Frequency Spectra from Solenoid Lattice Orbits

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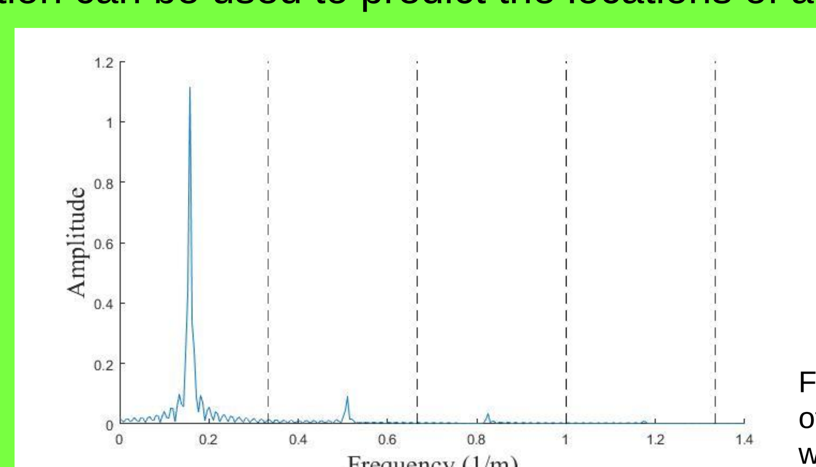
Background

- Multicharge state beams proposed to increase beam intensity in ion accelerators
- Want to understand how multicharge state beam dynamics
  - Center will be weighted average of composite single charge states
  - Positions become complex for solenoid linacs in the lab frame
- May be easier to view beam center in frequency domain for quasi-periodic linacs
  - Multicharge state spectra is superposition of composite single charge state spectra
  - Need understanding of single charge state spectra to create a model of multicharge state beams in the frequency domain
- Easiest to simulate the beam in Larmor frame then rotate into the lab frame
  - In frequency domain, the rotation becomes a splitting of the peaks by the rotation frequency

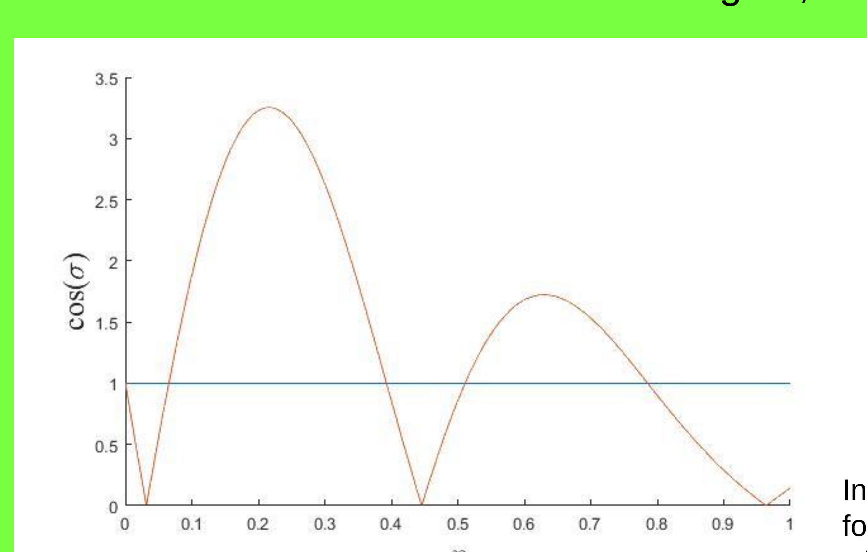
Hill's Equation Solution

- Integrated Hill's equation to get orbits for hard edge FOFO lattice in Larmor frame. Saw peaks in the spectra at $\frac{n}{2L} \pm \Delta f$ for all n. The step size was small enough so aliasing was not an issue.
- Therefore, mapping solution can be used to predict the locations of all the peaks



Frequency spectrum of solenoid lattice with L=1.5, η=1/3, κ=4

- Location of frequency of maximum amplitude can be determined using the instability criteria.
  - If parameters before the first unstable region, then first peak is maximum
  - If parameters between first and second unstable region, then second peak is maximum
  - Etc.



Instability criterion for solenoid lattice with $\sqrt{\kappa}L = 8$.

Realistic Models

Current models use hard edge, non-accelerating periodic lattices

- Accelerating Lattices
  - Fix $\frac{B}{\gamma\beta}$ to maintain same focusing strength
  - No change in frequency spectra
- Fringe Fields
  - Used thin solenoid field
  - Frequency shifted from hard edge equivalent model
- Quasi-Period Lattices
  - Removed every third solenoid
  - Some frequency peaks disappeared

Mapping Solution

- Transfer map for solenoid FOFO lattice gives simple method of generating orbits.

$$M=\begin{bmatrix} \cos(\sqrt{\kappa}L\eta) - \sqrt{\kappa}L(1-\eta)\sin(\sqrt{\kappa}L\eta) & \frac{1}{\sqrt{\kappa}}\sin(\sqrt{\kappa}L\eta) - L(1-\eta)\cos(\sqrt{\kappa}L\eta) \\ -\sqrt{\kappa}\sin(\sqrt{\kappa}L\eta) & \cos(\sqrt{\kappa}L\eta) \end{bmatrix}$$

$$\begin{bmatrix} x \\ x' \end{bmatrix}_{s=(n+1)L} = M \begin{bmatrix} x \\ x' \end{bmatrix}_{s=nL} \qquad \kappa = \left(\frac{qB}{2m\gamma\beta c}\right)^2$$

- κ is focusing strength, η occupancy, L period length
- Fixed step size allows frequency spectrum to be determined using discrete Fourier transform. But, the large step size causes aliasing.

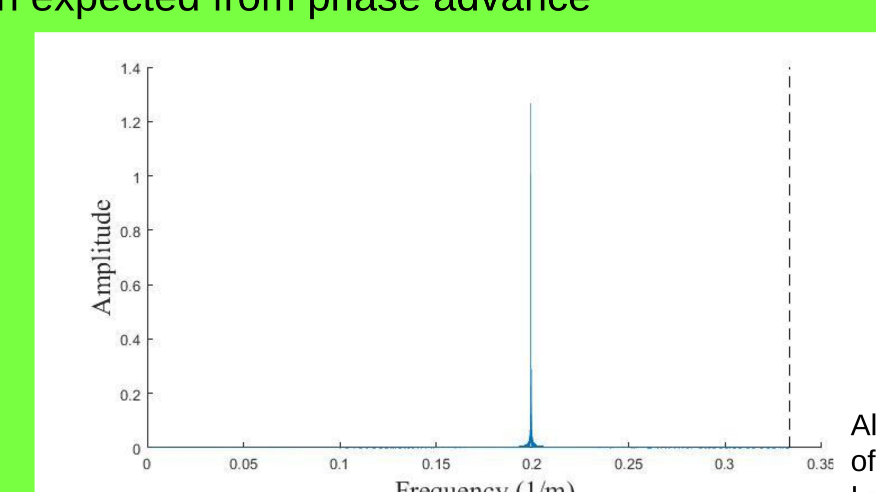
- Can determine phase advance per lattice period from transfer matrix

$$\cos(\sigma) = \cos(\sqrt{\kappa}L\eta) - \frac{1}{2}\sqrt{\kappa}L(1-\eta)\sin(\sqrt{\kappa}L\eta)$$

- Mapping gives orbit after every lattice period. Therefore the frequency should be given by:

$$f(\sigma) = \frac{\sigma}{2\pi L}$$

- Only one aliased peak, therefore possible peak locations at $\frac{n}{2L} \pm \Delta f$
- Peak matches location expected from phase advance



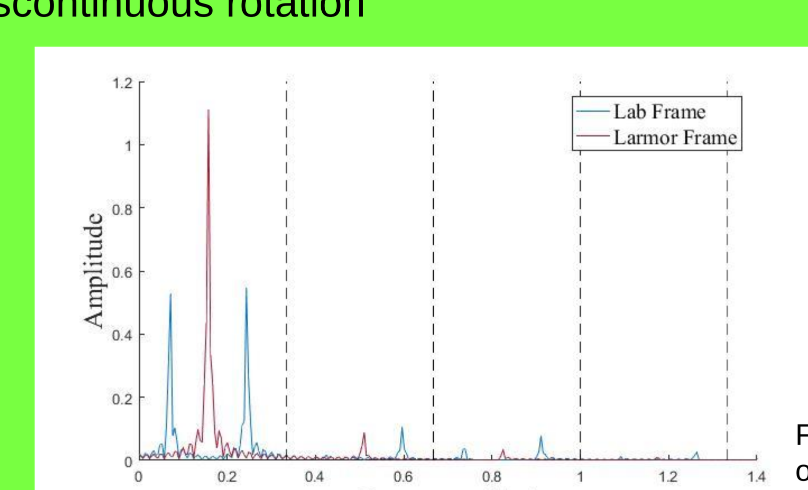
Aliased frequency spectrum of solenoid lattice with L=1.5, η=1/3, κ=4

Coordinate Transforms

- Single charge state solutions in the Larmor frame must be transformed into the lab frame before they can be superimposed
- Difficult to remove the discontinuous rotation of the Larmor frame in the continuous solution. It must be done in the time domain.
- The mapping solution has same rotation between each point, so multiplying by the negative Larmor frequency will remove the rotation.
  - Causes the frequencies to split

$$\cos(2\pi f s)\sum_n \cos(2\pi f_n s) = \sum_n [\cos(2\pi(f-f_n)s) + \cos(2\pi(f+f_n)s)]$$

- The split peaks correspond to the continuous solution in the lab frame despite not accounting for the discontinuous rotation



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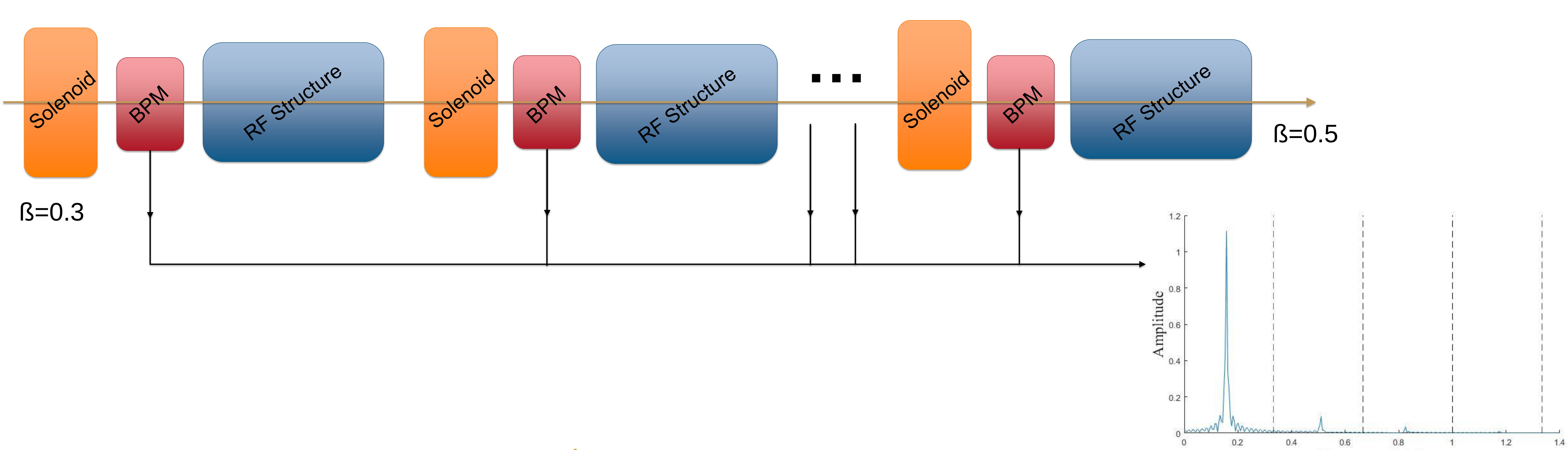

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The NSCL is funded in part by the National Science Foundation and Michigan State University.