derived systematic uncertainties that have dominated previous experiments, in this experiment, a beam obtained by

accelerating low-emittance muons produced by laser ioniza-

tion of muonium is required. In addition, to reduce decay

loss, the muons must be accelerated in a time sufficiently

shorter than the muon lifetime of $2.2 \,\mu$ s. The linear acceler-

ator was our choice because it is efficient in this respect. For

highly efficient acceleration, muons are accelerated from

thermal energy to relativistic energy using four RF struc-

tures, depending on the beam velocity. Figure 1 shows the

configuration of the muon linac. The muon linac starts with

an ultra-slow muon source which generates a muons by the

laser ionization of thermal muonium with an extremely small

momentum of 3 keV/c (kinetic energy W=25 meV). The gen-

erated muons are accelerated to 5.6 keV by electrostatic field and injected to a radio frequency quadrupole (RFQ). The

acceleration is performed up to 0.34 MeV by a 324 MHz RFQ. Then, the energy of the muon beam is boosted to

4.5 MeV with a 324 MHz interdigital H-type drift tube linac

(IH-DTL). Following the IH-DTL, a 1296 MHz disk-and-

washer coupled cell linac (DAW CCL) structures are used

to accelerate to 40 MeV. Finally, the muons are accelerated

from 40 MeV to 212 MeV by using a 2592 MHz disk-loaded

traveling wave structure (DLS). Details of the linac design

Considering the injection of accelerated muon beams into

the storage ring, beam parameters such as emittance and

momentum spread at the exit of linac are important factors,

since they directly affect the efficiency of injection into the

storage ring. The exact muon beam parameters required for

the experiment are currently being investigated in detail, but

the results of the latest injection simulation studies suggest

that a beam quality at the end of linac with a transverse

rms emittance of ~ 0.3 π mm mrad and a rms momentum

spread of $\sim 0.04\%$ are desirable. Hereafter, in this study,

these values will be used as reference values to evaluate

beam parameters. The muon spin properties are also very

important because they can be a source of systematic er-

rors in this experiment. Therefore, it is necessary to fully

understand how properties such as spin polarization and spin-momentum correlation, which is the correlation be-

are described in this separate paper [5].

END-TO-END SIMULATIONS AND ERROR STUDIES OF THE J-PARC MUON LINAC

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Abstract

A muon linac is under development for future muon g-2/EDM experiments at J-PARC. The linac provides a 212 MeV muon beam to an MRI-type compact storage ring. After the initial acceleration using the electrostatic field created by mesh and cylindrical electrodes, the muons are accelerated using four types of radio-frequency accelerators. To validate the linac design as a whole, end-to-end simulations were performed using General Particle Tracer. In addition, error studies is ongoing to investigate the effects on beam and spin dynamics of various errors in the accelerator components and input beam distribution. This paper describes the preliminary results of the end-to-end simulations and error studies.

INTRODUCTION

Muon g-2 is arguably one of the most important observables in modern particle physics. The long-standing existence of anomalies of more than three standard deviations between experimental values of muon g-2 measured by Brookhaven National Laboratory (BNL) [1] and the Standard Model (SM) predictions [2] has attracted many physicists, who believe that they suggest the existence of new physics beyond the SM. Recently, Fermilab's experimental group improved BNL's experimental equipment and new experiment now running in a similar method. The first result of the group was consistent with the previous experiment, and the discrepancy between the average of the two experiments and the SM prediction was updated to 4.2σ [3]. As a result, there is even more hope for the discovery of new physics. However, since the two experiments were performed using the same experimental method, it is very important to confirm the discrepancy by measuring with another new technique. Therefore, to verify the discrepancy, an experiment using a completely different method from the two previous experiments is being planned at the Japan Proton Accelerator Research Complex (J-PARC).

The J-PARC experiment aims to measure the muon g-2 with a precision of 0.1 ppm [4]. To reduce the muon beam-

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Figure 1: The configuration of muon linac.

tween the momentum of each of the rotation angles in the ZX plane (θ_{zx}) and in the ZY plane (θ_{zy}) with respect to the z-axis, are affected by acceleration with a linac. To this end, an end-to-end simulation using the General Particle Tracer (GPT) [6] are being developed to fully investigate the effects of acceleration by linac on the emittance, momentum spread, and spin properties of muon beams. With the current design, under error-free conditions (referred to as the ideal machine), a multi-particle simulation from the RFQ to the end of the linac was performed using GPT to investigate the aforementioned beam and spin characteristics and the following results was obtained. Figure 2 shows the evolution of the transverse rms emittance in linac. There is no significant increase during acceleration, and at the exit of linac, the horizontal and vertical normalized rms emittance are 0.278 π mm mrad and 0.219 π mm mrad, respectively, which are below the reference values. The rms momentum spread of 0.036% is also below the reference value. The spin depolarization is found to be 1.0×10^{-5} and has no effect on the statistical accuracy of the experiment. Moreover, there is no spin-momentum correlations in ideal machine case (absolute value of correlation coefficient is less than 0.02). These properties are greatly affected by the distribution characteristics of the RFQ input, i.e., laser ionization and electrostatic acceleration conditions, although it should be noted that this study focuses only on the accelerator components from the RFO to the DLS.



Figure 2: Transverse normalized rms emittance evolution in the ideal machine. The traces show horizontal (red) and vertical (blue) emittance.

ERROR STUDIES

Even if it works well in the ideal machine case, in real situation, various errors can cause beam characteristics to

deteriorate. Therefore, as the next step, the effects of misalignment such as transverse offset and rotation errors of the quadrupole magnets, and gradient errors of quadrupole magnets, and RF amplitude and phase fluctuations of the cavity on beam characteristics were investigated. For each case, the amplitude of the error was given, the maximum value of the error was set in five steps (20, 40, 60, 80, and 100%) relative to the amplitude, and the value of the error given to each element, quadrupole magnet or RF source, was randomly generated uniformly within the set maximum value. Hundreds of runs with a beam distribution of approximately 2×10^4 macro-particles were simulated for each step. The output beam parameters were statistically averaged over 100 runs. The effect of the errors on the ideal machine was evaluated by examining their impact on the normalized rms emittance, rms momentum spread, and polarization, at exit of the linac. The spin-momentum correlation is also being considered. Table 1 lists the elements and the amplitude of the errors examined in this studies.

 Table 1: The Elements and the Amplitude of the Errors

 Examined in This Studies

Error type	Description	Amplitude
1	Magnet Δx (mm)	0.1
2	Magnet Δy (mm)	0.1
3	Magnet $\Delta \theta_z$ (mrad)	5
4	Magnet gradient (%)	0.5
5	Cavity RF phase (deg)	1
6	Cavity RF amplitude (%)	1

RESULTS & DISCUSSION

While there are still aspects of the error study that need to be considered, this paper discusses preliminary results. Figure 3 shows normalized rms emittance ratio defined as $\varepsilon_e/\varepsilon_i$ between the error case ε_e and the ideal machine case ε_i at end of the linac at maximum amplitude (100%) for each error case. To achieve the reference value of transverse normalized rms emittance, $\varepsilon_e/\varepsilon_i$ must be less than 1.08 and 1.37 for x-plane and y-plane, respectively.

Figure 4 shows the ratio of the error case to the ideal machine case in the rms momentum spread at the end of linac at maximum amplitude (100%) for each error case. As can be seen in Figure 3, magnet alignment errors tend to strongly affect transverse emittance, while RF errors tend to strongly affect longitudinal emittance. Deviating from

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Figure 3: The normalized rms emittance ratio of the different single errors with 100% amplitude (preliminary results).

this trend, the error type 6 shows an increase in vertical (y) emittance. In addition, the emittance increase rate tends to be larger for y than for x. In IH -DTL, not only the longitudinal electric field but also the vertical electric field causes a shift in the vertical phase space distribution at the IH-DTL exit, and steering magnets are placed in MEBT2 to compensate for this. However, in this error study, the steering magnets were set to nominal values, and changes in RF amplitude may cause changes in the vertical phase space distribution, and the correction may not be optimal. It is possible that this trend was observed for these reasons, but a detailed study of the changes in beam parameters during the linac acceleration is required to determine this. Cases with 100% of amplitude exceeding the reference value of emittance are error types 1, 3, and 8 for which the tolerances are 60, 60, and 80% of the maximum error value.



Figure 4: The ratio of the error case to the ideal machine case in the rms momentum spread at the different single errors with 100% amplitude (preliminary results).

To achieve the reference value of rms momentum spread, the ratio of the error case to the ideal machine case should be less than 1.11, which corresponds to a tolerance of 0.2% of the cavity RF amplitude. It may be necessary to introduce a debuncher to increase tolerance. The maximum spin depolarization was found to be 3.9×10^{-5} with the error type 2, but has no effect on the statistical accuracy of the experiment. In the ideal machine case, no correlation between spin direction and momentum was observed, but it was found that a correlation can occur when a magnet error occurs.

CONCLUSION

A muon linac is under development for future experiments at J-PARC to measure muon g-2. The linac beam dynamics was studied with the reference values such as transverse rms emittance of ~ 0.3 π mm mrad and momentum spread of $\sim 0.04\%$. An end-to-end simulation was developed using GPT to investigate in detail how the muon beam parameters are affected by the acceleration by the linac. In the ideal case without errors, the reference values are expected to be achieved, and a detailed study including errors is in progress. The preliminary tolerances were examined based on the rate of increase in the normalized rms emittance and the rms momentum spread at the end of the linac. This study focused only on the beam parameters at the exit of the linac. However, it may be possible to increase tolerance by examining the changes in the beam parameters at each section of the linac to identify the causes of the beam quality degradation in detail and implementing appropriate countermeasures. The correlation between spin and momentum, which can be a source of systematic errors in the experiment, was also found to be possible due to errors. We plan to conduct a more detailed beam and spin dynamics study including these factors.

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