Science and Technology Facilities Council

Lancaster University

The Cockcroft Institute of Accelerator Science and Technology

M. Southerby[1,2], R. Apsimon[1,2]
1 Lancaster University, Bailrigg, Lancaster, LA1 4YR, UK
2 Cockcroft Institute, Daresbury Laboratory, Warrington, WA4 4AD, UK

LINAC2022 LIVERPOOL

TUPORI19

Beam Dynamics Framework Incorporating Acceleration Used to Define the Minimum Aperture of RF Cavity For FODO-like Focusing Scheme for Proton Radiotherapy Linac.

Transfer Map for an RF Cavity

$$M_{cav} = \begin{pmatrix} 1 & \dfrac{l_{cav}\sqrt{\gamma_{r0}^2-1}}{\gamma_{r1}-\gamma_{r0}}\ln\left(\dfrac{\sqrt{\gamma_{r1}^2-1}+\gamma_{r1}}{\sqrt{\gamma_{r0}^2-1}+\gamma_{r0}}\right) \\ 0 & \sqrt{\dfrac{\gamma_{r0}^2-1}{\gamma_{r1}^2-1}} \end{pmatrix}$$

- The M_{22} element can be equivalently written $\frac{\beta_{r0}\gamma_{r0}}{\beta_{r1}\gamma_{r1}}$ and is the Jacobian of the transformation through a cavity [1, 2].
- Liouville's theorem does not hold and areas in transverse phase space scale with $\beta_r\gamma_r$ as expected.

Twiss Parameter Matrix

$$\begin{pmatrix}\beta_{x1}\\ \alpha_{x1}\\ \gamma_{x1}\end{pmatrix} = \frac{\gamma_{r1}\beta_{r1}}{\gamma_{r0}\beta_{r0}}\begin{pmatrix} M_{11}^2 & -2M_{11}M_{12} & M_{12}^2 \\ -M_{11}M_{21} & M_{11}M_{22}+M_{12}M_{21} & -M_{12}M_{22} \\ M_{21}^2 & -2M_{21}M_{22} & M_{22}^2 \end{pmatrix}\begin{pmatrix}\beta_{x0}\\ \alpha_{x0}\\ \gamma_{x0}\end{pmatrix}$$

- Beta function is no longer periodic but increases with increasing longitudinal momentum.

Betatron Phase Advance

$$M_x = \sqrt{\frac{\beta_{r0}\gamma_{r0}}{\beta_{r1}\gamma_{r1}}}\begin{pmatrix} \sqrt{\dfrac{\beta_{x1}}{\beta_{x0}}}\left(\cos(\mu_x)+\alpha_{x0}\sin(\mu_x)\right) & \sqrt{\beta_{x0}\beta_{x1}}\sin(\mu_x) \\ \dfrac{(\alpha_{x0}-\alpha_{x1})\cos(\mu_x)-(1+\alpha_{x0}\alpha_{x1})\sin(\mu_x)}{\sqrt{\beta_{x0}\beta_{x1}}} & \sqrt{\dfrac{\beta_{x0}}{\beta_{x1}}}\left(\cos(\mu_x)-\alpha_{x1}\sin(\mu_x)\right) \end{pmatrix}$$

- Determinant of phase space map is a function of longitudinal momentum.

FODO-like Beam Line

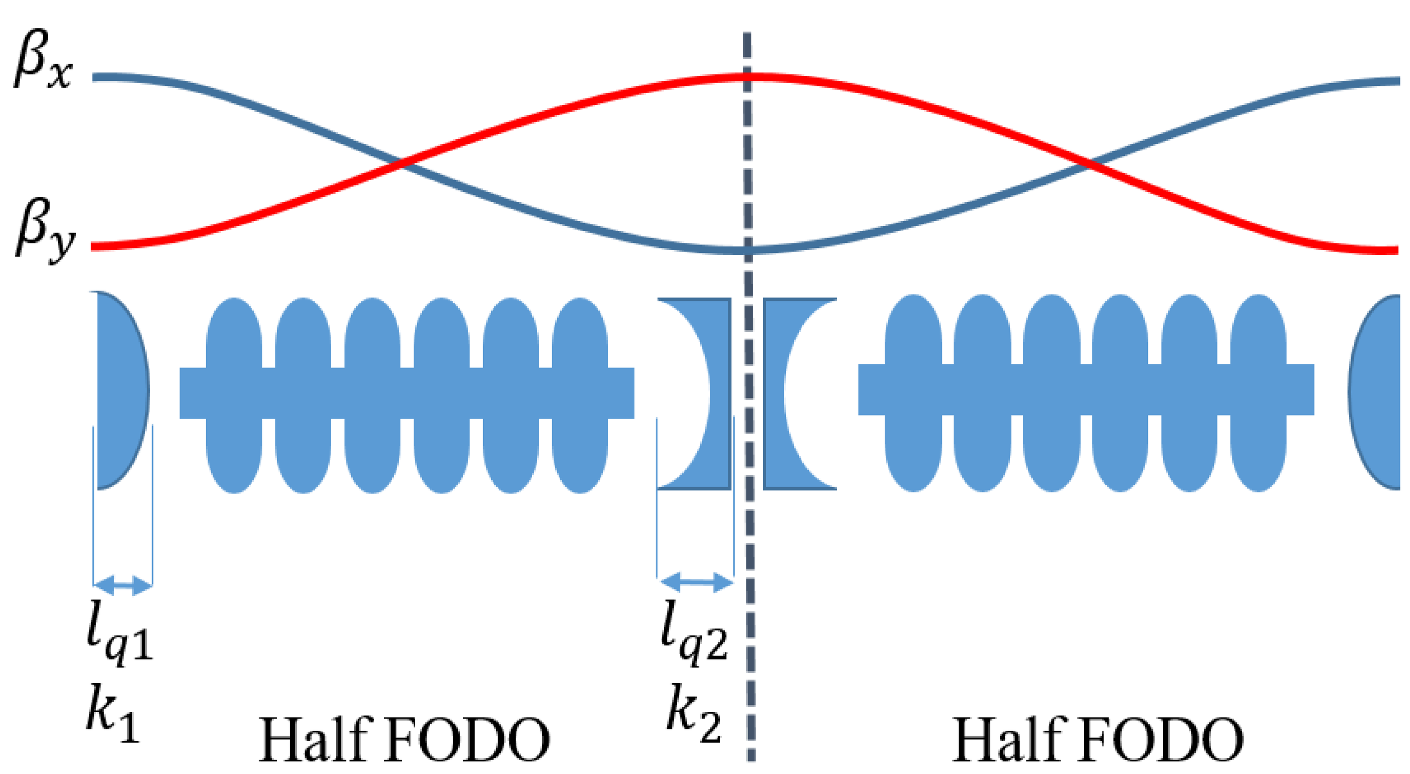

Fig. 1. Schematic showing how a FODO-like lattice is split into two half FODO elements.

The method requires solving for the value of k_1 that minimises the beam size at the cavity entrance. Requiring symmetry in both transverse planes forces constraints that allow the calculation of all FODO parameters.

$$k_{2n} = k_{2n+1} = k_1\frac{\gamma_{r0}^2-1}{\gamma_{rn}^2-1}, \qquad l_{qn+1} = l_{q1}\sqrt{\frac{\gamma_{rn}^2-1}{\gamma_{r0}^2-1}}$$

The cavity length is also determined and subsequent cavity lengths increase with a term very close to $\beta_r\gamma_r$. Therefore, cavity lengths require corrections to keep a beam synchronous.

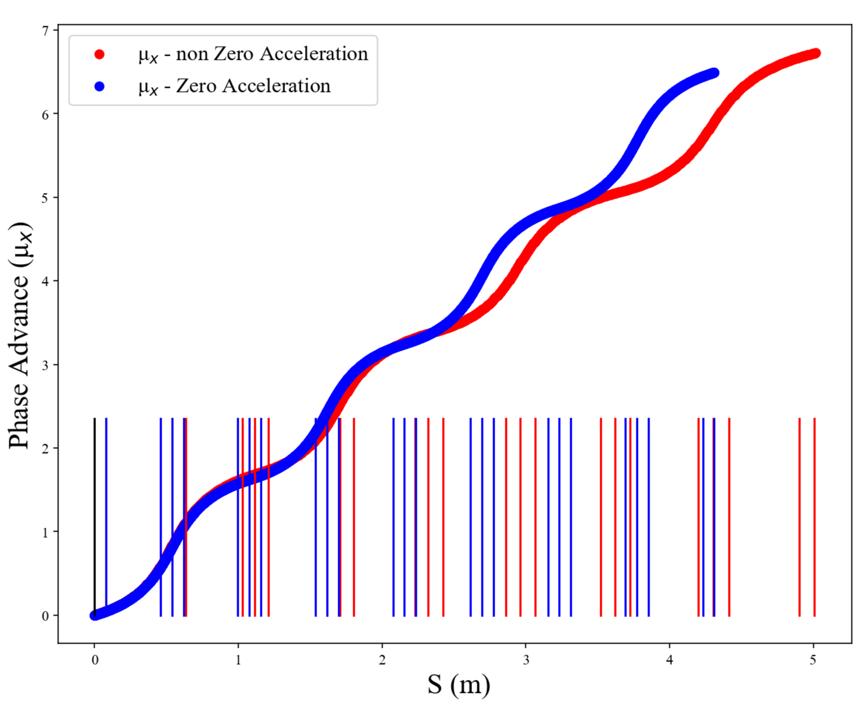

Fig. 2. Phase advance of a FODO-like lattice with and without acceleration.

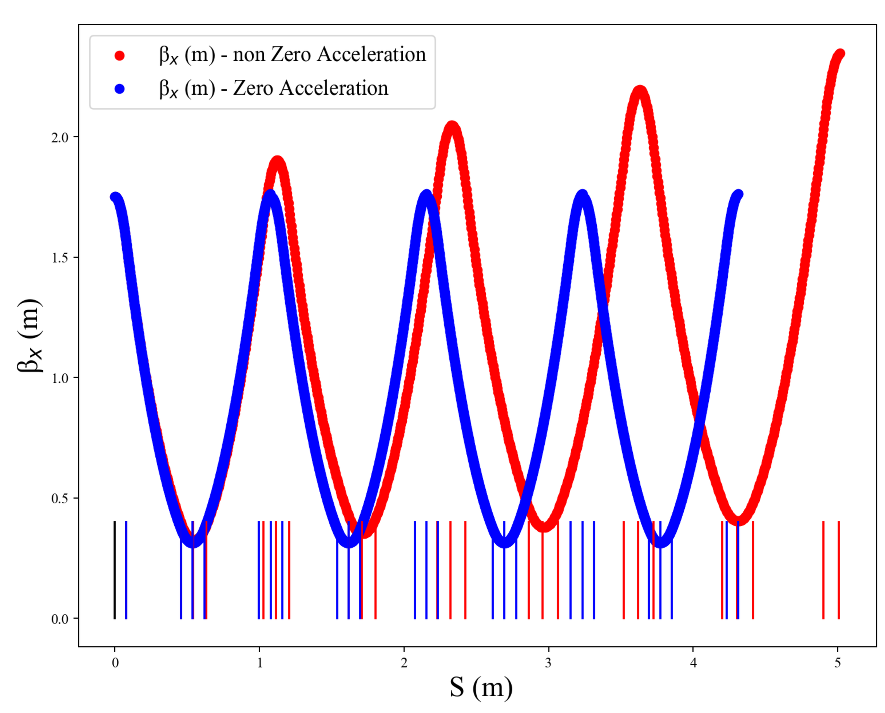

Fig. 3. Beta function for FODO-like lattice with and without acceleration with same initial beta.

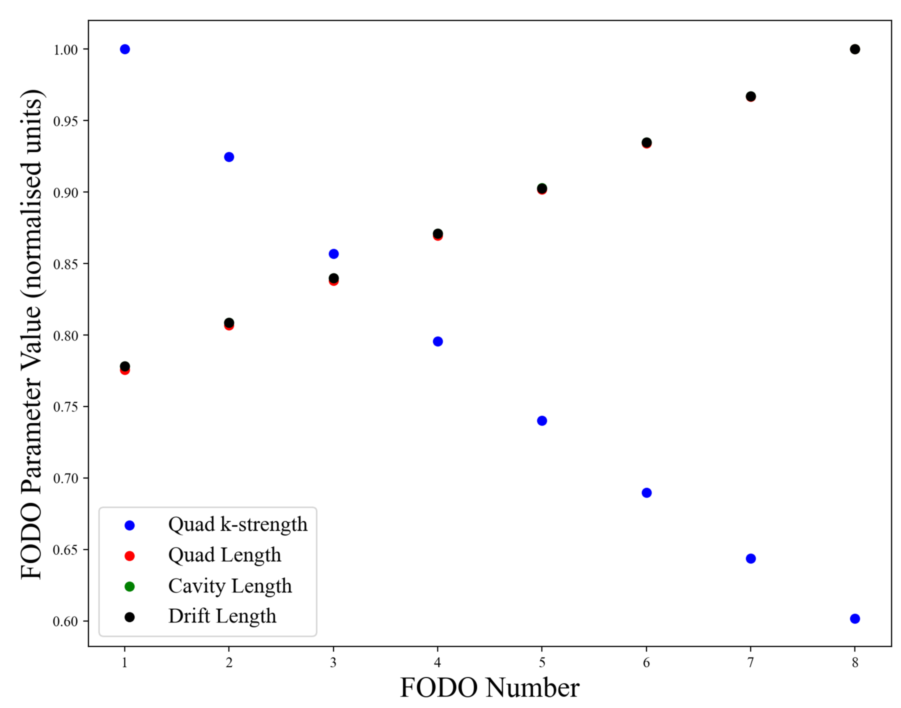

Fig. 4. The FODO parameters are shown as a function of half FODO number.

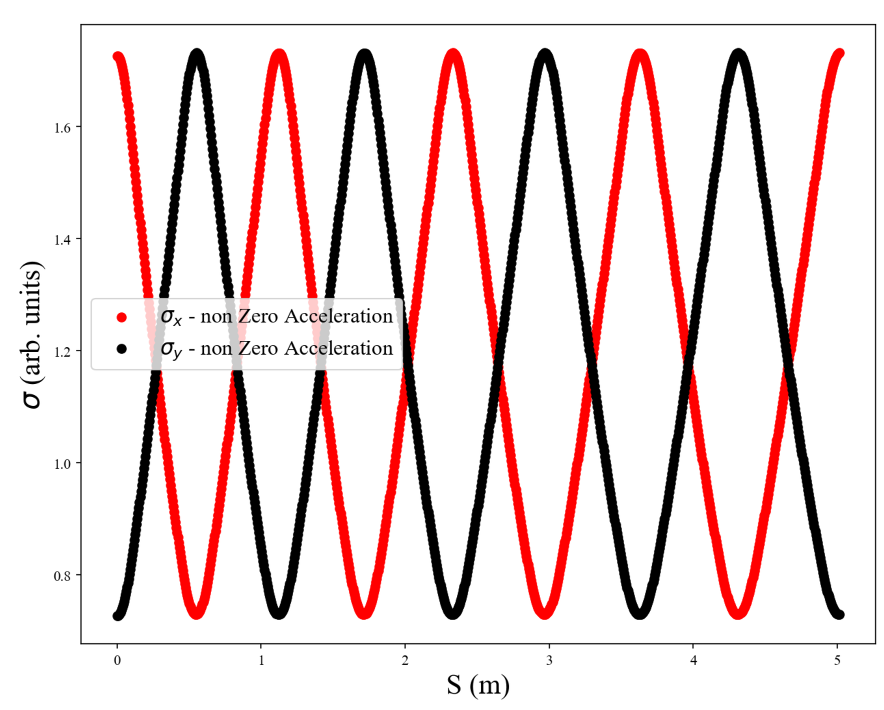

Fig. 6. Beam size for a FODO-like lattice factoring in acceleration.

References

[1] Wolski, A., 2014. *Beam dynamics in high energy particle accelerators*. World Scientific.
[2] Wolski, A., 2018. *Introduction to Beam Dynamics in High-Energy Electron Storage Rings*. Morgan & Claypool Publishers.