LONGITUDINAL BEAM DYNAMICS IN ARRAY OF EQUIDISTANT MULTICELL CAVITIES

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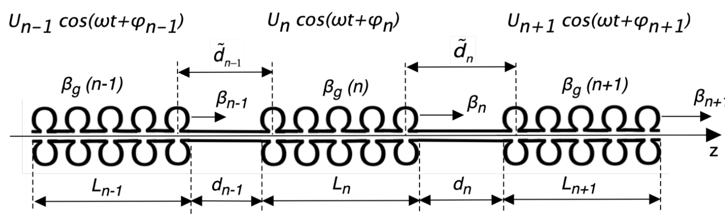
Abstract

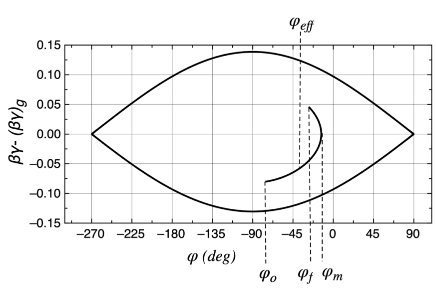
Linear accelerators containing the sequence of independently phase cavities with constant geometrical velocity along each cavity are widely used in practice. The chain of cavities with identical cell length is utilized within a certain beam velocity range, with subsequent transformation to the next chain with higher cavity velocity. Design and analysis of beam dynamics in this type of accelerators are usually performed using numerical simulations. In the present paper, we provide an analytical treatment of beam dynamics in such linacs based on Hamiltonian formalism. We begin our analysis with an examination of beam dynamics in an equivalent traveling wave of a single cavity, propagating within accelerating section with constant phase velocity. We then consider beam dynamics in arrays of cavities, utilizing an effective traveling wave propagating along with the whole accelerator with the velocity of synchronous (reference) particle. The analysis concluded with the determination of the matched beam conditions. Finally, we present a beam dynamics study in 805 MHz Coupled Cavity Linac of the LANSCE accelerator facility.

Accelerating Structure of Independently Phased Cavities

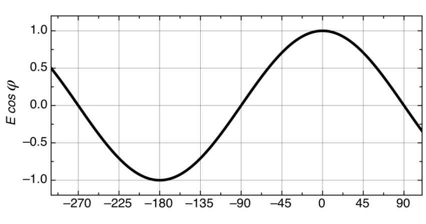


Accelerating structure of independently phased cavities: L_i is the cavity length, d_i is the distance between cavities, $\tilde{d}_i$ is the distance between centers of last and first cells of adjacent cavities, β_g is the geometrical velocity of cavity, β_i is the velocity of reference particle, U_i is the cavity voltage, and φ_i is the cavity RF phase.

Dynamics in a Single Cavity



Phase space trajectory of a particle in an RF structure with equidistant cells: φ_o is the initial phase, φ_f is the final phase, φ_{eff} is the effective phase, φ_m is the phase, at which the particle velocity is equal to geometrical velocity of cavity.



Equivalent traveling wave with amplitude E

Set of equations for particle dynamics in traveling wave

$$\frac{d\varphi}{dz}=\frac{2\pi}{\lambda}\left(\frac{1}{\beta}-\frac{1}{\beta_g}\right)$$

$$\frac{d\gamma}{dz}=\frac{qE}{mc^2}\cos\varphi$$

Hamiltonian of particle motion in traveling wave

$$H=\frac{2\pi}{\lambda}\left(\sqrt{\gamma^2-1}-\frac{\gamma}{\beta_g}\right)-\frac{qE}{mc^2}\sin\varphi$$

Energy gain in accelerating cavity

$$\gamma_f=\gamma_g\pm\sqrt{\frac{qE\lambda(\beta_g\gamma_g)^3}{\pi mc^2}}\sqrt{\sin\varphi_m-\sin\varphi_f}$$

Effective phase of particle in RF field of cavity

$$\cos\varphi_{eff}=\frac{mc^2(\gamma_f-\gamma_o)}{qE_oT(\beta)L_n}$$

Dimensionless time of particle acceleration in cavity

$$\Delta(\omega t)=\sqrt{\pi\beta_g\gamma_g^3\left(\frac{mc^2}{qE\lambda}\right)}\int_{\varphi_o}^{\varphi_f}\frac{d\varphi}{\sqrt{\sin\varphi_m-\sin\varphi}}$$

Particle phase slippage in RF cavity as a function of time of acceleration

$$\Delta(\omega t)\approx\sqrt{\frac{2\pi\beta_g\gamma_g^3mc^2}{qE\lambda|\sin\varphi_m|}}\{\arcsin[1+(\varphi_m-\varphi_f)\tan\varphi_m]-\arcsin[1+(\varphi_m-\varphi_o)\tan\varphi_m]\}$$

Dynamics in Array of Cavities

Velocity of reference particle

$$\beta_n=\frac{2\pi\tilde{d}_n}{\lambda(\varphi_n-\varphi_{n+1})}$$

Difference in RF phases in cavities

$$\varphi_n-\varphi_{n+1}=2\pi m-\Delta\varphi_n$$

Velocity of reference particle within the cavity

$$\beta_{s\_n}=\frac{\beta_{n-1}+\beta_n}{2}$$

Amplitude of equivalent traveling wave in linac

$$\bar{E}=E_{o\_n}T_n(\beta_{s\_n})\frac{L_n}{L_n+0.5(d_n+d_{n+1})}$$

Synchronous phase of reference particle

$$\cos\varphi_{s\_n}=\frac{mc^2}{qE_{o\_n}T_nL_n}\beta_{s\_n}\gamma_{s\_n}^3(\beta_n-\beta_{n-1})$$

Dimensionless frequency of longitudinal oscillations around reference particle

$$\frac{\Omega}{\omega}=\sqrt{\frac{q\bar{E}\lambda}{mc^2}\frac{|\sin\varphi_s|}{2\pi\beta_s\gamma_s^3}}$$

Normalized longitudinal acceptance of linac

$$\varepsilon_{acc}=\frac{2}{\pi}\lambda\beta^2\gamma^3\left(\frac{\Omega}{\omega}\right)\left(1-\frac{\varphi_s}{\tan\varphi_s}\right)$$

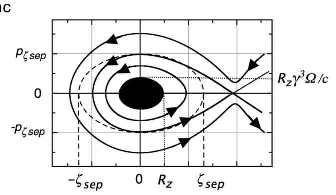
Matched beam parameters

$$(R_z)_{matched}=\sqrt{\frac{\varepsilon_z\lambda}{2\pi\gamma^3}\left(\frac{\omega}{\Omega}\right)}$$

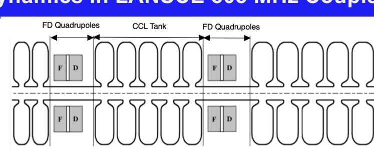
$$\left(\frac{p_\zeta}{mc}\right)_{matched}=\sqrt{2\pi\gamma^3\frac{\varepsilon_z}{\lambda}\left(\frac{\Omega}{\omega}\right)}$$

Beam Dynamics in LANSCE 805 MHz Coupled Cavity Linac

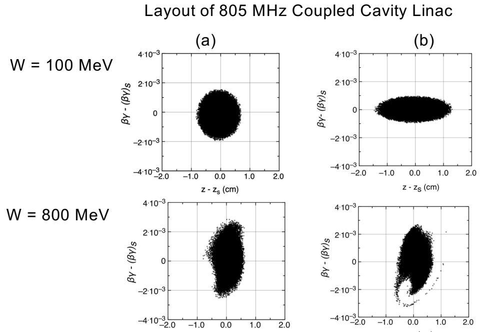
Accelerating tanks of 805 MHz Coupled Cavity Linac



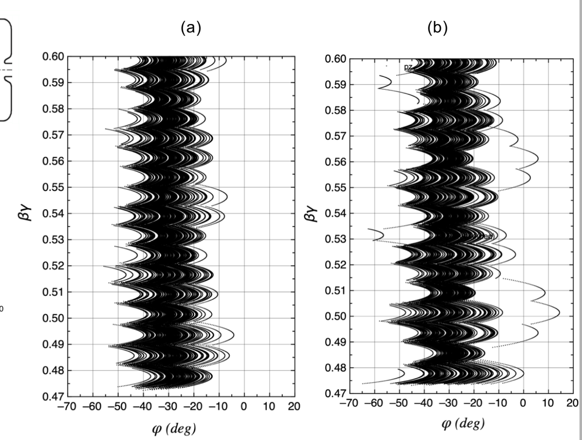
Elliptical approximation of separatrix and normalized longitudinal emittance of the matched beam.



Layout of 805 MHz Coupled Cavity Linac



Longitudinal phase-space distributions of the beam in 805 MHz linac: (a) matched beam, (b) mismatched beam.



Longitudinal beam dynamics in 805 MHz linac: (a) matched beam, (b) mismatched beam.