A NEW APPROACH TO SPACE CHARGE FOR LINAC BEAM DYNAMICS CODES

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Abstract

Apart from very computer time consuming PPI routines (Particle to Particle Interaction, e.g. [1]), all previous space charge routines require some kind of symmetry. A new routine, not requiring symmetry, is being developed. It offers fast computation and is little sensitive to statistical noise. It could become a good tool for studying halo formation phenomena.

Introduction

Almost all presently available space charge routines require some form of symmetry. The common SCHEFF routine [1][2], transforms the density distribution into a rotationally symmetrical one, where the field is computed and linearly corrected inside an elliptical cylinder. The outside density distribution is obtained using a different method, and discontinuities appear. Moreover it is very sensitive to statistical noise. In the SC3DELP [3] and MAPRO [4] routines, the bunched beam is assumed to have an ellipsoidal symmetry. The former SCHERM routine [2], represents the non-symmetrical longitudinal shape of the bunch with two or three ellipsoids. However it keeps in the transverse direction an elliptical profile. The MOTION routine [5] treats the two transverse directions x and y independently.

The present work suggests a new type of approach, offering fast computation without the need of strict symmetry.

Three dimensional representation of the bunch

The three dimensional representation of the charge distribution of the bunch is made with Hermite expansions.

As shown in [2], one can obtain the integrated charge density along one axis in the form of a Hermite series expansion :

$$r(x) = \sum_i A_i H_i\left(\frac{x}{a}\right)\exp\left(-\frac{x^2}{2a^2}\right) \qquad (1)$$

a is the r.m.s. dimension of the bunch along the x axis and A_i is defined as :

$$A_i = \frac{q}{i!\sqrt{2p}} \sum_{n=1}^{N} H_i\left(\frac{x_n}{a}\right) \qquad (2)$$

where q is the charge of the macroparticle. Analogous relations are valid in the y and z directions, with b and c as respective r.m.s. dimensions. In these expressions the coordinates are taken with respect to the centre of gravity of the bunch; N is the number of particles in the bunch. As $H_1(u) = u$ and $H_2(u) = u^2 - 1$ it turns out that $A_1 = A_2 = 0$.

Most functions of a coordinate can be represented with a Hermite series expansion. For instance, one may be interested in the variation a(z) of the transverse r.m.s. dimension in x of the bunch along the z axis. To do so one can compute the following function of z :

$$H_2\left[\frac{x(z)}{a}\right] r(z) = \sum_k A_{2,k} H_k\left(\frac{z}{c}\right)\exp\left(-\frac{z^2}{2c^2}\right) \qquad (3)$$

where

$$A_{2,k} = \frac{q}{k!\sqrt{2p}} \sum_{n=1}^{N} H_2\left(\frac{x_n}{a}\right) H_k\left(\frac{z_n}{c}\right) \qquad (4)$$

One obtains :

$$a^2(z) = a^2 + a^2 \frac{\sum_k A_{2,k} H_k\left(\frac{z}{c}\right)}{\sum_k A_k H_k\left(\frac{z}{c}\right)} \qquad (5)$$

A_k is given by :

$$A_k = \frac{q}{k!\sqrt{2p}} \sum_{n=1}^{N} H_k\left(\frac{z_n}{c}\right) \qquad (6)$$

Similarly, one could compute the variation of the centre of gravity in z along the x axis (ditto for the y axis). One gets :

$$z_0(x) = c \frac{\sum_i A_{i,1} H_i\left(\frac{x}{a}\right)}{\sum_i A_i H_i\left(\frac{x}{a}\right)} \qquad (7)$$

with

$$A_{i,1} = \frac{q}{i!\sqrt{2p}} \sum_{n=1}^{N} \frac{z_n}{i} H_i\left(\frac{x_n}{a}\right) \qquad (8)$$

and analogous relations for z(y). An example is given in Fig.1.

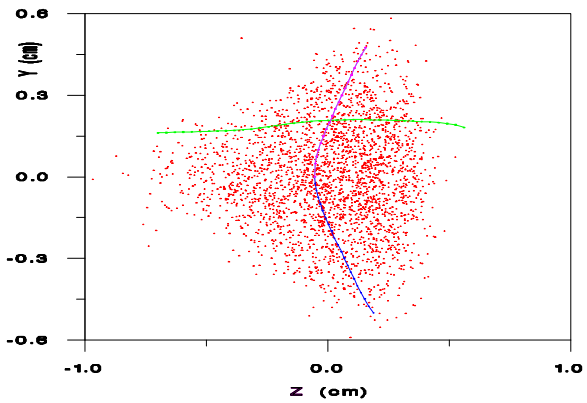

Fig.1. Projection on the (y,z) plane of the particle distribution in a bunch at the output of the 200 mA proton CERN RFQ2BThe variations of the y r.m.s. size along the z-axis and the centre of gravity in z along the y axis are shown.

In the above derivation the terms $A_{2,k}$ and $A_{i,1}$ of a two dimensional Hermite series expansion have been

introduced. Such an expansion can be extended to three dimensions, giving the local density distribution ρ (x,y,z) :

$$r\left(\frac{x}{a},\frac{y}{b},\frac{z}{c}\right)=\sum_{i,j,k} A_{ijk}\, H_i\left(\frac{x}{a}\right)H_j\left(\frac{y}{b}\right)H_k\left(\frac{z}{c}\right)\exp\left(-\frac{x^2}{2a^2}-\frac{y^2}{2b^2}-\frac{z^2}{2c^2}\right) \quad (9)$$

with

$$A_{ijk}=\frac{q}{(2p)^{3/2} i!\, j!\, k!}\sum_{n=1}^{N} H_i\left(\frac{x_n}{a}\right)H_j\left(\frac{y_n}{b}\right)H_k\left(\frac{z_n}{c}\right) \quad (10)$$

As above one has $A_{100}=A_{010}=A_{001}=A_{200}=A_{020}=A_{002}=0$. If the three axes are chosen such that $\overline{xy}=\overline{yz}=\overline{zx}=0$, the corresponding terms A_{110}, A_{011} and A_{101} are also zero. This last condition is not necessary; would it be needed one can straighten up the axes by rotation. The main term A_{000} is such that :

$$A_{000}=\frac{q\mathrm{N}}{(2p)^{3/2}} \quad (11)$$

Computation of the field distribution

The charge density distribution in a bunch can be considered to be a Gaussian :

$$r_0(x,y,z)=A_{000}\exp\left(-\frac{x^2}{2a^2}-\frac{y^2}{2b^2}-\frac{z^2}{2c^2}\right) \quad (12)$$

corrected by various terms (coefficients A_{ijk} in eq.9), each of them of total charge zero. In practice most of these terms are in the range of a few per cent or less of the fundamental term A_{000}, only a few ones are slightly above 10 %.

The field due to the Gaussian term (eq.12) is computed by numerical integration as explained in [2], but the macroparticle charge density contains a single term only and therefore computer time remains small. For the other terms the method introduces approximate expressions of the field, good in the part of the bunch where most of the particles lie, but less correct at large distances where the field is anyway very weak. According to the Laplace-Poisson relation :

$$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z}=\frac{r(x,y,z)-r_0(x,y,z)}{e_0} \quad (13)$$

and the following property :

$$\frac{d}{du}\left[H_i(u)\exp\left(-\frac{u^2}{2}\right)\right]=-H_{i+1}(u)\exp\left(-\frac{u^2}{2}\right) \quad (14)$$

one can take for the charge term , with $A_{ijk}\neq A_{000}$:

$$A_{ijk}H_i\left(\frac{x}{a}\right)H_j\left(\frac{y}{b}\right)H_k\left(\frac{z}{c}\right)\exp\left(-\frac{x^2}{2a^2}-\frac{y^2}{2b^2}-\frac{z^2}{2c^2}\right)$$

yielding to the following field components :

$$E_x=-a\frac{a}{e_0}A_{ijk}H_{i-1}\left(\frac{x}{a}\right)H_j\left(\frac{y}{b}\right)H_k\left(\frac{z}{c}\right)\exp\left(-\frac{x^2}{2a^2}-\frac{y^2}{2b^2}-\frac{z^2}{2c^2}\right) \quad (15)$$

$$E_y=-b\frac{b}{e_0}A_{ijk}H_i\left(\frac{x}{a}\right)H_{j-1}\left(\frac{y}{b}\right)H_k\left(\frac{z}{c}\right)\exp\left(-\frac{x^2}{2a^2}-\frac{y^2}{2b^2}-\frac{z^2}{2c^2}\right) \quad (16)$$

$$E_z=-g\frac{c}{e_0}A_{ijk}H_i\left(\frac{x}{a}\right)H_j\left(\frac{y}{b}\right)H_{k-1}\left(\frac{z}{c}\right)\exp\left(-\frac{x^2}{2a^2}-\frac{y^2}{2b^2}-\frac{z^2}{2c^2}\right) \quad (17)$$

a, b, c define the r. m. s. size of the bunch, and $a+b+g=1$. These α, β and γ, which depend on a, b and c but also on i, j and k, have to be optimized for each term. At large distance the real field vanishes as r^{-n} (with n>2) instead of exponentially. However, the error made is not significant.

A difficulty appears in the eq.(15), (16) and (17) for i, j or k = 0 : the integral of $H_0\left(\frac{x}{a}\right)\exp\left(-\frac{x^2}{2a}\right)$, which corresponds to the error function, does not go down to zero at infinity. It is therefore replaced by a polynomial equivalent to the above integral over most of the interval, but going to zero at infinity.

Preliminary results of the new method

Since the major interest of this new method is to compute space charge fields in the absence of any symmetry, comparisons have been made with a PPI routine in order to check its validity.

In order to explain the differences which could arise in these comparisons, it is essential to examine the limits of the PPI routine such as its sensitivity to statistical noise and to the so-called “stopping distance“, which avoids that the field becomes infinite when the macroparticles are too close to each other.

In Figures 2, 3 and 4 comparisons between the new space charge routine and previous ones for the CERN proton linac at 200 mA are shown for different positions along the machine for 5000 particles. In Fig.4 one clearly observe the differences in particle distributions between the results obtained with the new space charge routine and SCHEFF at the output of the CERN proton linac. The emittances and transmissions , however , yield to very similar values.

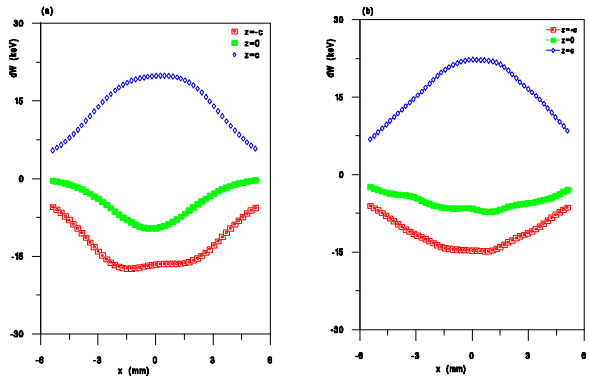

Fig. 2 The longitudinal impulse ΔW as a function of x in the bunch at a given point in the machine for the new routine (a) and a PPI routine (b). Curves are shown for the (z,y) plane with z=0,+c,-c .

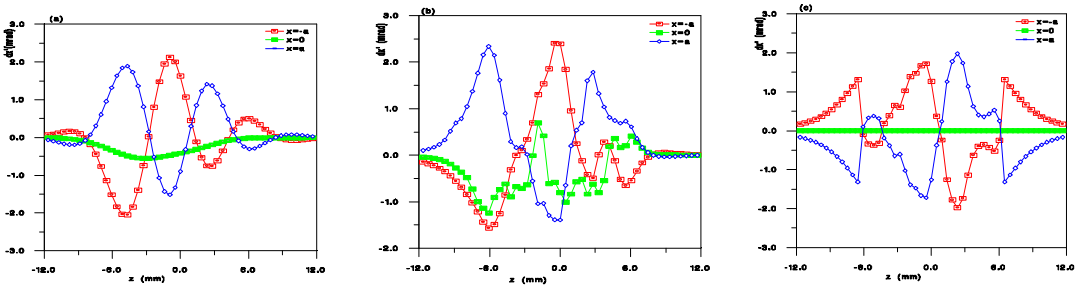

Fig. 3 The transverse impulse as a function of z in the bunch at a given point in the machine for the new routine (a), a PPI routine (b) and SCHEFF (c). The curves shown correspond to the differences between the total impulse and the Gaussian contribution for the (x,z) plane with x=0,+a,-a .

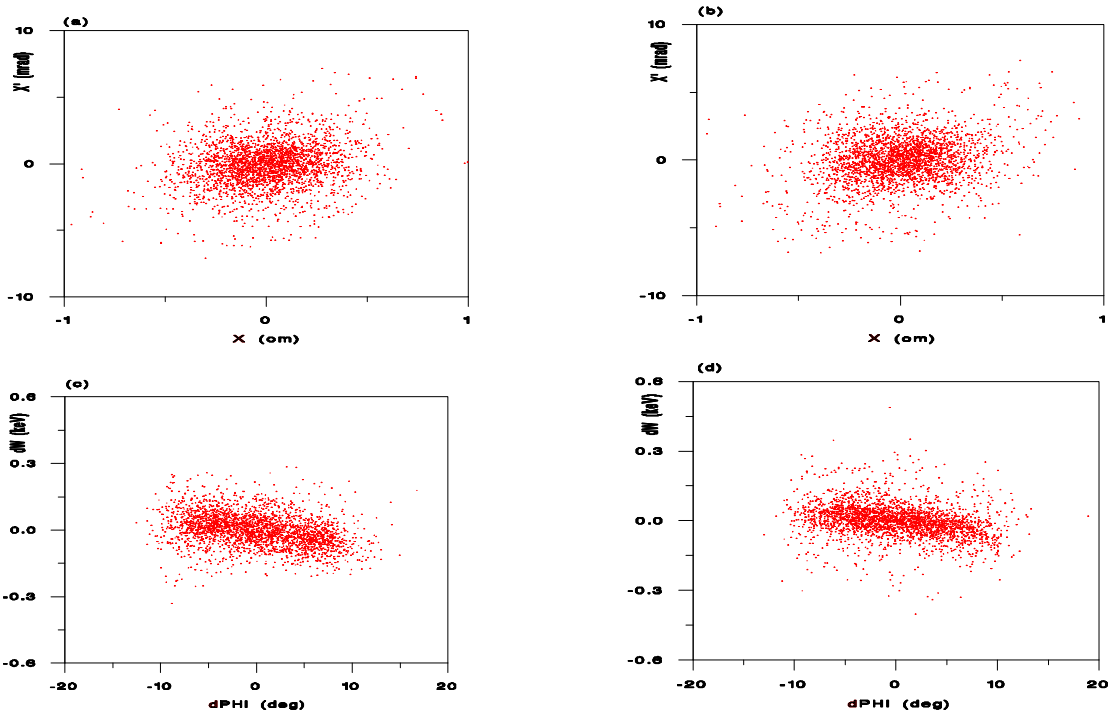

Fig. 4 The beam is shown in the (x,x') plane at a given point in the machine for the new routine (a) and for SCHEFF (b). In (c) and (d) the beam is shown in the longitudinal plane for the new routine and SCHEFF respectively.

Conclusion

This new space charge method is very promising the design and operation of high intensity linacs. The computing time needed with the new method is significantly shorter than any other presently available routine. Some possible refinements of the present method are still being studied. This routine might also be used for cyclotrons.

Acknowledgments

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