

FIELD DESCRIPTION IN AN RFQ AND ITS EFFECT ON BEAM DYNAMICS

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Abstract

In a Radio Frequency Quadrupole (RFQ) linac, the electric field distribution is generated by four poles arranged symmetrically around a central z-axis. It's a customary practice to use a formulation based on cylindrical harmonics. That leads to inaccurate description of the fields near the boundary walls. Here, we present an analysis of errors based on an alternate numerical method to map the field correctly in the entire space between the vanes. This is especially important for high currents when the beam tends to fill nearly all available cavity-space [1]. The difference in the field-description and its effect on beam dynamics in a typical RFQ end cell will be presented. Two ways for calculating harmonic coefficients, the harmonic analysis and the fit to the cell geometry, are described.

1 INTRODUCTION

Radio Frequency Quadrupole (RFQ) linacs are widely used in the accelerator community. They have the remarkable capability of simultaneously bunching low-energy ion beams and accelerating them up to the MeV range. Several types of vane-tip geometries have been considered, and it is important to know the electric fields produced by these various geometries. In R.F.Q. linacs, the impedance in the beam zone is such that the electrodes can be considered as isopotential surfaces, and the vector potential can be ignored. Solving the Laplace equation in cylindrical coordinates, the electric potential seems to be able to be represented by the expansion:

$$U(r, \theta, z) = \frac{V}{2} \left(\sum_p A_{0(2p+1)} r^{2(2p+1)} \cos[2(2p+1)\theta] + \sum_{m,n} A_{nm} I_{2n}(mkr) \cos(2n\theta) \cos(mkz) \right)$$

The first eight terms give satisfactory accuracy [2]. Different methods are found in literature for calculating the coefficients of the expansion. The most common ones are the projection on the base [3] and the fit to the cell geometry [4,5]. This paper is a discussion on the accuracy that can be achieved with these two different methods.

2 CALCULATION OF THE ELECTRIC POTENTIAL IN A RFQ CELL BY TOSCA

We have first simulated the electric potential in a RFQ cell with the code TOSCA¹ (Figure 1).

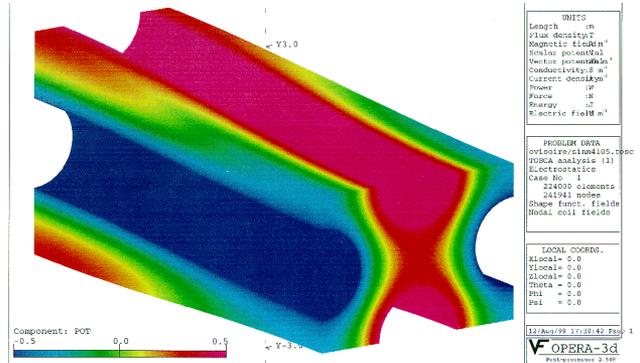


Figure 1: Electric potential in a RFQ cell.

This simulation will be the reference for validation and calculation in following stages. A typical RFQ end cell is chosen. The characteristics of the cell are:

- $\rho = 0.89$ (transverse radius of curvature)
- $L_c = 11$ (cell length)
- $m = 2$ (modulation factor)
- Sinusoidal profile in longitudinal direction

All distances are in R_0 units. The mesh of this simulation has been optimized for calculation of the peak surface electric field [6].

3 CALCULATION OF THE COEFFICIENTS BY PROJECTION ON THE BASE

To use the properties of harmonic analysis, we need to consider a limited space in which the boundaries satisfy the homogeneous conditions [7]. In cylindrical coordinates, the volume is a cylinder. The maximum value for the radius of this cylinder is "a", the minimum

¹ Vector Fields code for electromagnetic design

aperture radius. The coefficients can then be determined by the projection of the solution calculated by TOSCA on the lateral surface of this cylinder:

$$A_{0m} = \frac{16}{\pi V a^{2m} L_c} \int_0^{\frac{\pi}{2}} \int_0^{L_c} U(a, \theta, z) \cdot \cos(2m\theta) \cdot d\theta \cdot dz$$

$$A_{n0} = \frac{8}{\pi V I_0 (nka) L_c} \int_0^{\frac{\pi}{2}} \int_0^{L_c} U(a, \theta, z) \cdot \cos(nkz) \cdot d\theta \cdot dz$$

$$A_{nm} = \frac{16}{\pi V I_{2m} (nka) L_c} \int_0^{\frac{\pi}{2}} \int_0^{L_c} U(a, \theta, z) \cdot \cos(2m\theta) \cdot \cos(nkz) \cdot d\theta \cdot dz$$

Due to symmetry, only one quarter of a RFQ structure is necessary for the calculations. The results for the eight first order terms are shown in Table 1.

Table 1

A_{01}	A_{03}	A_{10}	A_{12}
0.9267	0.0012	0.6006	307.6125
A_{21}	A_{23}	A_{30}	A_{32}
-0.8337	-30753.6794	-0.0109	-7.0949

The difference between the real potential and the values obtained from the expansion derived coefficients is shown in Figure 2.

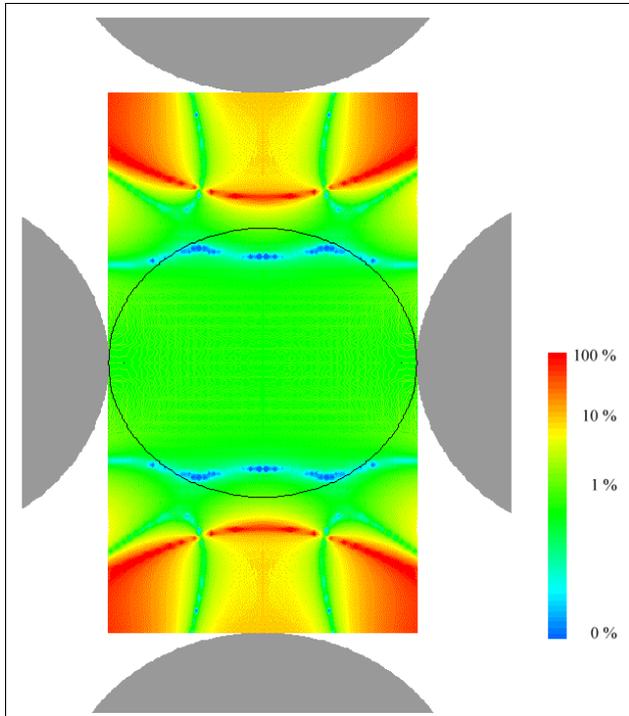


Figure 2: Difference between the real potential and the eight term potential. The accuracy is poor outside the cylinder used for calculations (black circle). A logarithmic scale is used.

It is apparent that the solution can't be extrapolated outside the volume used for integration. However, the accuracy is quite satisfactory within the cylinder as is shown in Figure 3.

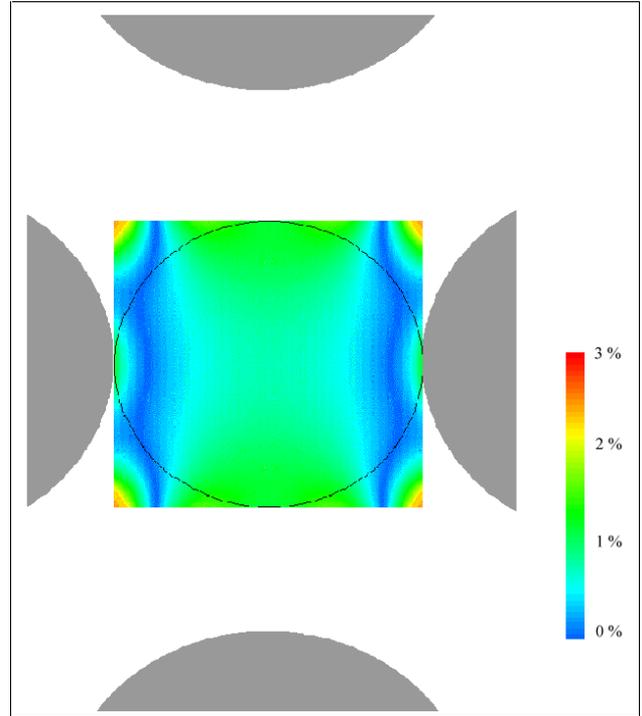


Figure 3: Same as in Figure 2 with a close-up view around the beam axis. A linear scale is used.

4 CALCULATION OF THE COEFFICIENTS BY FIT TO THE CELL GEOMETRY

The coefficients are calculated by the least-squares fit [8] using the isopotential surfaces defined by the conductors. This method is faster than the previous one for calculation of the expansion's coefficients. Table 2 contains the results of the fit obtained using the code MATHEMATICA².

Table 2

A_{01}	A_{03}	A_{10}	A_{12}
0.9587	0.0211	0.6064	3489.55
A_{21}	A_{23}	A_{30}	A_{32}
-0.5960	3493.32	-0.0093	-4.1060

The difference between the real potential and that obtained from the expansion using fit is shown in Figure 4.

² Environment for technical and mathematical computing.

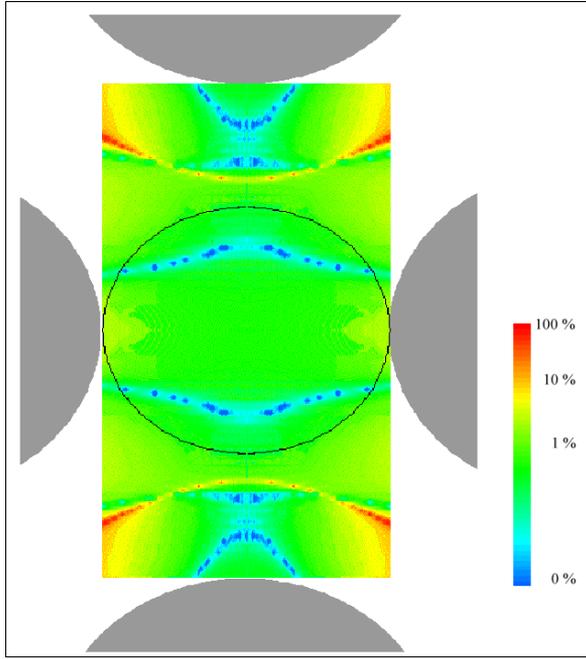


Figure 4: Difference between the real potential and the eight term potential with fit coefficients. A logarithmic scale is used.

The accuracy is better outside the cylinder used for projections (black circle). However, the deviation is about a few percent everywhere, which exceeds the tolerance for linacs ($\approx 1\%$ of maximum error).

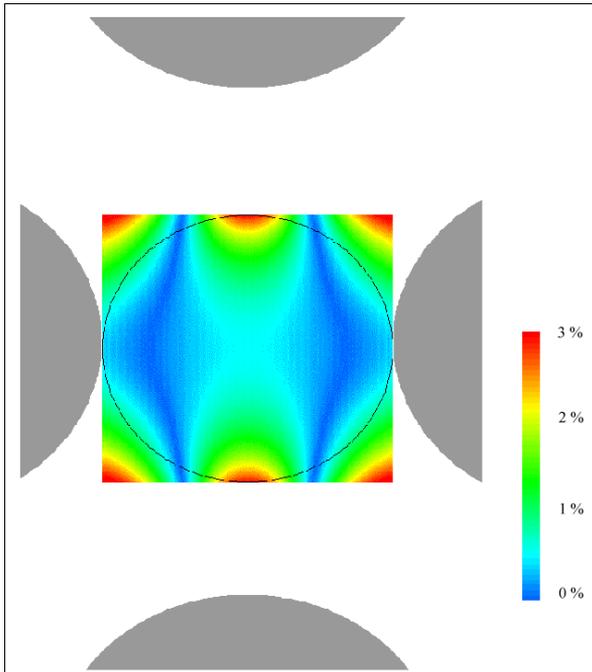


Figure 5: The same as in Figure 3 with a close-up view around the beam axis. A linear scale is used.

5 CONSEQUENCE ON BEAM DYNAMICS

In order to assess the influence of the difference induced by these two methods, beam dynamics simulation were done with particles that lay outside the cylinder of reference. 1035 particles of 1 MeV are uniformly distributed in a square with coordinates ($x = 0, a$; $y = a, m \times a$; $z = 0$) without any energy spread and zero transverse speed component for the injection. The transport is realized without any space charge and image effects. The intervane voltage is equal to 97 kV. The transmissions for three cases are summarized in Table 3.

Table 3

Real Fields	Projection	Fit
16.04 %	40.97 %	49.37 %

6 CONCLUSION

The projection on a cylindrical harmonic base gives the best accuracy of field around the beam axis. The fit to cell geometry by least-squares method doesn't give accuracy better than a few percent. However, regardless of the method used, cylindrical harmonics don't allow simulation with some accuracy in all region of the beam [2]. An alternative is to simulate the fields numerically. This is time consuming and needs a lot of memory to store the mesh for several cells. A good compromise would be to use 8 terms obtained by projection around the z-axis and a numerical mapping for the halo. A reduced mesh size is necessary in such case.

7 REFERENCES

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