# DETAILED ANALYSIS OF THE LONG-RANGE WAKEFIELD IN THE BASELINE DESIGN OF CLIC MAIN LINAC

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# Abstract

The baseline design of the accelerating structure of the CLIC main linac relies on strong damping of transverse higher order modes (HOMs). Each accelerating cell is equipped with four damping waveguides that causes HOM energy to propagate to damping loads. Most of the HOMs decay exponentially with a Q-factor of about 10, however there are modes with higher Q-factors. Though the amplitude of the high Q modes is nearly two orders of magnitude smaller than the dominating lowest dipole mode, their cumulative effect over the entire bunch train may be significant and dilute the beam emittance to unacceptable level. In this paper we report on an accurate calculation of the long-range wakefield and its overall effect on beam dynamics. We also discuss possible measures to minimise its effect in a tapered structure.

# **INTRODUCTION**

The CLIC baseline design of the main linac operates at 12 GHz [1]. Preserving the beam quality in linacs operating at high frequencies is primarily limited by the beam induced fields known as wakefields [2]. Inadequate suppression of wakefields may result in emittance dilution; hence it is essential to employ a damping mechanism. In the first section of this paper we present detailed calculation of the wakefields in the CDR [1] baseline design of the CLIC accelerating structure (CLIC AS) [3], which is based on strong waveguide damping of HOMs. We also investigate the presence of so called persistent wakefield [4] in this structure. In the CLIC scheme, the bunch train consists of 312 bunches [1]; in such multi-bunch acceleration, wakefield generated by a bunch often leads to kicking the following bunches. An analytical estimate of the effect of multi-bunch long range wakefields on the beam profile is presented in [5]. However, in [5], by considering the strong damping of the HOMs, it is assumed that only the first trailing bunch experiences a kick due to an off-axis driving bunch. Herein we calculate the very long range wakefields in CLIC AS which allows HOM kicks to be considered on all trailing bunches by applying the analytical model to estimate the long range multi-bunch wakefield effect. These results are discussed in the second section.

# WAKEFIELDS IN CLIC AS

The CLIC AS is designed with tapered irises so as to keep the accelerating gradient (nearly) constant. The wakefield is strongly damped by means of four waveguides attached to each accelerating cell [1, 3]. The time domain code GdfidL [6] was utilised to calculate the long-range wakefields excited in the structure. The wakefield can also be reconstructed using the frequencies, kicks and Q factors of the dominating modes. The lowest dipole mode has the strongest impact on the off-axis beam as it contains the largest kick factor. The damping waveguides are carefully designed, in particular to damp this mode with a  $Q \sim 10$  [3]. The envelopes of transverse wakefields of the sample cells, namely the first, middle and last are illustrated in the Fig. 1, and a disc of the CLIC AS is also shown inset. Beyond 1 m wake-length it is clear that only one mode dominates, which has low amplitude but high Q-factor. However, modes with high group velocity  $(v_g)$  and poor damping via these waveguides travel along the structure and are damped by means of evacuation from the structure volume through the input and output power couplers. Whereas the wakefield in the last cell does not seem to be affected by



Figure 1: Envelope of transverse wakefield in single cells of CLIC AS.

the group velocity of the mode, the wakefields in the first and the middle cells are, resulting in reduction of the wakefield envelope at about 4 m and at about 15 m respectively. In this case, the wakefield is calculated including the group velocity term ( $\beta$ ) as [1, 2]

$$W_{\perp}(s) = 2\sum_{p} K_{p} Exp\left[i\frac{\omega_{p}s}{c}\left(1 + \frac{i}{2(1 - \beta_{p})Q_{p}}\right)\right]\left(1 - \frac{\beta_{p}s}{L(1 - \beta_{p})}\right)$$
(1)

where  $K_p$  and  $\omega_p$  are the frequency and kick factor of the mode *p* respectively, *L* is the structure length and  $\beta = v_g/c$ . The modes with non-negligible  $\beta_p$  travel rapidly through the structure which expedites wake suppression, hence it is necessary to include this term in the calculation. In order to understand the modal contribution in the wake beyond 1 m, we study the transverse impedance (Fourier transform of the wake) in these cells. The impedance of these cells is shown in Fig. 2. As can be seen, the first mode around 17 GHz has the largest impedance (i.e. kick factor) and better damping (width of the peak) compared to the other modes. The modes at 21 GHz, 26 GHz, 40

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GHz and 55 GHz have non-negligible kicks. Among these, the mode at 21 GHz is poorly damped. Detailed Fourier analysis of the wakefield beyond 3 m indicates that only the 21 GHz mode contributes to the wakefield due to its poor damping. The impedance of the single cells beyond 3 m wake-length is shown in the inset of Fig. 2.



Figure 2: Transverse impedance in single cells of CLIC AS. Impedance for a window of 3m to 7m wake-length is shown inset.

The mode at 21 GHz has a detuning of  $\sim$ 1.5 GHz which assists in non-coherent addition of the wakefield. The wakefield excited in the CLIC AS is indicated in Fig. 3. The various modes contributing to the wakefield



Figure 3: Comparison of the envelope of transverse wakefield in CLIC AS using GdfidL (red curve) and reconstruction using single cell modes (black curve). Inset compares the reconstruction excluding the 21 GHz mode.

are visualized in detail in Figs. 1 and 2. From these modes we reconstruct the wakefield (eq. 1) of the CLIC AS. We learned that the 21 GHz mode has a Q of few hundreds (it varies from cell-to-cell) and  $v_g$  ranging from 10% c in the first cell, 2.8% c in the middle cell and down to 0% c in the last cell. The reconstruction of the wake using singlecell modes is compared with the GdfidL simulation in Fig. 3. As the 21 GHz mode travels through the structure, it couples from cell to cell. We utilised single band of circuit model [7] to compute the coupled mode frequencies of this mode. Very good agreement has been found between the reconstructed and GdfidL wakefields. If we artificially remove this mode (using reconstruction), the wakefield does reduce for s > 1 m, which is indicated in the inset of Fig. 3.

If a mode is excited in a structure close to the cut-off frequency of the waveguide, then it does not travel through the waveguide, hence it cannot be damped. The wakefield pertaining to this mode is defined as persistent wakefield which does not decay exponentially [4]. Lingering of such a wakefield-tail is not acceptable from the beam dynamics point of view. We investigated the possibility of persistent wakefield in the CLIC AS. We observe that the modes contributing significantly to the wakefield decay exponentially. The reconstruction plot for a sample (mid) cell is presented in Fig. 4.



Figure 4: Reconstruction of wakefield in the middle cell using modal sum method. The black curve represents reconstruction, the coloured curve GdfidL simulation.

The only mode that does not decay rapidly is the 21 GHz mode. Detailed study of the e.m. fields in this mode using HFSS revealed that this is a TE-type dipole mode concentrated near the iris of the cell; the fields in this mode are far away from the damping waveguide aperture, hence this mode is poorly damped. The wakefield of this mode can be reconstructed using eq. 1. It is clearly visible that the wakefield decays exponentially with a Q of a few hundred. The reconstruction of this mode confirms the fact that there is no visible persistent wakefield in the CLIC AS. The multi-bunch wakefield effect causing beam amplification is discussed in the next section.

## MULTIBUNCH WAKEFIELD EFFECT

In the CLIC scheme, the beam dynamics study defines an upper limit of 7 V/pC/mm/m wake on the first trailing bunch [1]. However, in multi-bunch acceleration, an offaxis driving bunch can cause serious deflections of the trailing bunches. In order to understand an overall effect of the wakefield on a multi-bunch beam profile, a matrix formulation was studied and is presented in [5]. This formulation involves two effects: an off-axis driving bunch deflecting a trailing bunch (direct effect), due to the deflection of this trailing bunch and, subsequent deflection of trailing bunches (indirect effect). In [5] it is assumed that every driving bunch exhibits wake only on the first trailing bunch. With this model a beam amplification factor was calculated which is caused by coherent jitter ( $F_c \sim 1$ ), r.m.s of jitter ( $F_{rms} \sim 4.9$ ) and a worst case amplification factor ( $F_w \sim 20$ ). Herein we extend this calculation to consider a more realistic case of very long range wake including the kicks on all trailing bunches even though the kicks have small amplitudes. For illustration a case of 3 bunches is assumed. The direct effect (*a*) and direct-indirect effect (*A*) matrix formulation in this case is [5]

$$a = \begin{pmatrix} 0 & 0 & 0 \\ ia_1 & 0 & 0 \\ ia_2 & ia_1 & 0 \end{pmatrix}; A = Exp[a] = \begin{pmatrix} 1 & 0 & 0 \\ ia_1 & 1 & 0 \\ \frac{i^2 a_1^2}{2} + ia_2 & ia_1 & 1 \end{pmatrix}.$$
(2)

In matrix a,  $a_1$  and  $a_2$  are the kicks experienced by the first and second trailing bunch respectively due to an offaxis leading bunch; which is nothing but a direct effect. Whereas  $(ia_1)^2/2$  is the indirect effect on the second trailing bunch as is shown in matrix A. The aforementioned amplification factors are defined as [5]

$$F_{c} = \frac{1}{n} \sum_{k} \left| \sum_{j} A_{kj} \right|^{2}; F_{rms} = \frac{1}{n} \sum_{k=0}^{n-1} \sum_{j=1}^{k} A_{kj} A_{kj}^{*}$$
(3)

 $(u, \lambda, v) = \text{SVD}[A]; \qquad F_w = \lambda_1^2$ (4)where SVD is the singular value decomposition of the matrix A. Using the matrix formulation, the amplification factors in CLIC AS are calculated and the results are presented in Fig. 4. Beyond 100 bunches (50 ns) the  $F_c$ does not change. This means a driving bunch kicks about 100 trailing bunches and beyond this its contribution is negligibly small. The consequence is that simulation over the time of an entire bunch train (47 m) is not needed. In Fig. 5 it can be observed that  $F_c$  does not change alarmingly compared to the previously calculated value [5] and the difference is about +5% which is within the beam dynamics limit [8]. The other two amplification factors are also within the acceptable limits [8]. In order to understand the sensitivity of the amplification factors we introduce +/-1% random errors in bunch spacing (r.m.s  $\sim 0.6\%$ ). Though this error changes the kicks on the bunches, the amplification factor  $F_c$  changes by only 1%. This is illustrated in Fig. 6.

#### **FINAL REMARKS**

The baseline design of the CLIC AS for the CDR meets the beam dynamics criterion of damping HOMs. The maximum amplification of the beam due to coherent jitter considering the multi-bunch long range effect is about 6% which is within the acceptable limit.

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Figure 6: Amplification parameters of CLIC AS with random errors of +/-1% in bunch spacing.

## REFERENCES

- [1] M. Aicheler, et. al, CLIC-CDR, SLAC-R-985, 2012.
- [2] P. B. Wilson, SLAC-PUB 4547, 1989.
- [3] A. Grudiev and W. Wuensch, LINAC10, MOP068, 2010.
- [4] N. M. Kroll and X. Lin, SLAC-PUB-6144, 1993.
- [5] D. Schulte, PAC09, FR5RFP055, 2009.
- [6] www.gdfidl.de
- [7] K. Bane and B. Gluckstern, SLAC-PUB-5783, 1992.
- [8] D. Schulte, private communication.

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