FIFTH-ORDER MOMENT CORRECTION FOR BEAM POSITION AND SECOND-ORDER MOMENT MEASUREMENT

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INTRODUCTION

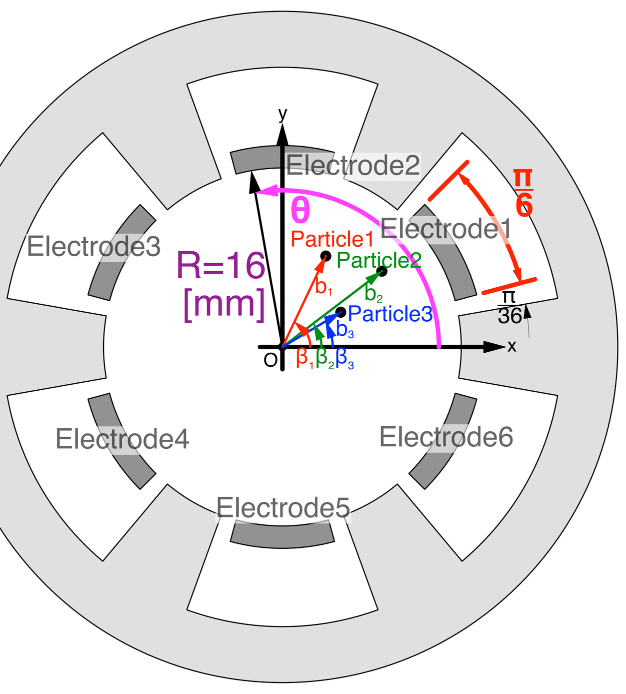
For measurements of beam position and second-order relative moments, six-electrode BPMs with circular cross-section have been installed at SPring-8 linac.

To obtain the relative attenuation factors between the BPM electrodes, we developed a beam-based calibration method, i.e., entire calibration. During the entire calibration, beams must be located at a position more than 4 mm from the BPM center.

We also developed a recursive correction scheme with up to fifth-order moments to improve the accuracy of the entire calibration when a beam was located far from the BPM center.

Previously, correction terms were usually expressed by the higher-order polynomials of the beam positions for obtaining (calculating) precise beam positions. Because the correction terms came from higher-order moments that appeared on the output voltages of BPM, we constructed a new correction scheme whose correction terms were expressed by higher-order moments.

This paper describes the theoretical features of the correction scheme, the simulation (calculation) by an image charge method, and the experiment results using electron beams at SPring-8 linac.



THEORETICAL FEATURES

- $E(\theta)$: Electric Field (Distribution) on the Inner Surface of BPM

$$E(\theta) \propto M + 2\sum_{n=1}^{\infty}\sum_{N=1}^{M}\frac{p_{Nn}\cos n\theta + q_{Nn}\sin n\theta}{R^n}$$

$$\propto 1 + 2\sum_{n=1}^{\infty}\frac{P_n\cos n\theta + Q_n\sin n\theta}{R^n} \qquad (1)$$

$$p_{Nn} = b_N^n\cos n\beta_N, q_{Nn} = b_N^n\sin n\beta_N$$

$$P_n = \frac{1}{M}\sum_{N=1}^{M}p_{Nn}, Q_n = \frac{1}{M}\sum_{N=1}^{M}q_{Nn}$$

- V_d (d=1,…,6) : Output Voltage form Electrode d

$$V_d \propto R\int_{(4d-3)\pi/12}^{(4d-1)\pi/12}E(\theta)\,d\theta = \frac{\pi}{12} + \sum_{n=1}^{\infty}\frac{c_{dn}P_n + s_{dn}Q_n}{R^n} \qquad (2)$$

$$c_{dn} = \int_{(4d-3)\pi/12}^{(4d-1)\pi/12}\cos n\theta\,d\theta, s_{dn} = \int_{(4d-3)\pi/12}^{(4d-1)\pi/12}\sin n\theta\,d\theta$$

Treat Moments up to 5th-Order

$$\begin{aligned}
&f_1 = c_{11} = -c_{31} = -c_{41} = c_{61}, 0 = c_{21} = c_{51},\\
&h_1 = s_{11} = s_{31} = -s_{41} = -s_{61}, 2h_1 = s_{21} = -s_{51},\\
&f_2 = c_{12} = c_{32} = c_{42} = c_{62}, 2f_2 = -c_{22} = -c_{52},\\
&h_2 = s_{12} = -s_{32} = s_{42} = -s_{62}, 0 = s_{22} = s_{52},\\
&0 = c_{13} = c_{23} = c_{33} = c_{43} = c_{53} = c_{63},\\
&h_3 = s_{13} = -s_{23} = s_{33} = -s_{43} = s_{53} = -s_{63},\\
&f_4 = -c_{14} = -c_{34} = -c_{44} = -c_{64}, 2f_4 = c_{24} = c_{54},\\
&h_4 = s_{14} = -s_{34} = s_{44} = -s_{64}, 0 = s_{24} = s_{54},\\
&f_5 = -c_{15} = c_{35} = c_{45} = -c_{65}, 0 = c_{25} = c_{55},\\
&h_5 = s_{15} = s_{35} = -s_{45} = -s_{65}, 2h_5 = s_{25} = -s_{55}.
\end{aligned} \qquad (3)$$

- Difference of Output Voltage Cn, Sn

$$C_1 = \frac{V_1 - V_3 - V_4 + V_6}{V_1 + V_3 + V_4 + V_6},$$

$$S_1 = \frac{V_1 + V_3 - V_4 - V_6}{V_1 + V_3 + V_4 + V_6}, \qquad (2)$$

$$C_2 = \frac{V_1 + V_3 + V_4 + V_6 - 2(V_2 + V_5)}{V_1 + V_3 + V_4 + V_6 + 2(V_2 + V_5)},$$

$$S_2 = \frac{V_1 - V_3 + V_4 - V_6}{V_1 + V_3 + V_4 + V_6},$$

$$S_3 = \frac{V_1 - V_2 + V_3 - V_4 + V_5 - V_6}{V_1 + V_2 + V_3 + V_4 + V_5 + V_6}.$$

We suppose that Pn, Qn can be expressed as a product of an nth power of effective aperture radius R^n_{CnPn}, R^n_{SnQn} and corrected difference $C^\dagger_n$, $S^\dagger_n$.

$$P_1 = \frac{R_{C1P1}}{2}C_1^\dagger, Q_1 = \frac{R_{S1Q1}}{2}S_1^\dagger, P_2 = \frac{R_{C2P2}^2}{2}C_2^\dagger, Q_2 = \frac{R_{S2Q2}^2}{2}S_2^\dagger, Q_3 = \frac{R_{S3Q3}^3}{2}S_3^\dagger. \qquad (5)$$

Where;

$$R_{C1P1} = \frac{\pi}{6f_1}R = 18.69, R_{S1Q1} = \frac{\pi}{6h_1}R = 32.37, R_{C2P2} = \sqrt{\frac{\pi}{9f_2}}R = 18.91,$$
$$\text{[mm]}\quad\text{[mm]}\quad\text{[mm]}$$

$$R_{S2Q2} = \sqrt{\frac{\pi}{6h_2}}R = 17.59, R_{S3Q3} = \sqrt[3]{\frac{\pi}{6h_3}}R = 16.57. \qquad (6)$$
$$\text{[mm]}\quad\text{[mm]}$$

Vd in Eq. (2) is substituted into Eq. (4). But Vd is expressed as the linear combination of Pn and Qn up to the infinite-order. How much order do we confine?

If we only confine the **fundamental** (smallest) order, i.e. **without correction**;

$$C_1^\dagger = C_1, S_1^\dagger = S_1, C_2^\dagger = C_2, S_2^\dagger = S_2, S_3^\dagger = S_3. \qquad (7)$$

If we confine the correction with up to **third-order** moments;

$$C_1^\dagger = C_1\left(1 + \frac{2P_2}{R^2_{C1P2d}}\right), S_1^\dagger = S_1\left(1 + \frac{2P_2}{R^2_{S1P2d}}\right) - \frac{2Q_3}{R^3_{S1Q3u}},$$

$$C_2^\dagger = C_2\left(1 - \frac{2P_2}{R^2_{C2P2d}}\right), S_2^\dagger = S_2\left(1 + \frac{2P_2}{R^2_{S2P2d}}\right), S_3^\dagger = S_3. \qquad (8)$$

Where;

$$R_{C1P2d} = \sqrt{\frac{\pi}{6f_2}}R = 23.16, R_{S1P2d} = \sqrt{\frac{\pi}{6f_2}}R = 23.16, R_{S1Q3u} = \sqrt[3]{\frac{\pi}{6h_3}}R = 16.57,$$
$$\text{[mm]}\quad\text{[mm]}\quad\text{[mm]}$$

$$R_{C2P2d} = \sqrt{\frac{\pi}{3f_2}}R = 32.75, R_{S2P2d} = \sqrt{\frac{\pi}{6f_2}}R = 23.16. \qquad (9)$$
$$\text{[mm]}\quad\text{[mm]}$$

If we confine the correction with up to **fifth-order** moments;

$$C_1^\dagger = C_1\left(1 + \frac{2P_2}{R^2_{C1P2d}} - \frac{2P_4}{R^4_{C1P4d}}\right) + \frac{2P_5}{R^5_{C1P5u}},$$

$$S_1^\dagger = S_1\left(1 + \frac{2P_2}{R^2_{S1P2d}} - \frac{2P_4}{R^4_{S1P4d}}\right) - \frac{2Q_3}{R^3_{S1Q3u}} - \frac{2Q_5}{R^5_{S1Q5u}},$$

$$C_2^\dagger = C_2\left(1 - \frac{2P_2}{R^2_{C2P2d}} + \frac{2P_4}{R^4_{C2P4d}}\right) + \frac{2P_4}{R^4_{C2P4u}}, \qquad (10)$$

$$S_2^\dagger = S_2\left(1 + \frac{2P_2}{R^2_{S2P2d}} - \frac{2P_4}{R^4_{S2P4d}}\right) - \frac{2Q_4}{R^4_{S2Q4u}}, S_3^\dagger = S_3.$$

Where;

$$R_{C1P4d} = \sqrt[4]{\frac{\pi}{6f_4}}R = 19.95, R_{C1P5u} = \sqrt[5]{\frac{\pi}{6f_5}}R = 17.50, R_{S1P4d} = \sqrt[4]{\frac{\pi}{6f_4}}R = 19.95,$$
$$\text{[mm]}\quad\text{[mm]}\quad\text{[mm]}$$

$$R_{S1Q5u} = \sqrt[5]{\frac{\pi}{6h_5}}R = 19.53, R_{C2P4d} = \sqrt[4]{\frac{\pi}{3f_4}}R = 23.73, R_{C2P4u} = \sqrt[4]{\frac{\pi}{9f_4}}R = 18.03,$$
$$\text{[mm]}\quad\text{[mm]}\quad\text{[mm]}$$

$$R_{S2P4d} = \sqrt[4]{\frac{\pi}{6f_4}}R = 19.95, R_{S2Q4u} = \sqrt[4]{\frac{\pi}{6h_4}}R = 17.39. \qquad (11)$$
$$\text{[mm]}\quad\text{[mm]}$$

SIMULATION

Variable : P_1 (Horizontal Position), Q_1 (Vertical Position) and P_{g2}

Regarded Other Relative Moments,

Q_{g2}, P_{g3}, Q_{g3}, P_{g4}, Q_{g4}, P_{g5} and Q_{g5} as Zero

$$P_2 = p_{G2} + P_{g2}, p_{G2} = P_1^2 - Q_1^2, Q_2 = q_{G2} = 2P_1Q_1,$$

$$P_3 = p_{G3} + 3p_{G1}P_{g2}, p_{G3} = P_1^3 - 3P_1Q_1^2, p_{G1} = P_1,$$

$$Q_3 = q_{G3} + 3q_{G1}P_{g2}, q_{G3} = 3P_1^2Q_1 - Q_1^3, q_{G1} = Q_1,$$

$$P_4 = p_{G4} + 6p_{G2}P_{g2}, p_{G4} = P_1^4 - 6P_1^2Q_1^2 + Q_1^4, \qquad (12)$$

$$Q_4 = q_{G4} + 6q_{G2}P_{g2}, q_{G4} = 4P_1^3Q_1 - 4P_1Q_1^3,$$

Explicit Expression

$$P_5 = p_{G5} + 10p_{G3}P_{g2}, p_{G5} = P_1^5 - 10P_1^3Q_1^2 + 5P_1Q_1^4,$$

$$Q_5 = q_{G5} + 10q_{G3}P_{g2}, q_{G5} = 5P_1^4Q_1 - 10P_1^2Q_1^3 + Q_1^5.$$

$E(\theta)$ Calculation : Method of Images with a Mirror Point Charge

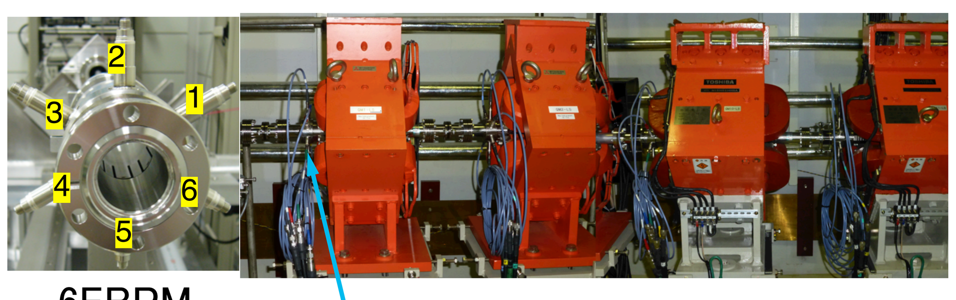
P_{g2} Calculation : Assume an **Electric Quadrupole**

Range of Variables

$$-4 \le \text{Set } P_1 \le 4 \text{ [mm] by 0.1 mm steps,}$$

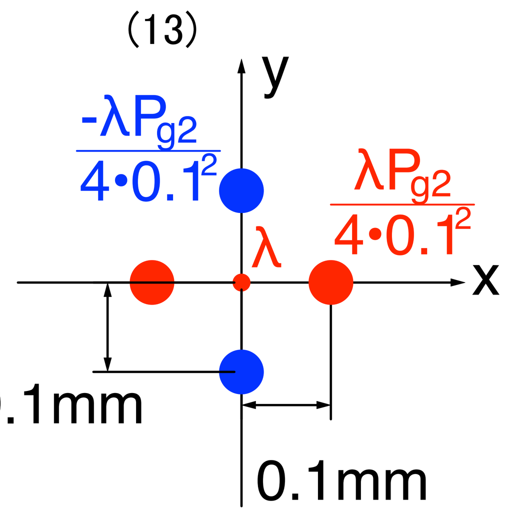
$$-4 \le \text{Set } Q_1 \le 4 \text{ [mm] by 0.1 mm steps,}$$

$$\text{Set } P_{g2} = -2, 11 \text{ [mm}^2\text{]}. \qquad (13)$$

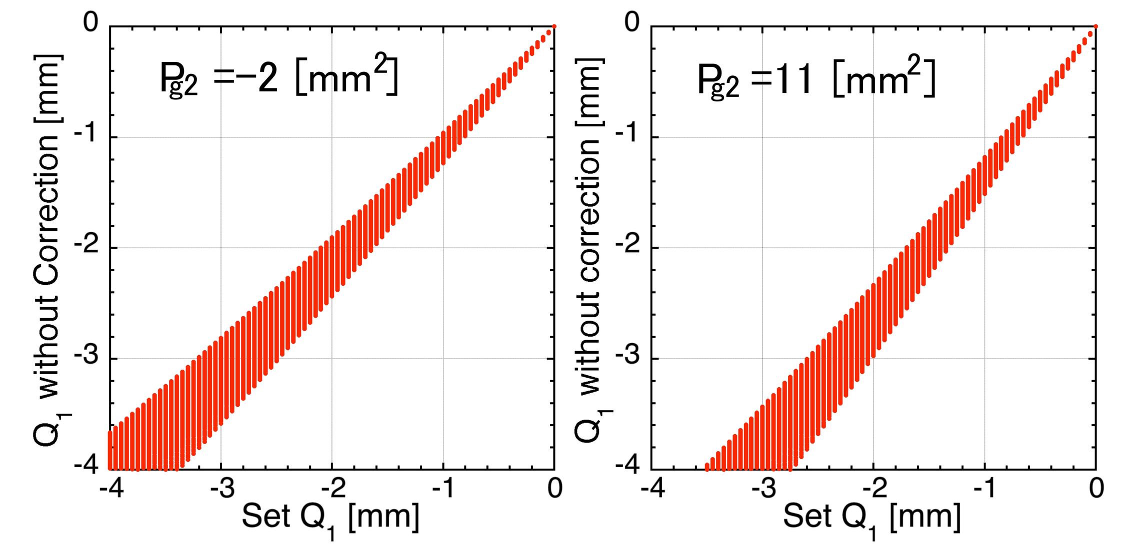


6EBPM.

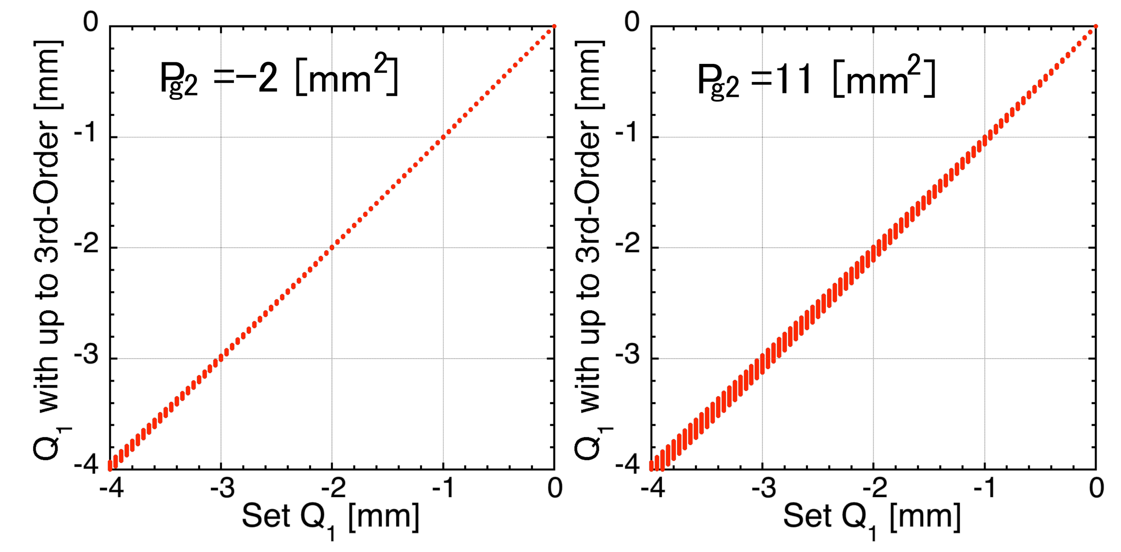
6EBPM used for the experiment.



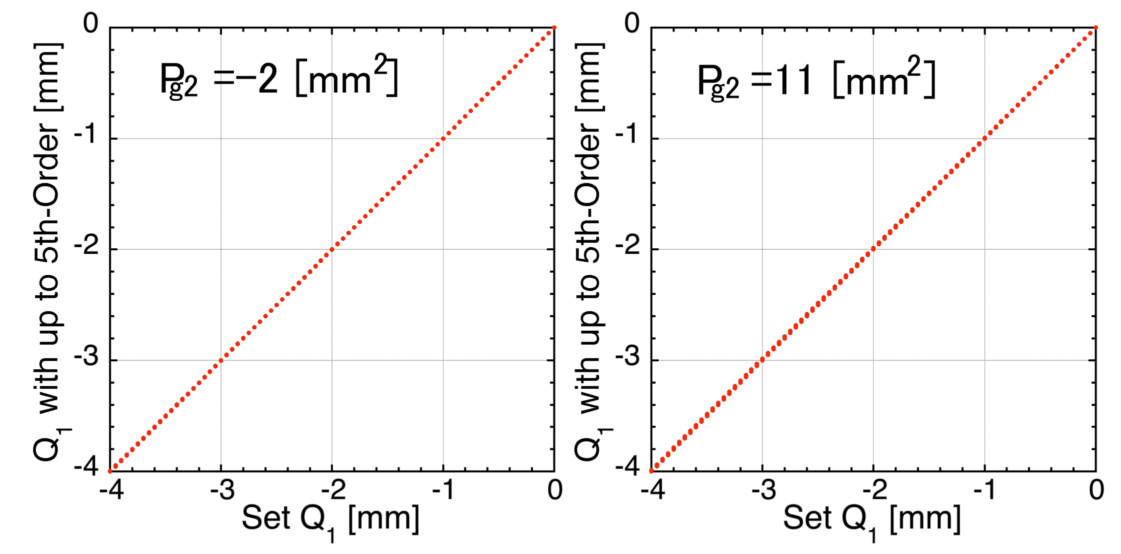
Electric Quadrupole.



Simulated Q_1 without correction using Eq. (7).

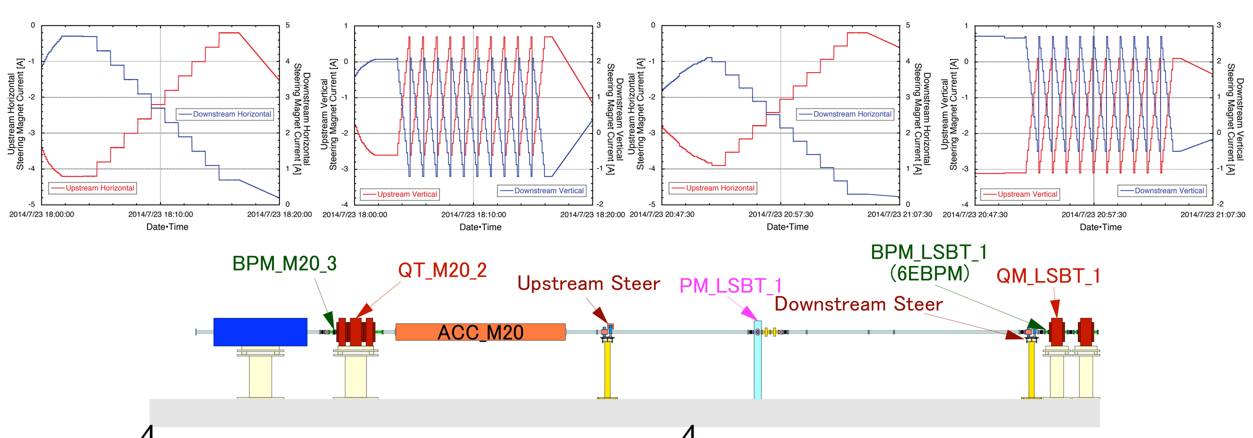


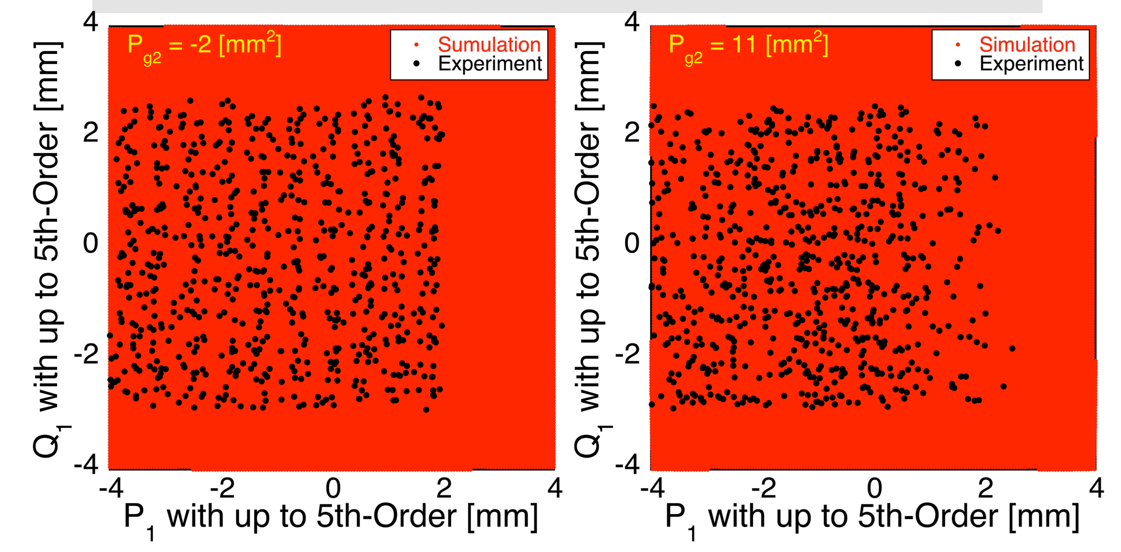
Simulated Q_1 with up to third-order moment correction using Eq. (8).



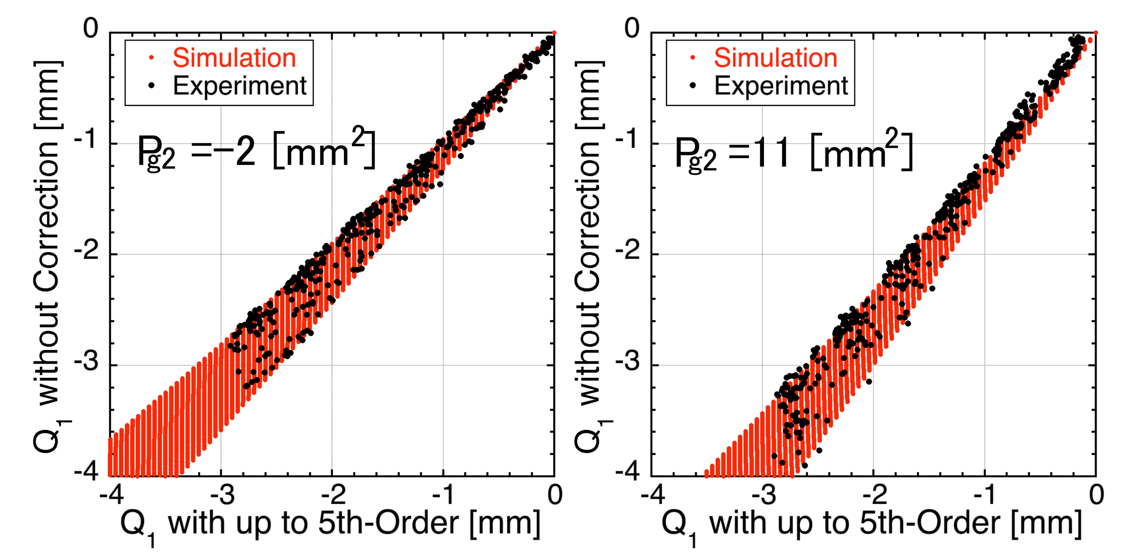
Simulated Q_1 with up to fifth-order moment correction using Eq. (10).

COMPARISON WITH EXPERIMENT

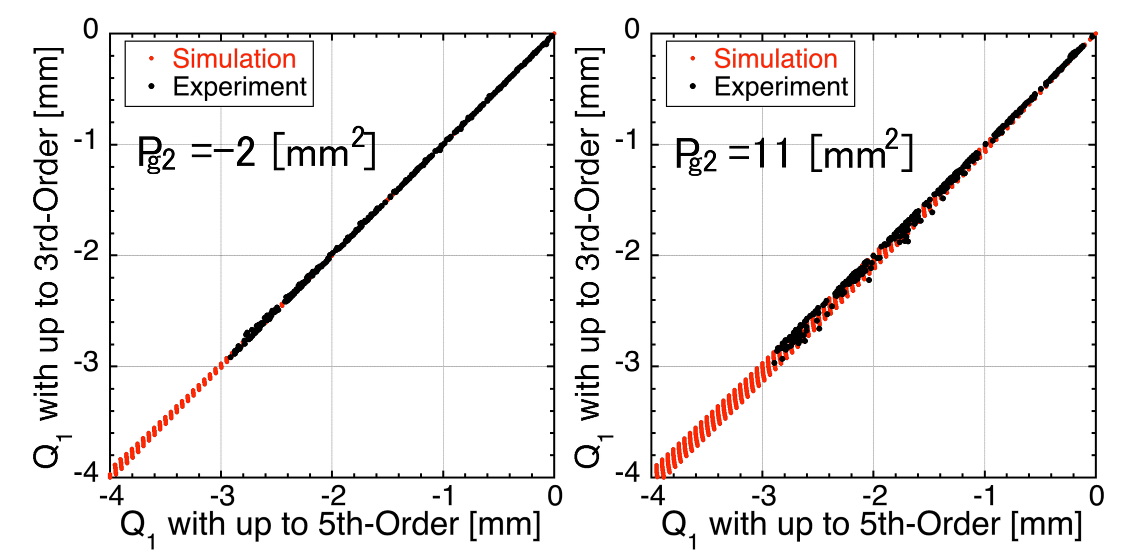




Horizontal positions (P_1) and vertical positions (Q_1). Small red circles show the result of sumulation with up to fifth-order moment correction, and large black circles show the result of experiment with up to fifth-order moment correction.



Correlation plot between Q_1 with up to fifth-order moment correction and Q_1 without correction.



Correlation plot between Q_1 with up to fifth-order moment correction and Q_1 with up to third-order moment correction.

In the figures, the small red circles (simulation) and the large black circles (experiment) show good agreement, and this consistency proves the validity of higher-order moment correction.