The Physics of Transverse Emittance Manipulations

Bruce Carlsten
Deputy Director, Office of Accelerator R&D and Production
August 30, 2022

U.S. DEPARTMENT OF ENERGY | Office of Science

Outline

- Review of rms beam quantities
  - What "symplectic" means for rms beam quantities
  - Examples of some non-symplectic beam interactions
- Examples of some transverse phase-space manipulations
  - Emittance compensation
  - Flat beam transforms
  - Emittance exchangers
  - Some ongoing applications

We represent beam properties with canonical variables

- Using $x' = dx/dz$ we rewrite the canonical variables in what we call phase (or trace) space variables

$$\left(x, x', y, y', c\Delta t, \frac{\Delta(\gamma\beta)}{\gamma_0}\right)$$

These are the variables we actually use (all deviations from some reference location and trajectory)

- We define beam parameters as ensemble averages of these quantities
  $x_{rms} = \sqrt{\langle x^2 \rangle}$, $x'_{rms} = \sqrt{\langle x'^2 \rangle}$ and so on
  – For a uniformly filled beam, the edge is given by $x_{edge} = 2x_{rms}$

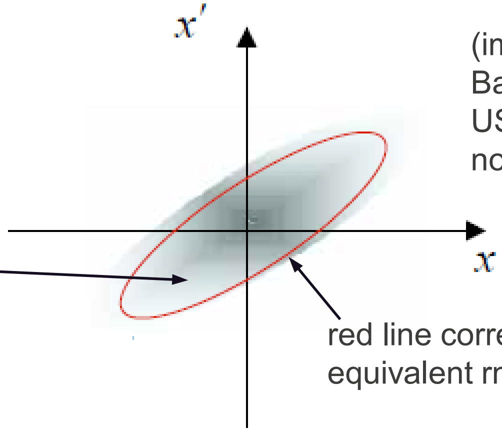
- **Early work from Sacherer and Lapostalle showed that the rms beam evolution very largely depends on only the beam's rms quantities**
  – allows us to define an "emittance" $\varepsilon_x = \gamma_0\beta_0\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$
  – and to think of the beam in terms on an equivalent hard-edged rms beam

"Emittance" is a measure of the quality of the beam – a low emittance is needed for many applications (i.e., X-ray free-electron lasers (XFELs))

The beam emittance is a measure of the area associated with the beam's phase space

- The area of the beam's transverse phase space is about

$$area \approx 4\pi \frac{\varepsilon_x}{\gamma_0 \beta_0} = 4\pi\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$



- **Ignoring collisions, Liouville's theorem states that phase space area is invariant – this leads to the fact the emittance written as $\varepsilon_x = \gamma_0 \beta_0 \sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$ is conserved under uncoupled linear forces (even with acceleration)**

- Because of this beam, physicists have been tempted to define a transverse beam temperature $T_{transverse} = \varepsilon_x^2 \frac{mc^2}{k_B \gamma_0 x_{rms}^2}$

What does symplectic mean in an rms or linear sense?

- Lorentz force law follows from a Hamiltonian:

$$H = c\sqrt{\left(\vec{p} - q\vec{A}(\vec{r},t)\right)^2 + m^2c^2} + q\phi(\vec{r},t)$$

- All (collisionless) electrodynamic motion satisfies Liouville's theorem
- **If the Hamiltonian is quadratic in beam coordinates (transformation is linear), then the motion is symplectic**:

$$J_6 = R^T J_6 R$$

- *If the Hamiltonian is higher order in beam coordinates, the rms symplectic condition no longer follows*:

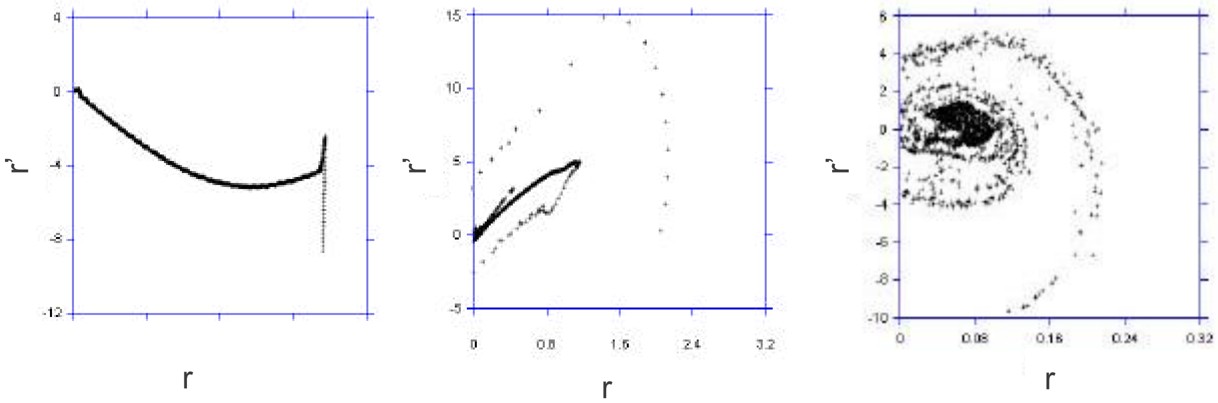
$$J_6 \neq R^T J_6 R$$

“Eigen-emittances” are conserved under symplectic transformations (Dragt)

- Phase-space manipulations are best thought of in terms of eigen-emittances
- **The actual beam emittances are equal to the eigen-emittances if there are no correlations between dimensions**
- The eigen-emittances are conserved under all forces linear with the beam coordinates (even with coupling between dimensions)
- Linear transformations can “swap” eigen-emittances
- Nonlinear forces (nonsymplectic forces) can change the eigen-emittances

There are three scenarios where rms phase-space area can change

1. The force is nonlinear. If the beam is stiff enough it doesn't mix (i.e., doesn't "wave-break") the phase-space structure can be manipulated (even "unwound"). Ion beams mix characteristically mix within a ¼ focusing ("betatron") period, electron beams become relativistic and stiffen quickly, suppressing mixing.



Evolution of beam phase space – the first configuration on the left is easy to unwind, the last configuration on the right would be hard to unwind

$$\varepsilon_{wave-breaking} \sim 8x_{rms}\sqrt{\frac{I/I_A}{\gamma_0\beta_0}}$$

Scott Anderson's thesis work (UCLA) studied wavebreaking

There are three scenarios where rms phase-space area can change

2. A non-sympletic interaction is introduced (force higher-order in beam coordinates)

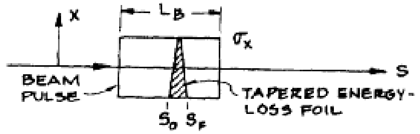


Figure 1. Geometry of the Tapered Energy-Loss Foil.

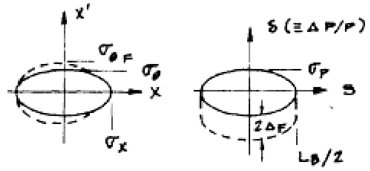


Figure 2. Transverse and Longitudinal Phase Space Occupied by Beam Before (solid) and After (dashed) the Tapered Energy-Loss Foil.

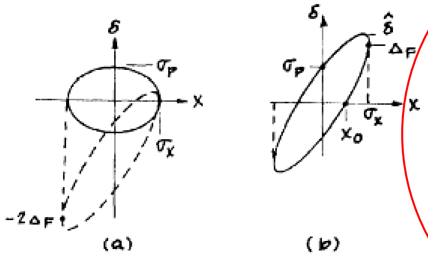


Figure 3. (a) x,δ Phase Space Occupied by Beam Before (solid) and After (Dashed) the Tapered Foil (b) After the Foil with Momentum Renormalized to the Center Momentum.

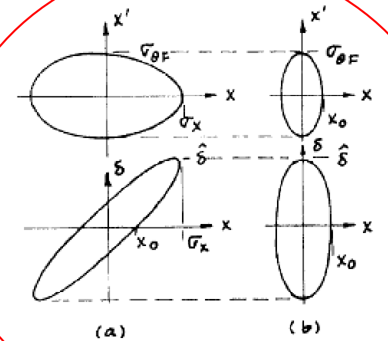


Figure 5. x,x' and x,δ Phase Space Occupied (a) At S_F, the Exit Side of the Tapered Foil and (b) At S_1, the Image Point with Zero Dispersion.

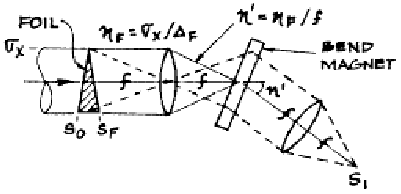


Figure 4. One-Dimensional Beam Transport System that Removes Dispersion and Has One-to-One Imaging of S_F at S_1. Focal length of each lens is f.

IEEE Transactions on Nuclear Science, Vol. NS-30, No. 4, August 1983

REDUCTION OF BEAM EMITTANCE BY A TAPERED-FOIL TECHNIQUE*

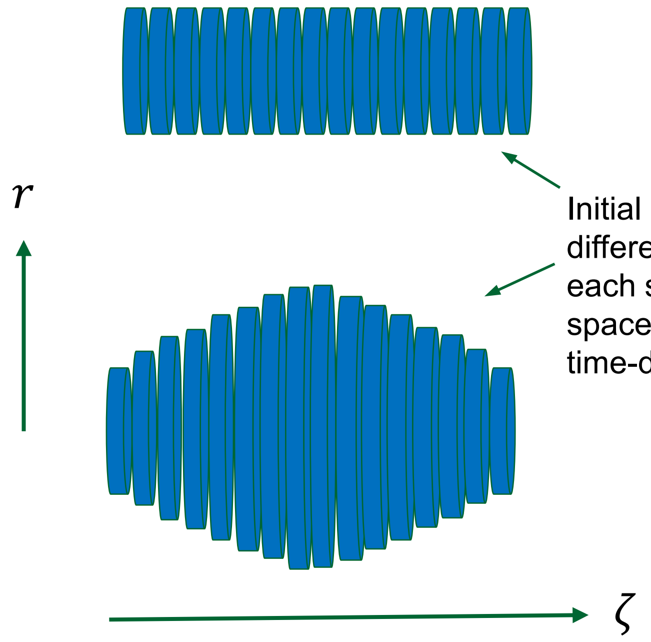
J.M. Peterson
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Peterson's idea was the first phase-space manipulation scheme I know about

First reported by Claud Bovet LBL-ERAN89, June 1970.

There are three scenarios where rms phase-space area can change

3. The beam has axial variations (slices) and while the emittance evolution of each one may be thermodynamic in nature, the ensemble average of all together doesn't have to be (different kind of force higher order in beam coordinates coupling longitudinal direction)



$$\ddot{r}_{slice} = \frac{F_{Space\ charge}(\zeta)}{\gamma m} = 2\frac{I(\zeta)/I_A}{r_{slice}\beta\gamma^3}c^2$$

We need to differentiate between "projected" rms quantities and "slice" rms quantities

Transverse oscillation period is the same for all slices

Serafini and Rosenzweig realized that the transverse plasma period only depends on external focusing:

$$r''_{slice} + K r_{slice} - \frac{K_{SC}(\zeta)}{r_{slice}} = 0$$

Slice equation of motion depends on position ζ within bunch

$$r_{slice,equil} = \sqrt{K_{SC}(\zeta)/K}$$

Slice equilibrium radius depends on position ζ within bunch

$$r_{slice} = r_{slice,equil} + \delta$$

Expand slice radius about the equilibrium radius to first order

$$\delta'' + 2K\delta = 0$$

The period of oscillation about the equilibrium radius is independent of position ζ within bunch

This is a remarkable result – the slices will oscillate at different amplitudes (leading to different angles in phase space) but will all stay in phase

(There are actually 2 emittance minimums – "Ferrario working point" freezes the emittance at the second minimum; C-X Wang)

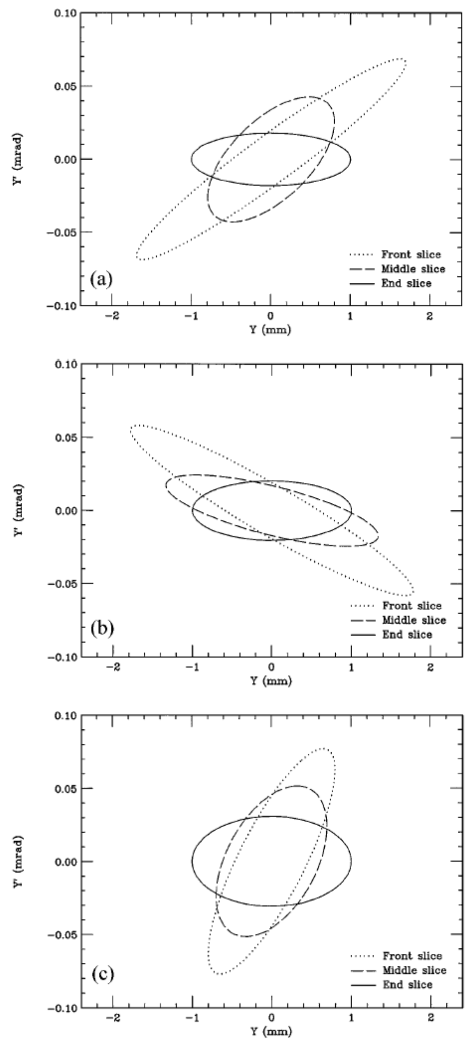
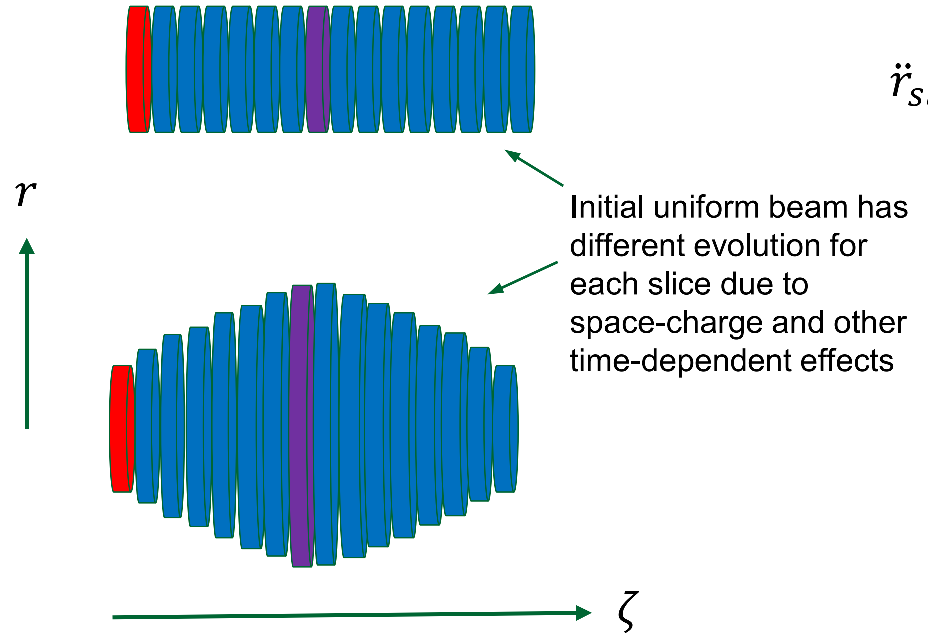


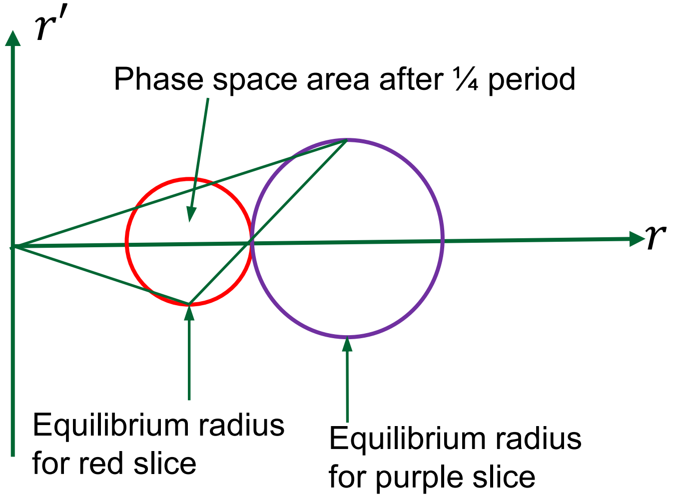
FIG. 4. Plots of the transverse phase space ellipses for each slice. The solenoid currents are (a) 102 A, (b) 106 A, and (c) 110 A.

Qui, Batchelor, Ben-Zvi, and Wang

Emittance compensation – 1/4 oscillation period (phase-space area has grown)

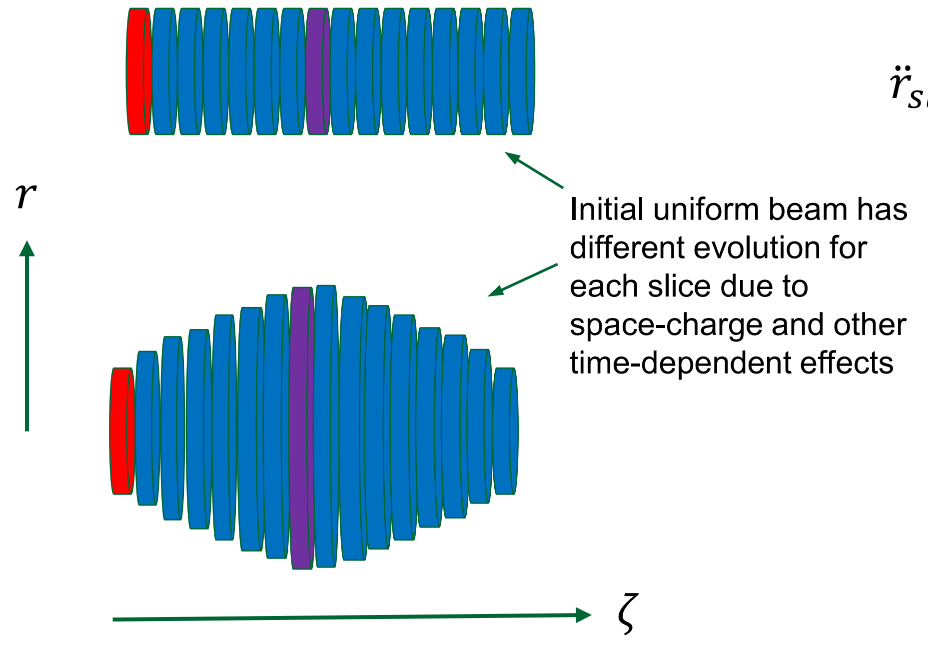


$$\ddot{r}_{slice} = \frac{F_{Space\ charge}(\zeta)}{\gamma m} = 2\frac{I(\zeta)/I_A}{r_{slice}\beta\gamma^3}c^2$$

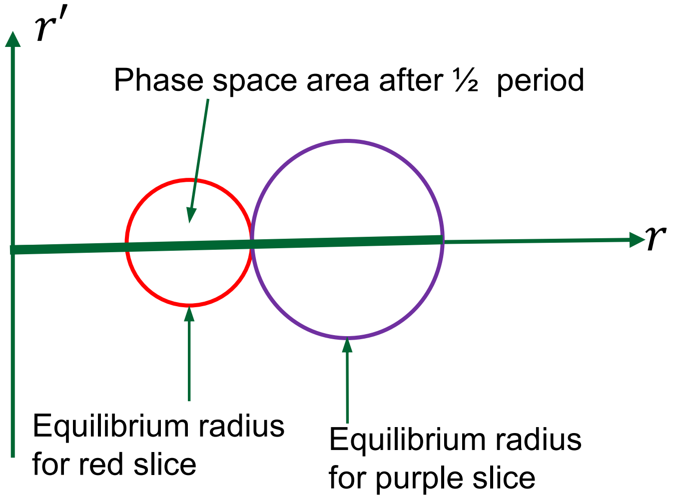


For this discussion – assume the beam expansion is limited by an axial magnetic field

Emittance compensation – ½ oscillation period (phase-space area has collapsed)

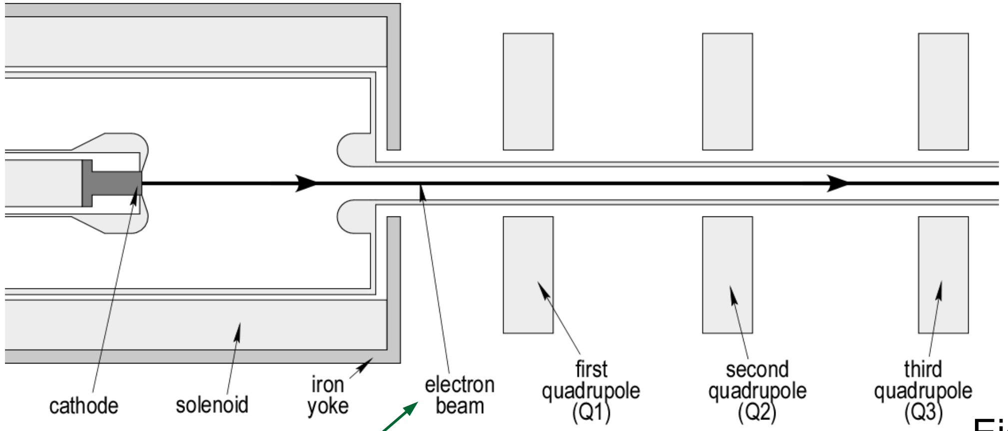


$$\ddot{r}_{slice} = \frac{F_{Space\ charge}(\zeta)}{\gamma m} = 2\frac{I(\zeta)/I_A}{r_{slice}\beta\gamma^3}c^2$$



For this discussion – assume the beam expansion is limited by an axial magnetic field

Flat beam transforms (FBTs) swap eigen-emittances



A "flat beam transform" is an example of manipulating the eigen-emittances by putting canonical angular momentum on the cathode (i.e., a magnetic field) and changing the unperturbed emittances ε_0 to one that is higher and one that is lower.

Eigen-emittances: $\varepsilon_{eig,-} = \dfrac{\varepsilon_0^2}{2\mathbf{L}}$

$\varepsilon_{eig,+} = 2\mathbf{L}$

Observed emittances: $\varepsilon_{beam} = \sqrt{\varepsilon_0^2 + \mathbf{L}^2}$

$$\mathbf{L} = \frac{e|B_{cath}|R_{cath}^2}{8\gamma\beta cm} = |a|\sigma_x^2 = \frac{1}{2}\left|\langle xy' - yx'\rangle\right|$$

$\mathbf{L}^2 >> \varepsilon_0^2$

Intrinsic emittance

FBT is protected from nonlinearities by symmetry and conservation of canonical angular momentum

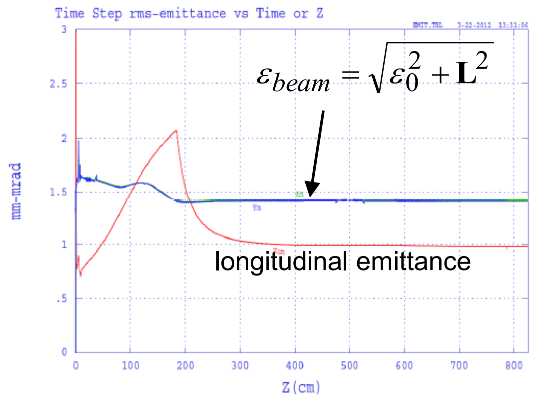
These are always zero

$$\sigma_{XY} = \begin{pmatrix} \langle xy\rangle & \langle xy'\rangle \\ \langle x'y\rangle & \langle x'y'\rangle \end{pmatrix}$$

Derbenev; Brinkmann, Derbenev, and Floettmann; and D. Edwards, et al.

FBT example with real physical components: SLAC LCLS injector

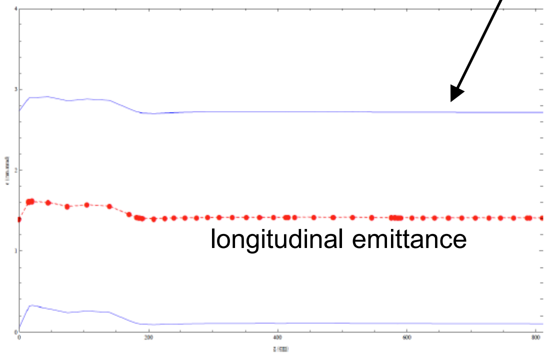
Emittances through RF gun



Emittances through FBT

$\varepsilon_{eig,+} = 2\mathbf{L}$

Eigen-emittances



$$\varepsilon_{eig,-} = \frac{\varepsilon_0^2}{2\mathbf{L}}$$

Wang and Duffy

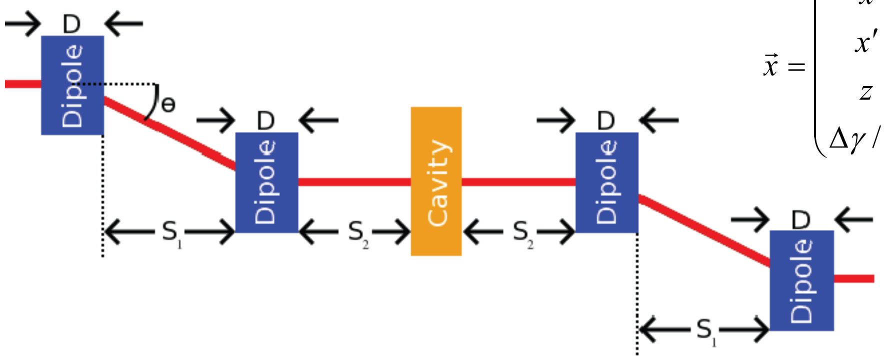
Example: 603 G on cathode ($\mathbf{L}$=1.3 μm; ε_0=0.4 μm), FBT at 100 MeV (past second linac section)

Recovers eigen-emittances of about 0.06 and 2.7 μm (250 pC)

Turns out to be a more optimized operating condition than the standard operating condition – final space-space volume is about 28% smaller – because the magnetic field through cathode suppresses nonlinear phase-space growth

Emittance EXchanger (EEX) swaps a transverse eigen-emittance with a longitudinal eigen-emittance

Cornacchia and Emma; Koeth;
Koeth and Fliller



$$\vec{x} = \begin{pmatrix} x \\ x' \\ z \\ \Delta\gamma/\gamma \end{pmatrix} \quad M_{dog} = \begin{pmatrix} 1 & L & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & \eta & 1 & \varepsilon \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad M_{kicker} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & k & 0 \\ 0 & 0 & 1 & 0 \\ k & 0 & 0 & 1 \end{pmatrix}$$

$$M_{dog} M_{kciker} M_{dog} = \begin{pmatrix} 0 & 0 & kL & \eta + \varepsilon kL \\ 0 & 0 & k & \varepsilon k \\ \varepsilon k & \eta + \varepsilon kL & 0 & 0 \\ k & kL & 0 & 0 \end{pmatrix}$$

This transfer matrix exchanges the x and z emittances:

with: $k\eta = -1$

$$T = M_{dog} M_{kciker} M_{dog} = \begin{pmatrix} 0 & B \\ A & 0 \end{pmatrix}$$

$$\sigma_{new} = \begin{pmatrix} 0 & A^T \\ B^T & 0 \end{pmatrix} \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_z \end{pmatrix} \begin{pmatrix} 0 & B \\ A & 0 \end{pmatrix} = \begin{pmatrix} A^T \sigma_z A & 0 \\ 0 & B^T \sigma_x B \end{pmatrix}$$

$$\sigma_{beam} = \begin{pmatrix} \sigma_x & 0 \\ 0 & \sigma_z \end{pmatrix}$$

It's worth noting an EEX switches "projected" emittances, "slice" emittances are lost

FBTs and EEXs might lead to customized phase spaces for some applications

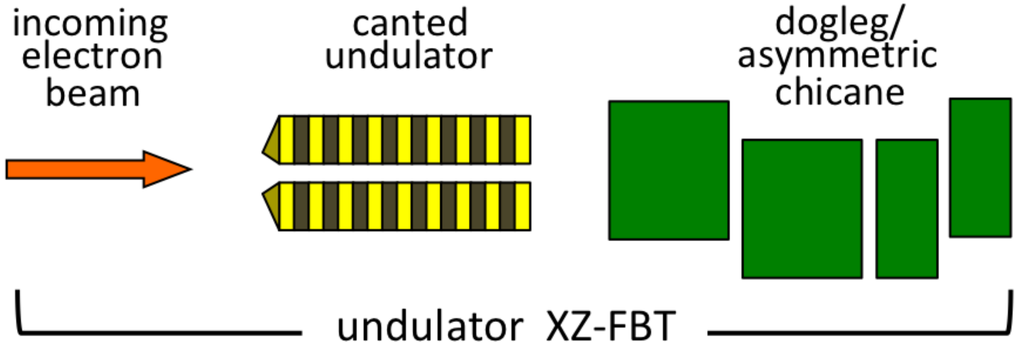
KJ Kim proposal: move excess transverse emittance into the longitudinal dimension which can tolerate a large eigen-emittance

1. Start with a super-short pancake of charge, emittances of 2.1/2.1/0.15 μm (for $\varepsilon_x/\varepsilon_y/\varepsilon_z$), all in a magnetized photoinjector (assume a total volume of about 0.7 μm^3 for the amount of charge in the bunch independent of split)
2. Use a FBT to adjust these numbers to 0.15/30/0.15 μm
3. Use an EEX to swap y and z and end up with 0.15/0.15/30 μm

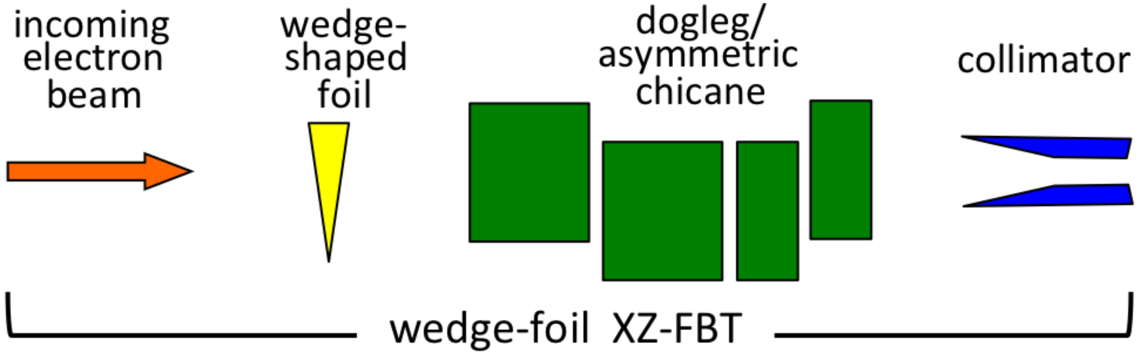
FBT: Possible application for high-frequency, high-power RF amplifiers (enable high-power sheet beams)

EEX: Actively pursued by Argonne (AWA) for increasing transformer ratio for wakefield accelerators and ASU for nano-bunching electron beam for compact XFEL concept

A nonsymplectic approach can enable a 2-stage partitioning architecture (with a FBT first stage)



Here, incoherent synchrotron radiation provides an x-energy correlation (ISR quantum fluctuations limit the amount of emittance transfer because of increased energy spread)



Here, energy loss in a wedge-shaped foil provides an x-energy correlation (energy straggling limits the amount of emittance transfer because of increased energy spread)

LCLS example: 0.27/0.19 μm with 250 pC; about factor of 6 reduction in transverse emittance product (overall phase space volume is increased by a factor of about 3)

Carlsten and Bishofberger

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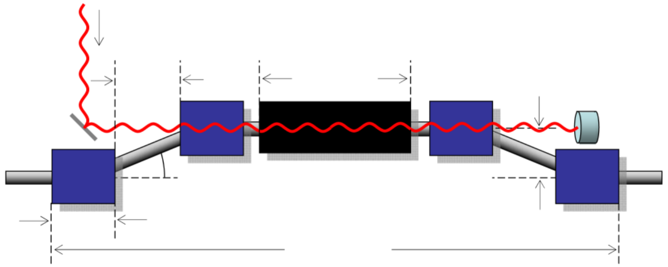
Summary

- The understanding of emittance and phase-space partitioning and manipulation has matured greatly over the last three decades
  - Single-stage, multi-stage, symplectic, non-symplectic schemes are all well understood using concept of eigen-emittances
- Emittance compensation is still the only widely used scheme to manipulate the beam's transverse phase space
  - Emittance compensation is unique in that it uses the beam's own nonlinear forces to compensate for themselves
  - Emittance compensation has been essential in realizing the potential of (most) X-ray FELs
  - EEX manipulations may have important future applications
- Comparable methods (that we didn't talk about) that manipulate the beam's longitudinal phase space are commonly used and some are now integral parts of X-ray FEL designs

Lasers are used to manipulate the beam's longitudinal phase space



Several schemes have been developed:

- Laser heater – gives the beam in an XFEL extra longitudinal energy spread to suppress the microbunch instability (Huang)
- HGHG – high gain harmonic generation – to seed shorter wavelengths for XFELs (Yu)
- EEGH – echo enabled harmonic generation – to seed shorter wavelengths for XFELs (Stupakov)

These approaches don't modify a beam's transverse phase space at all, but improve XFEL performance in other ways

REVIEWS OF MODERN PHYSICS, VOLUME 86, JULY–SEPTEMBER 2014

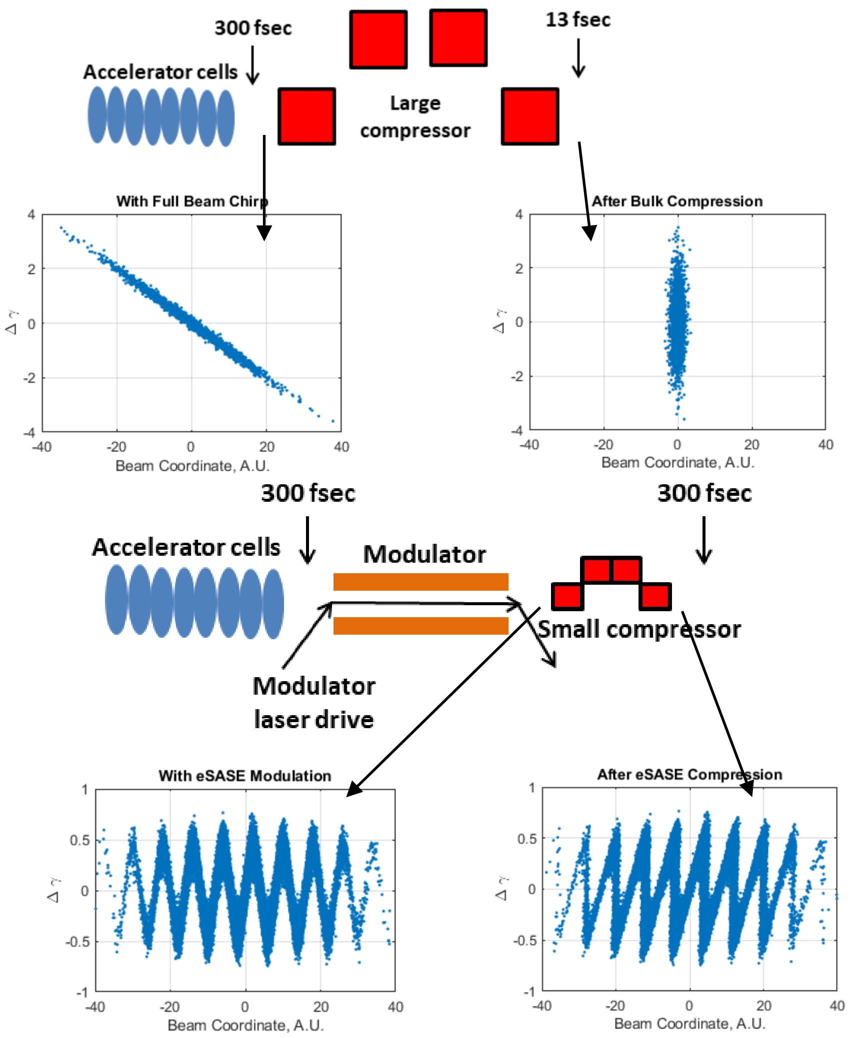
Beam by design: Laser manipulation of electrons in modern accelerators

Erik Hemsing,* Gennady Stupakov,† and Dao Xiang‡
SLAC National Accelerator Laboratory, Menlo Park, California 94025, USA

Alexander Zholents§
Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439, USA

(published 14 July 2014)

LABC version of e-SASE



Anisimov, Carlsten, and Marksteiner

- We're developing an alternative XFEL accelerator compression architecture called laser assisted bunch compression (LABC).
- In our design, we use LABC to eliminate the second bunch compressor (BC). Our final current is the same current as before, 3 kA.
- Lack of second BC reduces coherent synchrotron radiation, eliminates microbunch instability, and suppresses undulator resistive wall wake, all major degradation mechanisms for XFELs

inspired by eSASE - Zholents

Abstract

Review the physics of phase space manipulations in general and emittance compensation specifically. All the key elements of several schemes will be discussed to overview the broad topic of transverse phase space manipulations, focusing on the underlying beam physics. The special schemes of non-symplectic manipulations will be discussed and the impact of the techniques for developed for electrons will be discussed for broader applications as for ion beams.