UNDULATOR AND RF-SYSTEM FOR ION LINEAR ACCELERATORS

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Abstract

The paper continues the study of an undulator linear accelerator (UNDULAC). The various versions of UNDULAC with transverse and longitudinal RF fields are investigated. The suggested method of beam focusing and acceleration can be considered as the special case of those by means of two nonsynchronized waves. The comparison with two wave approximation method, where the accelerating wave synchronizes and the focusing wave doesn't synchronize with beam, is described. This comparison is suitable for demonstration of the capabilities of the new method.

Introduction

It is known, that one wave can not simultaneously accelerate and focusing the particles of beam. In a normal drift tube linac (DTL) where one synchronous wave is present, longitudinal beam stability is provided, but then transverse beam defocusing occurs inevitably. The transverse beam stability can be ensured by quadruple magnets, installed in the drift tubes, or by.a.focusing wave which has a phase velocity different from the equilibrium particle velocity (RF-focusing). In a conventional low energy ion DTL, it is difficult to install focusing magnets of sufficient strength in the very short drift tubes. That is why RF-focusing is used. The simplest RF-focusing principle for periodical structure was suggested by Good [1] and Faynberg [2] and was named an alternating phase focusing (APF). Such focusing can be realized by changing the drift tube lengths and applying some gaps to one period. The beam dynamics theories for APF have been proposed in many papers [3-5]. The main idea consists of the study of space harmonics influence on beam dynamics. The accelerating harmonic synchronizes with beam, whereas the focusing harmonics have phase velocities different from equilibrium particle velocity. In the paper [4] it was shown that the two travelling waves approach can be applied to three main types of APF. In contrast to the two wave approximation, a standing wave approach for many harmonics was used in the theory discussed in [5].

The other way to create three-dimensional potential well by means of two nonsynchronous waves was mentioned in the paper [6]. Summary field $\mathbf{E}=\mathrm{Re}\{\mathbf{E}_1 \mathrm{e}^{ik_1 z-i\omega_1 t}+\mathbf{E}_2 \mathrm{e}^{ik_2 z-i\omega_2 t}\}$ can accelerate particles efficiently with simultaneous achievement of longitudinal and transverse beam stability, if k_1, k_2 and ω_1, ω_2 are linked by a relation $\omega_1+\omega_2=v_s(k_1+k_2)$. In the low energy ion accelerator it is difficult to create resonator system for the two varied frequencies. However in the case, when one of frequencies $\omega_i=0$ (a static field of undulator), the problems with RF-system tuning do not arise. The idea to apply a combination of undulator field and RF field for acceleration and focusing of ion beams was discussed in [7] for magnetostatic undulator (UNDULAC-M) and electrostatic undulator (UNDULAC-E).

Longitudinal beam dynamics

At first, we discuss the motion equation for a particle in the UNDULAC-M. The coordinates and the kinetic momentums of the particles can be expressed by the summation of two different type of motion: the slowly varying $\mathbf{R}_\mathrm{C}, \mathbf{p}$ and the rapidly oscillating $\tilde{\mathbf{R}}, \tilde{\mathbf{p}}$. By averaging over rapid oscillations, we obtain the equation, that describes the slow evolution of $\mathbf{R}_\mathrm{C}$

$$\frac{\mathrm{d}^2 \mathbf{R}_\mathrm{C}}{\mathrm{d}t^2}=-\frac{\mathrm{e}^2}{2m^2}\nabla<\mathbf{A}_\Sigma^2>, \qquad (1)$$

where $\mathbf{A}_\Sigma=\mathbf{A}_v+\mathbf{A}_0$ is the total vector potential of RF field and periodical field of undulator. Taking into account only the main space harmonics of the magnetostatic undulator $\mathbf{a}_0=\frac{\mathrm{e}\mathbf{A}_0}{mc}=\mathbf{a}_{0,1}\mathrm{e}^{ik_0 z}$ and electromagnetic field in RF structure $\mathbf{a}_v=\frac{\mathrm{e}\mathbf{A}_v}{mc}=\mathbf{a}_v \mathrm{e}^{ikz-i\omega t}$ with $k<<k_0$, the equation (1) can be rewritten in the form

$$\frac{\mathrm{d}^2 \mathbf{r}}{\mathrm{d}\tau^2}=-\frac{1}{4}\nabla_\mathbf{r} U_\mathrm{M}, \qquad (2)$$

where $\mathbf{r}=\frac{2\pi}{\lambda}\mathbf{R}_\mathrm{C}$, $\lambda=\frac{2\pi}{k}$ - the RF-field wavelength, $\lambda_0=\frac{2\pi}{k_0}$ - the undulator period, $\tau=\omega t$. The potential function $U_\mathrm{M}=U_{1,\mathrm{M}}(\mathbf{r}^\perp)+U_{2,\mathrm{M}}(\mathbf{r}^\perp,\psi)$

$$U_{1,\mathrm{M}}=\left|\mathbf{a}_{0,1}\right|^2+\left|\mathbf{a}_v\right|^2;\ U_{2,\mathrm{M}}=2\,\mathrm{Re}\left(\mathbf{a}_{01}\mathbf{a}_v^* \mathrm{e}^{i\psi}\right), \qquad (3)$$

$\psi=z/\beta_s-\tau+\psi_0$ is the particle phase in the composite wave field, ψ_0 - the initial phase, $\beta_s=\lambda_0/\lambda$ - the normalized velocity of the synchronous particle.

The same equation can be obtained for the UNDULAC-E, where the potential function $U_\mathrm{E}=U_{1,\mathrm{E}}\left(\mathbf{r}^\perp\right)+U_{2,\mathrm{E}}\left(\mathbf{r}^\perp,\psi\right)$

$$U_{1,\mathrm{E}}=\left|\mathbf{e}_{0,1}\right|^2+\left|\mathbf{e}_v\right|^2;\ \ U_{2,\mathrm{E}}=2\,\mathrm{Re}\left(\mathbf{e}_0\cdot\mathbf{e}_v\,\mathrm{e}^{i\psi}\right) \qquad (4)$$

Here $\mathbf{e}_{v,0} = \mathrm{e}\,\mathbf{E}_{v,0}\lambda / 2\pi mc^2$ are the dimensionless amplitudes of the basic RF-field harmonic and the first electrostatic field harmonic.

The acceleration rate is proportional to the amplitudes of the RF and the undulator fields. The energy increase is maximum, when $\mathbf{B}_v^{\perp} \| \mathbf{B}_0^{\perp}$ or $\mathbf{E}_v \| \mathbf{E}_0$. Therefore the choice of the magnetic (electrostatic) undulator type and field orientation depend on the RF-structure type [7].

In order to achieve effective beam bunching and capture the function and undulator period λ_0 have to grow and the synchronous phase has to decrease from $\pi / 2$. For example, if the synchronous phase $\psi_s = \pi / 2 - \mu z$, where μ is the factor of sliding, the undulator period is growing and can be calculated by the formula

$$\lambda_0(z) = \left[\lambda_0^3(0) + \frac{3}{2}\lambda_v^3 \int_0^z U_2\left(\mathbf{r}^{\perp}, \mu z\right) \mathrm{d}\, z \right]^{1/3}. \quad (5)$$

The acceleration gradient of the synchronous particle $\mathrm{d} W_s / \mathrm{d}\, z = \mathrm{e}\, T_{\mathrm{M,E}} E_v \cos\psi_s$, where E_v is basic nonsynchronous harmonic amplitude, $T_{\mathrm{M,E}}$ - an acceleration efficient factor for UNDULAC-M and for UNDULAC-E

$$T_{\mathrm{M}} = \frac{\mathrm{e}\, B_0 \lambda}{2\pi mc}, \quad T_{\mathrm{E}} = \frac{\mathrm{e}\, E_0 \lambda^2}{2\pi mc^2 \lambda_0}. \quad (6)$$

It is interesting to compare this acceleration method with APF linac for resonant periodical structure, where RF field has one synchronous harmonic E_s and a number of nonsynchronous harmonics E_n. The equation of motion for APF linac can be received in the smooth approximation as above

$$\frac{\mathrm{d}^2 \mathbf{r}}{\mathrm{d}\,\tau^2} = \mathrm{Re}\left(\mathbf{e}_s\, \mathrm{e}^{i\psi}\right) - \frac{1}{4}\nabla_{\mathbf{r}} \overline{U}_{\mathrm{E}}, \quad (7)$$

where $\mathbf{e}_s$ is the amplitude of synchronous harmonic. The potential function $\overline{U}_{\mathrm{E}}$ is analogous to (4) and consists of two parts:

$$\overline{U}_{\mathrm{E}} = \overline{U}_{1,\mathrm{E}}\left(\mathbf{r}^{\perp}\right) + \overline{U}_{2,\mathrm{E}}\left(\mathbf{r}^{\perp}, 2\psi\right),$$

$$\overline{U}_{1,\mathrm{E}} = \frac{1}{4}\left|\mathbf{e}_s\right|^2 + \sum_{n \neq \pm s} \alpha_{ns} \left|\mathbf{e}_s\right|^2, \quad (8)$$

$$\overline{U}_{2,\mathrm{E}} = \frac{1}{2}\mathrm{Re}\left\{ \mathbf{e}_s^* \mathbf{e}_{3s}\, \mathrm{e}^{2i\psi} + 2 \sum_{s \neq \pm s, 3s} \alpha_{ns} \mathbf{e}_n \mathbf{e}_{n-2s}^*\, \mathrm{e}^{2i\psi} \right\},$$

e_n is the nonsynchronous harmonic amplitudes of RF-field, $\alpha_{ns} = s^2 / (n - s)^2$.

For two travelling waves $E_s \cos(k_s z - \omega t)$ and $E_n \cos(k_n z - \omega t)$, $\overline{U}_{2,\mathrm{E}} = 0$. In this approach the acceleration gradient is proportional $\mathrm{e}_{v,s}\cos\psi$ and is independent on the nonsynchronous harmonic $\mathrm{e}_{v,n}$. For the resonant structure only a standing wave approach must be used. In this case the all back waves are considered. The conditions of longitudinal beam stability are changed because of $\overline{U}_{2,\mathrm{E}} \neq 0$. If the amplitude of the nonsynchronous harmonics are large, additional maxima and minimuma in the longitudinal potential well occur. The phase trajectories are deformed and new separatrixs appear on the phase plane. In the APF linac the velocities of the nonsynchronous harmonics are approaching to equilibrium velocity, when the injection ion beam energy is low. In this case the resonant capture of particles with $v \neq v_s$ is possible and overlap of a synchronous resonance with another one on the phase plane takes place. As a result longitudinal stochastic instability occurs [8]. The magnitudes of the nonsynchronous harmonic amplitudes must be chosen to retain large longitudinal acceptance. But the most important circumstance, which limiting the amplitudes of harmonics in APF linac, is the realization of transverse beam focusing.

Transverse focusing of the beam

Let us consider first the transverse beam focusing in UNDULAC. The choice of the RF-field harmonic amplitude and the undulator field harmonic amplitude is not arbitrary because it is necessary to keep up the focusing of the beam simultaneously with acceleration. The total effect can be found only from the analysis of equation

$$\nabla_{\perp} U_{\mathrm{M,E}} = 0. \quad (9)$$

Equilibrium trajectories may exist for all particle phase, if two conditions are valid

$$\nabla_{\perp} U_{1,\mathrm{M,E}} = 0, \nabla_{\perp} U_{2,\mathrm{M,E}} = 0. \quad (10)$$

In a simple case the potential functions U_{ME} can be found without considering higher harmonics of the RF-field and undulator field (3), (4). The motion around an equilibrium trajectory is stable if the potential has an absolute minimum. The analysis of the transverse stability for undulators of different types shows, that trajectory of the particle with any phase is stable in the transverse movement, if $B_0 = \alpha_{\mathrm{M}} \frac{\lambda}{\lambda_0} B_v$, $E_0 = \alpha_{\mathrm{E}} E_v$, where $\alpha_{\mathrm{M,E}} \cong 1$. It is possible to express the acceleration efficient factor T_{ME} in (6) with the amplitude of the main nonsynchronous RF-field harmonic.

$$T_{\mathrm{M,E}} = \frac{\mathrm{e}\, E_v \lambda}{2\pi mc^2 \beta_s} \alpha_{\mathrm{M,E}}. \quad (11)$$

The acceleration rate decreases when λ_0 and the beam velocity grow. This fact indicates that UNDULAC is more suitable for the low energy region.

The exact magnitude $\alpha_{\mathrm{M,E}}$ can be found after analysis of decisions of (9) and (10) where the influence of the higher harmonics must be accounted for. It is important also to study non-linear oscillations of the beam particles to investigate the coupling resonances. As a rule the higher harmonic amplitudes reduce the quality of the bunch and must be minimized.

For APF linac it is necessary to have large amplitudes of the higher harmonics to supply the transverse beam focusing. When only one synchronous E_s and one nonsynchronous E_n harmonics of the travelling wave are taken into account, the condition stability for a small oscillations can be derived from equation (7), where $\overline{U}_{1,\mathrm{E}} = \overline{\alpha}_{ns}|\mathbf{e}_n|^2$, $\overline{U}_{2,\mathrm{E}} = 0$,

$$\overline{\alpha}_{ns}\,\mathrm{e}_n^2\, g - \beta_s\,\mathrm{e}_s \sin\psi > 0,$$

$$\overline{\alpha}_{n,s} = \frac{k_s^2}{(k_n - k_s)^2}, \quad g \cong 1. \qquad (12)$$

The amplitude E_s and the acceleration rate can be large when the synchronous phase $\psi_s \cong 0$ and the longitudinal acceptance is small. The trajectory of the particle with any phase ψ_s is stable in the transverse direction, when

$$E_s < \frac{\mathrm{e}\,E_n^2\lambda}{2\pi mc^2\beta_s}\overline{\alpha}_{ns} g. \qquad (13)$$

To express the acceleration rate with nonsynchronous harmonics amplitude E_n, we use

$$\frac{\mathrm{d}W_s}{\mathrm{d}z} = T_{\mathrm{RF}} E_n \cos\psi_s, \qquad (14)$$

where the factor $T_{\mathrm{RF}} = \frac{\mathrm{e}\,E_n\lambda}{2\pi mc^2\beta_s}\overline{\alpha}_{ns} g.$ This magnitude coincides with (11) for UNDULAC.

For the resonator structure it is necessary to use a standing wave approach and to take into account a number of nonsynchronous harmonics. As it was showed above in this case $\overline{U}_{2,\mathrm{E}} \neq 0$, the acceleration rate and the longitudinal acceptance may be inferior to the two wave approximation. The analysis of the transverse stability for APF linac may be implemented if the potential function $\overline{U}_{\mathrm{E}}$ is used. As in the UNDULAC the higher nonsynchronous harmonics reduce the longitudinal and transverse acceptances in the APF linac.

Examples

As it was shown above, the longitudinal or the transverse RF-field can be used for acceleration and focusing of ion beams in the UNDULAC. There is no need for drift tubes. It simplifies the design of RF-system and makes it possible to operate at higher frequency or in a lower-β region than usual ion RF-accelerator. Many versions of unconventional design permit the increase of the ion beam intensity in the UNDULAC.

The current may be increased for a ribbon beam with the large cross-section. Study of the ribbon ion beam interaction with the RF-field in the plane electrostatic undulator was carried out in the paper [9].

For UNDULAC-M, it is impossible to obtain the large cross-section area of the beam. However, there is an opportunity to accelerate more then one beam in the magnetic channel because there are no drift tubes. The task is to choose a special symmetry of the transverse RF-field and periodical magnetostatic field. The RF system must have a small transverse dimension to fit inside the undulator. Therefore, it is preferable to use a multielectrode line where transverse electromagnetic waves(TEM or TE) travel [10].

The one more interesting method for increasing of the ion beam intensity in UNDULAC exists. How was shown the potential function $U_{\mathrm{E,M}}$ depends on the particle charge squared, and the motions of positive and negative charged ions are identical. This fact can be used for compensation of the space charge by acceleration of ions with different signs of the charge in the same bunch. Study of the possibility of simultaneous acceleration of both positive and negative ions with the identical charge-to-mass ratio in UNDULAC is great interest because in all kinds of RF accelerators (DTL, RFQ, APF), this opportunity can not be realized [11].

Conclusion

A use of the undulator and RF resonator system to accelerate low energy ion beams promises to be a very perspective practice. The conditions to achieve both the transverse focusing and large longitudinal acceptance are found. The acceleration rate for UNDULAC is comparable with the acceleration rate for APF linac, but the definite advantages exist for UNDULAC. For acceleration of ions it possible to use not only longitudinal, but also transverse RF- fields. There is no need to have the drift tubes in the RF-structure, where only one nonsynchronous wave is travelling. It simplifies the design of RF-system and permits to reduce a RF power losses in the walls and to increase the ion beam intensity. Three methods for increasing of the ion beam intensity in UNDULAC are suggested.

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