MEASUREMENT OF HOM-PROPAGATION THROUGH CAVITY CHAINS IN TERMS OF S-PARAMETERS

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Abstract

The propagation of HOM-energy along an accelerator channel can be described in terms of frequency dependent scattering (S-) parameters of the individual elements of the channel. These S-parameters can be measured for each element (cavities, couplers, etc.) separately. Once they are known, it is possible to predict the behaviour of any arbitrary combination of elements. As long as only one waveguide mode propagates in the connecting pipes, standard RF calibration schemes are applicable methods to measure the three S-parameters of the representing two-port. In the presence of additional modes - corresponding to higher frequencies - S-matrices of higher dimensions have to be determined. Therefore we have been developing an experimental method which allows for determination of S-parameters in the regime of waveguide ports with several propagating modes. The principles of the method as well as results from measurements of normal conducting TESLA cavity models are presented.

Introduction

The TESLA HOM-damping scheme consists of two couplers attached to either side of each cavity and a single absorbing element in a 8-cavity module [1]. The latter is intended to dissipate HOM-power propagating through the accelerator. This leads to the question of how to measure RF power transmission in a complicated structure at frequencies that may allow for the appearance of more than only the fundamental waveguide mode. Therefore the problem exceeds the capabilities of the usual two-port S-parameter measurement, which is only appropriate for a single propagating mode. Even then the question of de-embedding the test devices properties from the measurement results, being modified by the necessary coaxial line-waveguide-adaptors, remains, but it is similar to calibration problems in pure coaxial setups. If more modes are present in the waveguides there was to our knowledge no practicable method available to measure a multidimensional S-matrix at an arbitrary (for a given number of modes) fixed frequency (or a spectrum of them).

We performed measurements in the frequency range with only the fundamental mode propagating (2.25 GHz to 2.95 GHz for 78 mm diameter TESLA beam pipe) using a standard Through-Short-Delay-calibration method (eg. [3]). For higher frequencies we have been developing an alternate method that has been tested now with two waveguide modes for a device measurement and with three modes for a calibration of an adaptor at single frequency points (see [4] for details).

Measurements with one mode

Fig. 1 shows results from single mode S-parameter measurements of two 9-cell cavities (compare [2] for details) using a TSD-method for the adaptor calibration. With the knowledge of the individual S-matrices one can calculate the result expected for two cavities chained together. This calculation is plotted in Fig. 1 together with the measured transmission of the chain.

In Fig. 2 the calculated transmission through four identical cavities is plotted. One observes a behaviour well known from filter cascades: The slopes increase with the length of the chain.

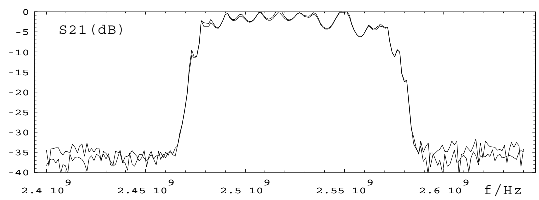

Fig. 1 Transmission through a chain of two TESLA 9-cell copper cavities: Calculated from single cavity measurements and measured directly (two curves)

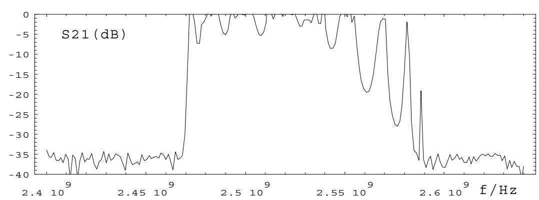

Fig. 2 Transmission through a chain of four TESLA 9-cell copper cavities, calculated from single cavity measurement

Measurements with more than one mode

In the case of more than one propagating mode the S-matrix of an adaptor with one coaxial line (index 0) and n waveguide ports may be written as:

$$\underline{\underline{\hat{A}}} = \left(\begin{array}{c|ccc} A_{00} & A_{01} & \cdots & A_{0n} \\ \hline A_{01} & A_{11} & \cdots & A_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ A_{0n} & A_{1n} & \cdots & A_{nn} \end{array} \right) = \left(\begin{array}{c|c} A_{00} & \vec{A}^{T} \\ \hline \vec{A} & \underline{\underline{A}} \end{array} \right) \qquad (1)$$

Herein the scalar A_{00} describes the reflection at the coaxial port, the vector the coupling from the coaxial line to each waveguide mode and the submatrix the reflection at the waveguide flange, that may couple every mode to each other. The matrix is symmetric due to the reciprocity of the device. Like in the single mode case, the problem of determining the properties of a device splits into the calibration step - i.e. determination of the adaptors - and the measurement once the adaptors are known. Considering the number of unknowns (10 in the case of two modes at two waveguide ports) it becomes clear, that a single measurement with two completely known adaptors, which gives three numbers (two reflection, one transmission quantity), is not able to provide a sufficient amount of information. Thus one has to use different pairs of known adaptors for a number of subsequent device measurements. To keep the calibration effort as small as possible we take only two fixed adaptors and combine them with various delay line lengths (see Fig. 3). In the same manner we use a short (which is one of the very few reliable broadband standards in waveguide technique) and different delay line lengths to calibrate the two adaptors.

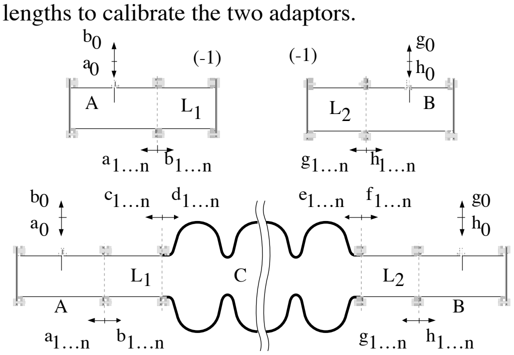

Fig. 3 Schematic drawing of setups used for adaptor calibration with delayed shorts and for measurement. Small letters denote the signals at all connection planes, index 0 corresponds to the coaxial line.

Basic Equations

If we consider a setup with two adaptors A and B, a test device C and two connecting waveguides with lengths L_1 and L_2 (see Fig. 3), we are able to write down all signals, related by S-matrices:

$$\begin{pmatrix} b_0 \\ \vec{b} \end{pmatrix} = \begin{pmatrix} A_{00} & \vec{A}^T \\ \vec{A} & \underline{\underline{A}} \end{pmatrix} \begin{pmatrix} a_0 \\ \vec{a} \end{pmatrix}, \quad \begin{pmatrix} g_0 \\ \vec{g} \end{pmatrix} = \begin{pmatrix} B_{00} & \vec{B}^T \\ \vec{B} & \underline{\underline{B}} \end{pmatrix} \begin{pmatrix} h_0 \\ \vec{h} \end{pmatrix} \qquad \text{(2a,b)}$$

$$\begin{pmatrix} \vec{a} \\ \vec{d} \end{pmatrix} = \begin{pmatrix} 0 & \underline{\underline{E}}(L_1) \\ \underline{\underline{E}}(L_1) & 0 \end{pmatrix} \begin{pmatrix} \vec{b} \\ \vec{c} \end{pmatrix}, \quad \begin{pmatrix} \vec{e} \\ \vec{h} \end{pmatrix} = \begin{pmatrix} 0 & \underline{\underline{E}}(L_2) \\ \underline{\underline{E}}(L_2) & 0 \end{pmatrix} \begin{pmatrix} \vec{f} \\ \vec{g} \end{pmatrix} \qquad \text{(2c,d)}$$

$$\begin{pmatrix} \vec{c} \\ \vec{f} \end{pmatrix} = \begin{pmatrix} \underline{\underline{C}}_{11} & \underline{\underline{C}}_{12} \\ \underline{\underline{C}}_{12}^T & \underline{\underline{C}}_{22} \end{pmatrix} \begin{pmatrix} \vec{d} \\ \vec{e} \end{pmatrix} \qquad \text{(2e)}$$

All the submatrices are (n x n)-dimensional, especially holds for the waveguide of length L and the phase constants γ_j:

$$\underline{\underline{E}}(L) = \begin{pmatrix} e^{\pm \gamma_1 L} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & e^{\pm \gamma_n L} \end{pmatrix} \qquad (3)$$

Using an additional abbreviation

$$\begin{pmatrix} \underline{\underline{H}}_{11} & \underline{\underline{H}}_{12} \\ \underline{\underline{H}}_{12}^T & \underline{\underline{H}}_{22} \end{pmatrix} = \begin{pmatrix} \underline{\underline{E}}(L_1) & 0 \\ 0 & \underline{\underline{E}}(L_2) \end{pmatrix} \begin{pmatrix} \underline{\underline{C}}_{11} & \underline{\underline{C}}_{12} \\ \underline{\underline{C}}_{12}^T & \underline{\underline{C}}_{22} \end{pmatrix} \begin{pmatrix} \underline{\underline{E}}(L_1) & 0 \\ 0 & \underline{\underline{E}}(L_2) \end{pmatrix} \qquad (4)$$

and with respect to the fact, that the complete setup is just a coaxial line two port with a (2 x 2)-S-matrix

$$\begin{pmatrix} b_0 \\ g_0 \end{pmatrix} = \begin{pmatrix} \Gamma_1 & T \\ T & \Gamma_2 \end{pmatrix} \begin{pmatrix} a_0 \\ g_0 \end{pmatrix} \qquad (5)$$

one finds after some calculations in order to eliminate all signal quantities (see [4]):

$$\begin{pmatrix} \Gamma_1 & T \\ T & \Gamma_2 \end{pmatrix} = \begin{pmatrix} A_{00} & 0 \\ 0 & B_{00} \end{pmatrix} + \begin{pmatrix} \vec{A}^T & 0 \\ 0 & \vec{B}^T \end{pmatrix} \begin{pmatrix} \underline{\underline{H}}_{11} & \underline{\underline{H}}_{12} \\ \underline{\underline{H}}_{12}^T & \underline{\underline{H}}_{22} \end{pmatrix} \left[1 \pm \begin{pmatrix} \underline{\underline{A}} & 0 \\ 0 & \underline{\underline{B}} \end{pmatrix} \begin{pmatrix} \underline{\underline{H}}_{11} & \underline{\underline{H}}_{12} \\ \underline{\underline{H}}_{12}^T & \underline{\underline{H}}_{22} \end{pmatrix} \right]^{(\pm 1)} \begin{pmatrix} \vec{A} & 0 \\ 0 & \vec{B} \end{pmatrix} \qquad (6)$$

We shall refer to (6) as the "complete model".

Geometric Series Expansion

Equation (6) can be rewritten using

$$(\underline{\underline{1}} \pm \underline{\underline{M}})^{(\pm 1)} = (\underline{\underline{1}} + \underline{\underline{M}} + \underline{\underline{M}}^2 + \underline{\underline{M}}^3 + \ldots) \qquad (7)$$

(we skip the discussion of the mathematical conditions)

$$\begin{pmatrix} \Gamma_1 & T \\ T & \Gamma_2 \end{pmatrix} = \begin{pmatrix} A_{00} & 0 \\ 0 & B_{00} \end{pmatrix} + \begin{pmatrix} \vec{A}^T & 0 \\ 0 & \vec{B}^T \end{pmatrix} \left[\begin{pmatrix} \underline{\underline{H}}_{11} & \underline{\underline{H}}_{12} \\ \underline{\underline{H}}_{12}^T & \underline{\underline{H}}_{22} \end{pmatrix} + \begin{pmatrix} \underline{\underline{H}}_{11} & \underline{\underline{H}}_{12} \\ \underline{\underline{H}}_{12}^T & \underline{\underline{H}}_{22} \end{pmatrix} \begin{pmatrix} \underline{\underline{A}} & 0 \\ 0 & \underline{\underline{B}} \end{pmatrix} \begin{pmatrix} \underline{\underline{H}}_{11} & \underline{\underline{H}}_{12} \\ \underline{\underline{H}}_{12}^T & \underline{\underline{H}}_{22} \end{pmatrix} + \right.$$
$$\left. + \begin{pmatrix} \underline{\underline{H}}_{11} & \underline{\underline{H}}_{12} \\ \underline{\underline{H}}_{12}^T & \underline{\underline{H}}_{22} \end{pmatrix} \begin{pmatrix} \underline{\underline{A}} & 0 \\ 0 & \underline{\underline{B}} \end{pmatrix} \begin{pmatrix} \underline{\underline{H}}_{11} & \underline{\underline{H}}_{12} \\ \underline{\underline{H}}_{12}^T & \underline{\underline{H}}_{22} \end{pmatrix} \begin{pmatrix} \underline{\underline{A}} & 0 \\ 0 & \underline{\underline{B}} \end{pmatrix} \begin{pmatrix} \underline{\underline{H}}_{11} & \underline{\underline{H}}_{12} \\ \underline{\underline{H}}_{12}^T & \underline{\underline{H}}_{22} \end{pmatrix} + \cdots \right] \begin{pmatrix} \vec{A} & 0 \\ 0 & \vec{B} \end{pmatrix} \qquad (8)$$

as a geometric matrix series. This expansion is useful as well as an approach for the numerical solution of (6) with a set of measurement data as for its physical interpretation. We denote (8) as the "reduced model". To simplify discussion, we restrict ourselves to the calibration problem, which is a special case of (6) (set all elements of C to 0 except the upper left block which is the negative identity). Then the complete model is

$$\Gamma_1(L_1) = A_{00} \pm \vec{A}^T \underline{\underline{E}}^2(L_1) \left[\underline{\underline{1}} + \underline{\underline{A}}\, \underline{\underline{E}}^2(L_1) \right]^{(\pm 1)} \vec{A} \qquad (9)$$

and its reduced version reads like:

$$\Gamma_1(L_1) \approx A_{00} \pm \vec{A}^T \underline{\underline{E}}^2(L_1)\, \vec{A} + \vec{A}^T \underline{\underline{E}}^2(L_1)\, \underline{\underline{A}}\, \underline{\underline{E}}^2(L_1)\, \vec{A} \qquad (10)$$

Evaluating this in the case of two modes

$$\begin{aligned} \Gamma_1(L_1) = \; & A_{00} \pm \left(A_{01}^2 e^{\pm 2 i \gamma_1 L_1} + A_{02}^2 e^{\pm 2 i \gamma_2 L_1} \right) + \\ & + \left(A_{01}^2 A_{11} e^{\pm 4 i \gamma_1 L_1} + A_{02}^2 A_{22} e^{\pm 4 i \gamma_2 L_1} + \right. \\ & \left. + 2 A_{01} A_{02} A_{12} e^{\pm 2 i \gamma_1 (L_1 + L_2)} \right) \pm \cdots \end{aligned} \qquad (11)$$

shows that each term describes a possible signal path from initial incidence to final detection. The same holds for (8) but the expressions are much more complicated. With the arithmetic derivation of (6) we just did a summation over all signal parts, written in a very compact way. To solve a set of equations (6) with measurement data, we fit the data depending on L_1, L_2 in the reduced model using the set of oscillations with wave numbers, given by the combinations of the known phase advances. The amplitudes of the lowest and therefore dominant frequencies are functions easy to be solved for the S-parameters (compare (11)) (due to some quadratic expressions some of the signs remain ambiguous). This procedure works as well for an adaptor calibration as for a complete measurement; in the latter case we have to fit with respect to two parameters.

Measurement setup

The main effort in the setup had to be spent in the realization of the various delay line lengths. This has been done by building two adaptor systems sliding in two fixed waveguides. They are driven by stepping motors with spindles that allow for a nominal position resolution of 6.25 µm. The RF equipment consists of a HP8753C-6 GHz-network analyzer. The components are computer controlled using *LabVIEW™*, the data evaluation is done with *Mathematica™*.

Calibration results with three modes

One of the adaptors has been measured at 4.5 GHz with three propagating modes (TE_{11}, TM_{01}, TE_{21}). The results are:

$$\hat{\underline{\underline{A}}} = \left(\begin{array}{c|ccc} \pm 0.145 + 0.356\,I & 0.003 \pm 0.080\,I & 0.022 \pm 0.110\,I & \pm 0.026 \pm 0.012\,I \\ \hline 0.003 \pm 0.080\,I & 0.459 \pm 0.517\,I & \pm 0.306 \pm 0.202\,I & \pm 0.071 + 0.061\,I \\ 0.022 \pm 0.110\,I & \pm 0.306 \pm 0.202\,I & 0.199 \pm 0.438\,I & \pm 0.161 + 0.147\,I \\ \pm 0.026 \pm 0.012\,I & \pm 0.071 + 0.061\,I & \pm 0.161 + 0.147\,I & \pm 0.562 \pm 0.773\,I \end{array} \right)$$

We insert these parameters into the reduced and the complete model (Fig. 4) plotted against L_1 and add the measurement points. We observe a sufficient agreement of the reduced model and a very good one of the complete model. This may be explained by the limited amount of wave numbers contributing in the reduced model, whereas the complete model covers all of them up to an infinite degree of multiple reflection.

Introducing a normalized error function

$$E = \frac{1}{N} \sum_{j=1}^{N} \left| \frac{m_j(L_j) \pm \Gamma_1(L_j)}{m_j(L_j)} \right|^2 \tag{12}$$

we studied the error-sensitivity of the result by adding some random offset within a certain part of each parameter value. Fig. 5 shows the result of 100 attempts together with the error function of the unperturbed S-parameters. We found the majority of attempts revealing an increased error, confirming that the unperturbed S-parameters are a very good (but not optimal) approximation to the real values.

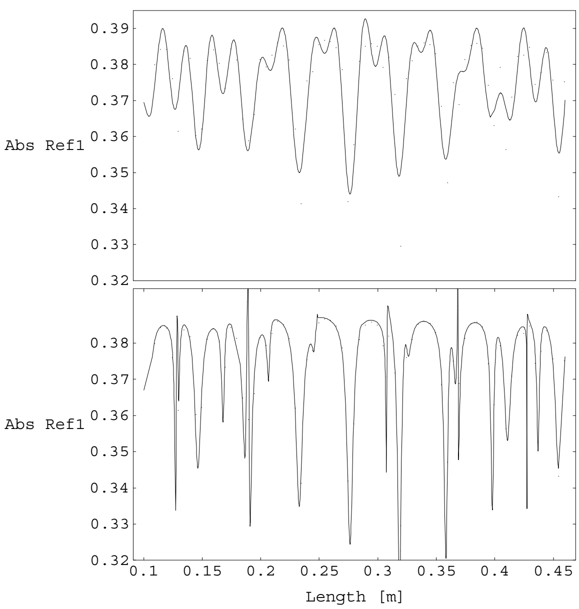

Fig. 4 Value of input reflection factor of adaptor A at 4.5 GHz against L_1; measured points (dots) together with reduced (upper curve) and complete model

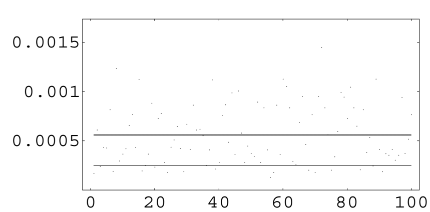

Fig. 5 Normalized error function (12) of adaptor S-parameters (normal line) with homogeneously distributed random offset of 3% for 100 attempts (dots); dashed line: average of all attempts

Measurement results with two modes

The S-matrix of the TESLA 9-cell copper cavity has been measured at 3.0968 GHz. At this frequency the TE_{11} and the TM_{01}-mode propagate. After the calibration runs of the adaptors (we skip these results) we find from (8):

$$\underline{\underline{\hat{C}}} = \left(\begin{array}{cc|cc} 0.309 + 0.287\,I & \pm 0.050 \pm 0.049\,I & 0.310 \pm 0.735\,I & \pm 0.034 \pm 0.016\,I \\ \pm 0.050 \pm 0.049\,I & \pm 0.264 \pm 0.898\,I & \pm 0.019 + 0.039\,I & 0.004 \pm 0.004\,I \\ \hline 0.310 \pm 0.735\,I & \pm 0.019 + 0.039\,I & 0.469 + 0.356\,I & \pm 0.076 \pm 0.047\,I \\ \pm 0.034 \pm 0.016\,I & 0.004 \pm 0.004\,I & \pm 0.076 \pm 0.047\,I & \pm 0.395 \pm 1.030\,I \end{array} \right)$$

The reason of the value of C_{44} being about 20% greater 1 is not yet clear. Probable causes might be the small number of measurement points (13 for L_1 and L_2, leading to an 13x13-array) or a temperature drift, that has been observed during the measurement time of about 1 hour. Again, we tested the complete model and found a sufficient though not extremely good agreement (Fig. 6, 7).

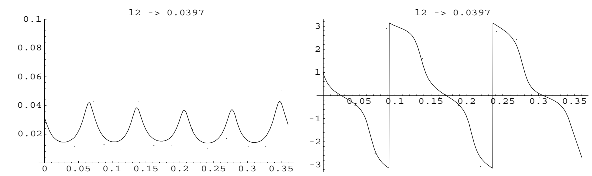

Fig. 6 Typical plot of the transmission of the complete setup for fixed L_2 against L_1: value (left) and argument (right curve)

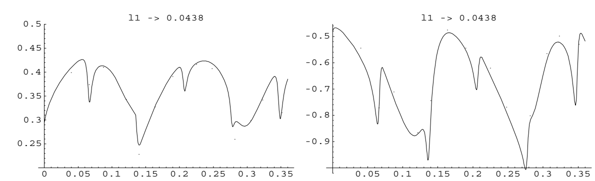

Fig. 7 Typical plot of the input reflection at port B of the complete setup for fixed L_1 against L_2 (comp. Fig. 6)

Conclusion and Outlook

The TSD-calibration technique is a useful tool to calibrate coax-waveguide-transitions if only one mode is propagating. To expand measurements in the frequency range of several waveguide modes we have been developing a new method for multimode S-parameter measurements showing encouraging results in first tests. These evaluations will continue to specify the capabilities of the method. In further investigations we shall try to resolve additionally degenerated modes, especially different polarisations.

Support

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