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CONSERVATION OF QUALITY FACTOR FOR SUPERCONDUCTING CAVITY AND HEARTBEAT UNDER RELATIVISTIC MOTION

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Abstract

The conservation of quality factor under relativistic motion is applied to the superconducting cavity as well as the heartbeat of mammal. The quality factor of the superconducting cavity is conserved under relativistic motion. The frequency of the cavity decreases and the decay time increases as the velocity and acceleration are increased. The quality factor of the superconducting cavity is comparable with the total heartbeat of the mammal. The quality factor for the heartbeat of the mammal representing the total number of heartbeat is also conserved under relativistic motion. Therefore, the heart rate is inversely proportional to the life expectancy under relativistic motion.

INTRODUCTION

Superconducting niobium cavity was developed well and was shown to have very high quality factor. Thermionic emission, field emission, and generalized electron emission were studied [1-5]. The superconducting cavities of the quarter-wave resonator (QWR) and the half-wave resonator (HWR) for RAON accelerator were developed [6-9]. Generalized Doppler effect was investigated for arbitrary velocity and acceleration [10]. The heart rate and life expectancy of mammal were studied [11, 12]. In this research we show the quality factor conservation for the superconducting cavity and the heartbeat of the mammal. The frequency-energy relation, Doppler effect, and energy conservation are presented. The frequency and the decay time of the superconducting cavity in reference frame is shown as a function of velocity and acceleration. The heart rate and lifespan of the mammal in reference frame are also calculated as a function of velocity and acceleration.

FREQUENCY-ENERGY RELATION

We postulate the conservation of frequency-energy relation. The product of frequency and energy for a particle is always conserved as

$$E(i)f(i) = \text{constant}\,, \tag{1}$$

where E is the energy and f is the frequency. The frequency-energy relation can also be expressed as

$$E(x)f(x) = E(v)f(v) = E(a)f(a)\,, \tag{2}$$

where x is the position, v is the velocity, and a is the acceleration.

DOPPLER EFFECT

Frequency shift for constant velocity and acceleration can be calculated from Doppler effect.

The frequency for constant velocity in reference frame is shifted to

$$f(v) = f_o\sqrt{1-(v/c)^2}\,, \tag{3}$$

where c is the speed of light.

The frequency for constant acceleration in reference frame is shifted to

$$f(a) = f_o\sqrt{1-(2ax/c^2)}\,. \tag{4}$$

The frequency decreases as the velocity and acceleration are increased.

ENERGY FOR MOTION

From the frequency-energy relation and Doppler effect, the energy of a body can be calculated. The frequency-energy relation for constant velocity is

$$E(v) = E(0)\frac{f(0)}{f(v)}\,. \tag{5}$$

The energy for constant velocity can be expressed as

$$E(v) = \frac{m_o c^2}{\sqrt{1-(v/c)^2}}\,, \tag{6}$$

where m_o is the rest mass of the body.

The frequency-energy relation for constant acceleration becomes

$$E(a) = E(0)\frac{f(0)}{f(a)}\,. \tag{7}$$

The energy for constant acceleration can be expressed as

$$E(a) = \frac{m_o c^2}{\sqrt{1-\frac{2ax}{c^2}}}\,. \tag{8}$$

The energy for constant velocity is the same as that of special relativity. The energy of the body increases as the velocity and acceleration are increased.

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ENERGY CONSERVATION

The frequency-energy relation proves the energy conservation. Let us think that the body is located at the height of h and it falls down to the ground under gravitational acceleration. From the frequency-energy relation and energy expressions we can get

$$\frac{m_0 c^2}{\sqrt{1-(\frac{v}{c})^2}} = \frac{m_0 c^2}{\sqrt{1-\frac{2gh}{c^2}}}, \tag{9}$$

where g is the gravitational acceleration.
From Eq. (9) the velocity can be expressed as

$$v^2 = 2gh. \tag{10}$$

Eq. (10) shows that the energy is conserved while the potential energy is changed to the kinetic energy.

QUALITY FACTOR FOR SUPERCONDUCTING CAVITY

The quality factor conservation for the cavity is proved in this section. The quality factor conservation for heartbeat is proved in this section. The quality factor of the superconducting cavity can be achieved even higher than 3×10^9. The quality factor of the superconducting cavity decreases as the accelerating electric field is increased.The intensity of the radio frequency for the superconducting cavity decreases as follows

$$I = I_o e^{-\frac{2\pi f t}{Q}}. \tag{11}$$

where I_o is the initial intensity and t is the time.

The quality factor for the superconducting cavity is measured as follows

$$Q = \frac{2\pi f \tau_{3.01dB}}{\ln 2}. \tag{12}$$

The quality factor for the cavity is usually measured when the intensity reduces to the half of the initial intensity.
The quality factor is expressed as follows

$$Q(0) = \frac{E}{\Delta E}. \tag{13}$$

where E is the energy and ΔE is the width of the energy.

The quality factor for constant velocity becomes

$$Q(v) = \frac{E\sqrt{1-(v/c)^2}}{\Delta E\sqrt{1-(v/c)^2}}. \tag{14}$$

The quality factor for constant acceleration becomes

$$Q(a) = \frac{E\sqrt{1-(2ax/c^2)}}{\Delta E\sqrt{1-(2ax/c^2)}}. \tag{15}$$

The frequency for constant velocity in reference frame is

$$f(v) = f(0)\sqrt{1-(\frac{v}{c})^2}. \tag{16}$$

The decay time for constant velocity in reference frame is

$$\tau(v) = \frac{\tau(0)}{\sqrt{1-(\frac{v}{c})^2}}. \tag{17}$$

The frequency for constant acceleration in reference frame is

$$f(a) = f(0)\sqrt{1-\frac{2ax}{c^2}}. \tag{18}$$

The decay time for constant acceleration in reference frame is

$$\tau(a) = \frac{\tau(0)}{\sqrt{1-\frac{2ax}{c^2}}}. \tag{19}$$

For the same quality factor of the superconducting cavity, the decay time increases as the resonance frequency is decreased. The resonance frequency for the cavity is inversely proportional to the decay time for the constant velocity and acceleration. The quality factor for the cavity is always conserved for the constant velocity and acceleration.

QUALITY FACTOR FOR HEARTBEAT

The quality factor conservation for the heartbeat of mammal is proved in this section. The total number of the heartbeat for the mammal is about constant and is closed to 3×10^9, which can be achieved by the superconducting cavity.
The quality factor of the mammal showing the total number of heartbeat can be simplified as

$$f_{heart}(0)t_{life}(0) = 3\times10^9. \tag{20}$$

The heart rate for constant velocity is

$$f_{heart}(v) = f_{heart}(0)\sqrt{1-(v/c)^2}. \tag{21}$$

The life expectancy for constant velocity is

$$t_{life}(v) = \frac{t_{life}(0)}{\sqrt{1-(v/c)^2}}. \tag{22}$$

The heart rate for constant acceleration is

$$f_{heart}(a) = f_{heart}(0)\sqrt{1-(2ax/c)^2}. \tag{23}$$

The life expectancy for constant acceleration is

$$t_{life}(a) = \frac{t_{life}(0)}{\sqrt{1-(2ax/c)^2}}. \tag{24}$$

The total number of the heartbeat is not changed by motion. The total number of the heartbeat is the same in rest, constant velocity, and constant acceleration. Therefore, the quality factor of the heartbeat is always conserved, which shows the total number of the heart beat is conserved for the rest, the constant velocity, and the constant acceleration.

Figure 1 shows the light mirror. (a), (b), and (c) show the light path in one-dimensional motion for stationary motion, constant velocity, and constant acceleration, respectively. (d), (e), and (f) show the light paths and light clocks for

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two-dimensional motion. (d) shows the circular mirror for stationary motion, (e) shows the elliptical mirror for constant velocity, and (f) shows the egg curve mirror for constant acceleration. (g), (h), and (i) show the light clock for three-dimensional motion. (g) shows the spherical mirror for stationary motion, (h) shows the ellipsoidal mirror for constant velocity, and (i) shows the egg shape mirror for constant acceleration. From the light clock, the Doppler shift can be derived for constant velocity and acceleration by knowing that the speed of light is constant [10].

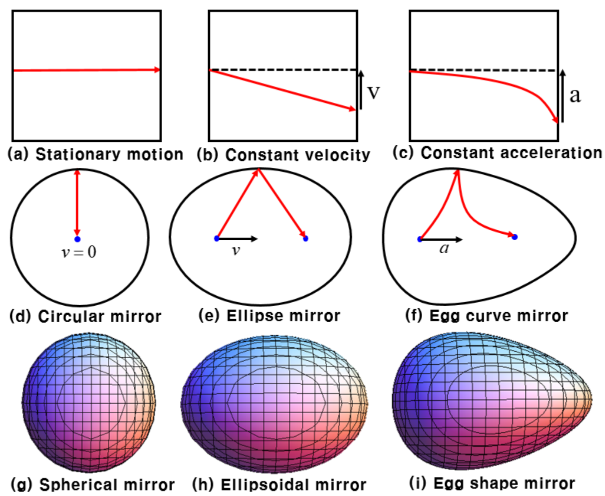

Figure 1: Light mirror. (a), (b), and (c) show the light path in one-dimensional motion for stationary motion, constant velocity, and constant acceleration, respectively. (d), (e), and (f) show the light paths and light clocks for two-dimensional motion. (d) shows the circular mirror for stationary motion, (e) shows the elliptical mirror for constant velocity, and (f) shows the egg curve mirror for constant acceleration. (g), (h), and (i) show the light clock for three-dimensional motion. (g) shows the spherical mirror for stationary motion, (h) shows the ellipsoidal mirror for constant velocity, and (i) shows the egg shape mirror for constant acceleration.

The frequency for the superconducting cavity and the heartbeat of the mammal are shown as a function of velocity in Fig. 2. The quality factors for the cavity and the heartbeat of the mammal are almost the same. The resonance frequency for the cavity and the heartbeat of the mammal decreases as the relative velocity is increased. Because of the quality factor conservation, the decay time of the cavity is inversely proportional to the resonance frequency and the life expectancy of the mammal is also inversely proportional to the heart rate.

Figure 3 shows the frequency for the superconducting cavity and the heartbeat of the mammal as a function of acceleration. The resonance frequencies for the cavity and the heartbeat of the mammal decrease as the acceleration is increased. Because of the quality factor conservation, the decay time of the cavity is inversely proportional to the resonance frequency and the life expectancy of the mammal is also inversely proportional to the heart rate.

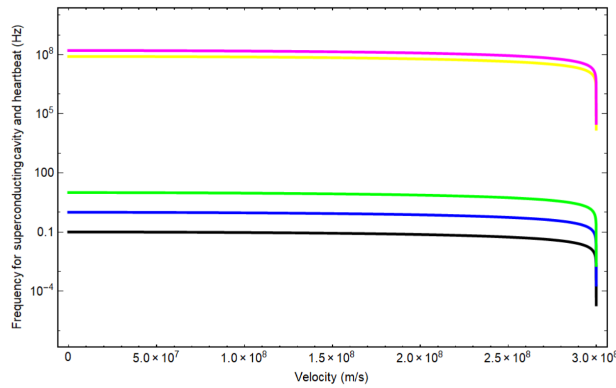

Figure 2: Frequency for the superconducting cavity and heartbeat as a function of velocity. The frequencies for 0.1, 1, and 10 Hz correspond to 6, 60, and 600 beats/min, respectively. The other two frequencies correspond to the resonance frequency, 81.25 MHz, for the quarter-wave resonator (QWR) and the frequency, 162.5 MHz, for the half-wave resonator (HWR), respectively [6, 13]. The quality factors for the cavity and the heartbeat of the mammal are almost the same. The frequency for the superconducting cavity and the heartbeat of mammal decreases as the velocity is increased.

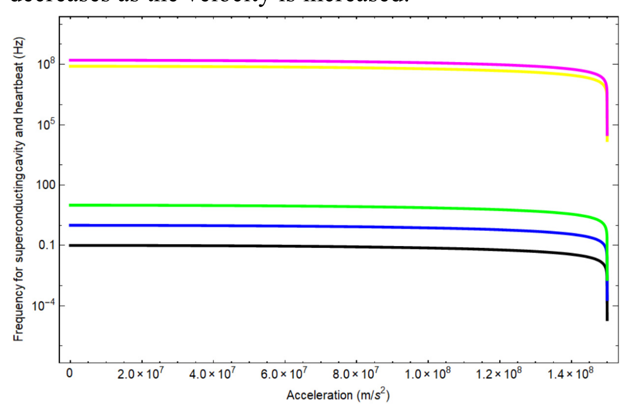

Figure 3: Frequency for the superconducting cavity and heartbeat as a function of acceleration. The resonance frequency for the cavity and the heartbeat of the mammal decreases as the acceleration is increased. Because of the quality factor conservation, the decay time of the cavity is inversely proportional to the resonance frequency and the life expectancy of the mammal is also inversely proportional to the heart rate.

SUMMARY

We have shown the conservation of the quality factor for the superconducting cavity and the heartbeat of the mammal under relativistic motion. The frequency-energy relation shows the Doppler effect and energy conservation. The frequency and the decay time of the superconducting cavity are derived as a function of velocity and acceleration. The heart rate and the lifespan of the mammal are also derived as a function of velocity and acceleration. Because of the conservation of the quality factor, the resonance frequency of the superconducting cavity is inversely proportional to the decay time and the heart rate of the mammal is also inversely proportional to the life expectancy under relativistic motion.

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