcusing quadrupoles (FODO).



Nowadays, the FODO cell is typically used at booster synchrotrons, but third generation storage rings need more complex lattices.





So far we have assumed that the energy of the particles does **not change** and it is **the same** for all of them. But...

- **RF cavities**: change the energy depending on arrival time.
- Radiation: produces energy loss at every bending magnet.
- Particle interactions: produce energy exchange with residual gas particles or among them.



The RF cavities produce longitudinal focusing:





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We will see that more energetic particles take more time to go around, they get retarded but then the RF removes energy from them...



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We will see that more energetic particles take more time to go around, they get retarded but then the RF removes energy from them...



# **However**, the dynamics in the longitudinal plane (synchrotron oscillations) is much slower than the transverse plane:



The energy deviation with respect to the reference particle  $\delta = \frac{\Delta E}{E}$  is assumed to be constant. Which is a good approximation in many cases.



~ "

If a particle has an energy deviation, it follows circles with different radius:

$$\rho = \frac{E}{qcB} = \rho_0(1+\delta)$$

Both dipoles and quadrupoles will have different effect, but the first order effect comes from the dipoles and adds a term to the **horizontal plane** equation:

$$\frac{d^2x}{ds^2} + \left(k(s) + \frac{1}{\rho_0^2(s)}\right)x = \frac{\delta}{\rho_0(s)}$$



An energy deviation produces a dispersion orbit around the machine equal to  $\delta D(s)$ .

$$\frac{d^2 D}{ds^2} + \left(k(s) + \frac{1}{\rho_0^2(s)}\right) D = \frac{1}{\rho_0(s)}$$

The **betatron oscillations** go around  $\delta D(s)$ :





In general the dispersion is positive  $(D_x(s)>0)$ , so, particles with more energy  $(\delta > 0)$  take a longer way and take more time to do one turn!




The beam contains particles with a distribution of energies, the RMS value  $\sigma_{\delta} = RMS(\delta)$ :



Assuming that it is Gaussian and not correlated with the action J distribution:

$$\sigma_{x} = \sqrt{\varepsilon_{x}\beta(s) + \sigma_{\delta}^{2}D^{2}(s)}$$
$$\sigma'_{x} = \sqrt{\frac{\varepsilon_{x}}{\beta(s)} + \sigma_{\delta}^{2}D'^{2}(s)}$$



Additionally, depending on  $\delta$  the quadrupoles have also a different focusing strength.

Energy deviations causes the beta function, the phase and also the tune to change accordingly. The chromaticity  $\xi$  is defined as:

$$Q_x = Q_{x,0} + \delta \xi_x$$
  

$$Q_y = Q_{y,0} + \delta \xi_y$$



# The chromaticity effect can be **visualized** as a different focal length for every quadrupole:



The chromaticity due to dipoles and quads (also called natural chromaticity) is always negative ranging from -30 to -100 (unitless) in both planes.

Z.Martí



The synchrotron radiation takes energy  $U_0$  every turn:



The proton has a mass around 1GeV, while electrons' is 0.5keV. For present accelerators, protons are assumed to almost no radiate.



# The RF cavities need to provide at least the energy loss per turn $U_0$ .





**However**, the energy compensation in the RF changes the momentum direction of the particle towards the design momentum!



Loss in a random direction

Energy gain in the RF



Gain in a predefined direction



# As a result, turn after turn, the trajectory angle x' gets reduced:



As the path gets closer to design value, the revolution time also gets closer to the design value.



As the revolution time gets closer to the design value, the RF gives less extra energy to that particle:



In all directions, an initially off energy and/or off center particle tends to end in the design orbit and energy.



This effect is called Radiation Damping, it is present in any particle accelerator, but almost only is noticeable in a reasonable time scales in lepton machines:

$$U_0 = \frac{q^2 \gamma^4}{3\epsilon_0 \rho}$$

Here we will deal mainly with electron accelerators, where the energy loss per turn can be a considerable part (0.01%) of the particle energy.



## Equilibrium Beam Size

## **However**, the beam size is not zero around the closed orbit.



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This is because the radiation emission happens in photons.



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At some point the radiation quantization effects interrupt the damping:





- The damping is a result of the dipole+RF effect, it damps in the 3 planes.
- The quantum exitation happens at the dipoles. The effect in the transverse plane depends on the dispersion at that point:





Damping

#### The equilibrum emittance can be expressed as a function of the dispersion and the bending radius: Anti damping

 $\varepsilon_{x} = C_{q} \frac{\gamma^{2} \oint \frac{\pi_{x}(s)}{\rho^{3}(s)} ds}{P_{x} \oint \frac{1}{\rho^{2}_{x}(s)} ds}$  $C_q$  is a constant,  $P_x$  is the horizontal partition number and  $\mathcal{H}_{\mathbf{x}}$  is the dispersion invariant:

 $\mathcal{H}_{\boldsymbol{\chi}}(s) = \beta(s) D_{\boldsymbol{\chi}}'^{2}(s) + 2\alpha(s) D_{\boldsymbol{\chi}}(s) D_{\boldsymbol{\chi}}'(s) + \gamma(s) D_{\boldsymbol{\chi}}^{2}(s)$ 



This makes a big difference in stability and in the beam size limitation:  $\varepsilon_{\chi}$ ,  $\varepsilon_{\gamma}$  and  $\sigma_{\delta}$ :

- Hadrons synchrotrons (Protons and Ions): where beam size is determined by the injected beam.
- Lepton synchrotrons (electrons mainly): beam size does NOT depend on the initial beam distribution.



### Wait a minute... when we look at the beam size in our CCD we are actually seeing a cloud of electrons performing quantum jumps?... Yes!





## Instabilities

### The beam electromagnetic field can reflect in abrupt transitions of the vacuum chamber. Or from a different point of view, image current

radiates!



These are called wake-fields. The more particles in the bunch the bigger the wake-field.



Hence, the wake-fields may cause instabilities at big currents.

If the beam is centered, no transverse fields are generated, but if it moves transversally, it will kick transversally in the same direction.





It takes some time for the electromagnetic field to reflect in the chamber. Hence, the particles in the bunch will be influenced by the wake field but with a delay.

The head of the bunch will affect the tail...



Transverse Head Tail Instability:

### A warning:

Beam instabilities is a complicated thing to study, if you only can remember one thing from all this, let it be: chromaticity must be corrected to positive values, otherwise...



## Head tail instability

#### Transverse Head Tail Instability:





Transverse Head Tail Instability:

The Head transverse betatron oscillations produce a force that increases or damps the Tail transverse oscillations. After half synchrotron oscillation the roles are inverted!



#### Transverse Head Tail Instability:





Head tail is just one type of instability. A mathematically rigorous analysis shows that to avoid the more important instability sources, chromaticity should be small and positive.

However, in a storage ring with quadrupoles **chromaticity** is **big** and **negative**!



On top of that, large chromaticity has other

harmful effects. Since typical values for  $\sigma_{\delta}$  are 10<sup>-3</sup> and natural chromaticities are around -100, this makes the beam to have a big tune spread so that it can hit

harmful resonances.





Indeed, **chromaticity** is **corrected** in any accelerator. To do so, **sextupole** magnets are used. They are similar to **quadrupoles**, but have 6 poles instead:





## Chromaticity

Like quadrupoles, the force (field) is zero at the center, but in the sextupole it has the same sign at both sides:





## Chromaticity

Sextupole have parabolic magnetic field on axis:



Placing sextupoles at locations with dispersion D(s) allows to correct the chromaticity:





Sextupoles allow for stable beams with small  $Q_{\gamma}$ tune shifts with energy. *j* + 1 As a pay off, more resonant  $j + \frac{2}{3}$ lines appear and the system becomes non  $j + \frac{1}{2}$  $\bar{3}\sigma_{\delta}$  $j + \frac{1}{3}$ linear: stability is not guaranteed at big amplitudes.  $i + \frac{1}{3}i + \frac{1}{2}i + \frac{2}{3}i$ *i* + 1

 $Q_{x}$ 



# Linear systems behave independently of the amplitude (action). Nonlinear systems don't:





### You all know other nonlinear systems:



The rabbit fractal:  $z_{n+1} = z_n^{(2)} + c$  c = -0.123 + 0.745iNonlinear

Here colors indicate how divergent are the points...



## In accelerators we try to see which particles are likely **not** to diverge...

**ALBA dynamic aperture** 



The accelerator map:  $\begin{pmatrix} x_{n+1} \\ x'_{n+1} \\ y_{n+1} \\ y'_{n+1} \end{pmatrix} = Map \begin{pmatrix} x_n \\ x'_n \\ y_n \\ y'_n \end{pmatrix}$ 



The non linear effects depend on how strong the sextupoles are and where they are located.

In modern light sources, with every time smaller emittances, and higher quadrupolar fields, stronger chromatic effects and stronger sextupolar fields, sextupoles is part of the lattice design.



## Nonlinear Lattice design

The working point is chosen to avoid resonances, but the best location depends on the details of every synchrotron lattice:





### The end

#### Thanks! **Questions**?

