A New Spherical Pulse Compressor Working with Degenerated “Whispering Gallery” Mode

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BOC

Barrel-shape Open Cavity

Working with “Whispering Gallery” mode.
Barrel cavity short theory.

The eigen-frequency of the Barrel cavity with \( E_{mnq} \) oscillation is the solution of the next equation:

\[
k\alpha = \nu_{mn} + \frac{(q-1/2)\alpha}{\sin \theta}
\]

\( \nu_{mn} \) is a root of the Bessel function that for the big \( m \) can be approximated as:

\[
\nu_{mn}^0 = m - \mu_n^0 \quad (n=1,2,...),
\]

\[
-\mu_n^0 = [(n-0.25)1.5\pi]^{2/3}, \quad \mu = \left(\frac{m}{2}\right)^{1/3}.
\]

The optimal radius \( r_0 \), when the external caustic has the smallest height comes from:

\[
r_0 = 2a \sin^2 \theta
\]

where \( \alpha \) and \( \theta \) are derived from:

\[
\sin \alpha = \sqrt{r_0 \over a} \sin \theta \quad \cos \theta = {m \over \nu_{mn}}
\]

Finally the height of the external caustic and Q-factor of the cavity are:

\[
z_{q-1} = 2 \sqrt{(q-1/2) \frac{a \sin \theta}{k \sin 2\alpha}}
\]

\[
Q_E = \frac{a}{\sigma_s}
\]
In this work, we use a spherical cavity, but for convenience, I will use the definitions in BOC.
Traditional BOC:

- Cavity is surrounded by rectangular waveguide to couple power.
- Power is coupled through coupling apertures.

New design:

- We plan to use a coupling slot instead of the apertures, which can be machined more precisely.
- The cavity and the mode launcher can be designed separately, which can simplify the design.
For Whispering Gallery mode, both BOC and SC theory can be used:

Resonant frequency of $\text{TM}_{mnq}$ for BOC:

$$ka = v_{mn} + \frac{(p - 1/2)\alpha}{\sin \theta}$$

When $m=24$, $n=1$, $q=1$

$$a = 120.3 \text{ mm}$$

$$Q_0 = 200,000$$

Resonant frequency of $\text{TE}_{mnp}$ for spherical cavity:

$$f = \frac{u_{np}}{2\pi a\sqrt{\varepsilon\mu}}$$

When $n=24$, $p=1$

Simulation result:

<table>
<thead>
<tr>
<th>$\text{TM}_{24,1,1}$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius /mm</td>
<td>120.3</td>
</tr>
<tr>
<td>Frequency /MHz</td>
<td>11995.8</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>199,374</td>
</tr>
</tbody>
</table>
We need to reduce the size of mode launcher, but there’s a limitation because of the index $m$. For $m=24$, the minimum radius of the input port is 104.85 mm, the cut-off radius of $\text{TE}_{24,1}$ mode.

We have to keep the Q-factor high enough, so the cavity radius should not be reduced.

The solution is to decrease the index $m$, but increase the index $p$ correspondingly.

$m$ is the number of oscillations along $\varphi$ direction on equator plane. $p$ is the number of oscillations along $z$ direction.
For different modes in BOC, as long as $n=1$, $m+p=25$:

- For elliptical BOC, their frequencies are different.
- The smaller the difference between $a$ and $b$ is, the smaller the frequency difference will be.
- If $a=b$, the frequency difference becomes 0. All these modes are degenerated.
Analysis of TM$_{9,1,16}$ mode

The mode launcher is still too big. So we continue to reduce index $m$. But $m$ cannot be too small, otherwise the surface field in mode launcher will be too high. Besides, the smaller the index $m$ is, the more degenerated high-order modes there will be. For example:

<table>
<thead>
<tr>
<th>Working mode</th>
<th>High-order modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM$_{9,1,16}$</td>
<td>TM$_{18,1,7}$</td>
</tr>
<tr>
<td>TM$_{8,1,17}$</td>
<td>TM$<em>{16,1,9}$, TM$</em>{24,1,1}$</td>
</tr>
<tr>
<td>TM$_{7,1,18}$</td>
<td>TM$<em>{14,1,11}$, TM$</em>{21,1,4}$</td>
</tr>
<tr>
<td>TM$_{6,1,19}$</td>
<td>TM$<em>{12,1,13}$, TM$</em>{18,1,7}$, TM$_{24,1,1}$</td>
</tr>
<tr>
<td>TM$_{5,1,20}$</td>
<td>TM$<em>{10,1,15}$, TM$</em>{15,1,10}$, TM$_{20,1,5}$</td>
</tr>
</tbody>
</table>

For now, index $m$ of 9 is chosen:

Neighboring modes.

<table>
<thead>
<tr>
<th>Eigenmode</th>
<th>Frequency (GHz)</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>11.9249 +j 0.000183232</td>
<td>32540.4</td>
</tr>
<tr>
<td>Mode 2</td>
<td>11.9249 +j 0.000183227</td>
<td>32541.4</td>
</tr>
<tr>
<td>Mode 3</td>
<td>11.9958 +j 3.00849e-05</td>
<td>199365.</td>
</tr>
<tr>
<td>Mode 4</td>
<td>12.0073 +j 4.80147e-05</td>
<td>125038.</td>
</tr>
<tr>
<td>Mode 5</td>
<td>12.0490 +j 3.62833e-05</td>
<td>166040.</td>
</tr>
</tbody>
</table>
Analysis of TM_{9,1,16} mode - cavity

There's some mode mixture.

19 oscillations on the wall
The Q-factor is around 194,000 (~2.5% reduced).
Analysis of TM_{9,1,16} mode – mode launcher

Model:

Best result for now:
Mode launcher size comparison

Mode launcher circles of the three pulse compressor schemes:

The use of degenerated mode reduces the size of mode launcher significantly.
Summary

- This work is to design a spherical pulse compressor using degenerated “whispering gallery” mode.

- Some investigations on “whispering gallery” mode and its degenerated modes are presented.

- The structure is being designed. Some preliminary results are presented.

Thank you!

Meet me at THPO109.
Thank you!