

## Multibunch beam dynamics in a superconducting linac injector

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### Abstract

Superconducting linear collider schemes envisage the generation and acceleration of trains of bunches with high charge per bunch and repetition rate. The designs of injectors that make use of RF guns or classical thermionic guns followed directly by superconducting bunching cavities are faced with the problem of the interaction of a train of not fully relativistic, high charge and repetition rate bunches with an accelerating structure. A new numerical model has been developed to treat such cases. It integrates coupled Newton and Maxwell equations by a slowly varying envelope approximation for the time evolution of the modes amplitude and a current density description of the beam. Space charge and RF focusing forces, beam loading and build-up effects of higher order modes under beam-tube cutoff frequency are taken into account as well. The application to a particular design is worked out.

### Introduction

When treating the evolution of high charge, not fully relativistic electron bunches in RF fields of an accelerating cavity, it is necessary to take into account also the field induced by the beam in the fundamental and higher order modes, and the variation of bunch sizes due to both the RF fields and space charge.

For single bunches the problem has been already tackled using PIC codes, which describe the bunch as an ensemble of particles and track their motion, coupled to the E.M. field evolution. The case of long bunch trains would consume an unbearable amount of computer time if treated by a mere extension of the single bunch case.

We have therefore devised a simple model [1] that uses a current density description of the beam and slowly varying envelope approximation (SVEA) for the evolution of the cavity normal modes. Motion and field equations are coupled together through the driving current term. The SVEA approximation supposes small field perturbations produced by any single bunch, that add up to give an envelope of any field mode slowly varying on the time scale of its period. Because the characteristic cavity reaction time is of the order of  $\tau = 2Q/\omega \gg T$  we fulfill the SVEA hypothesis. This approximation leads to only first order equations for the field amplitude, thus reducing the numerical computing time.

A fast running code (HOMDYN) has been developed and has been tested by comparing single bunch results with that of the reference PIC code ITACA [2]

The code allows to follow the evolution of both the longitudinal and transverse envelopes of each bunch in a train. By slicing the bunch in a succession of cylinders, each subject to the local fields, one obtains also the energy spread and the

emittance degradation due to phase dependent RF effects. The present version deals only with TM monopole modes: an extended version comprehensive of dipole modes is under development.

Because of the long interaction time the field equations require also an excitation term represented by an on axis localized generator in order to take into account the cavity refilling from bunch to bunch passage. The evolution of the field amplitude during the bunch to bunch interval is given by the analytical solution of the equation driven by the generator only, which connects successive numerical integrations applied during any bunch transit.

As a first example of application we consider the Tesla Test Facility (TTF) Injector design under development in Saclay for the TTF collaboration [3].

### The TTF Injector case

In Fig 1 is shown a sketch of the TTF Injector: a thermoionic Gun with an electrostatic accelerating column provides a 250 KeV beam 800  $\mu$ sec long that a Sub-harmonic Buncher transforms in a train of bunches with 216.7 MHz repetition rate. The Capture Cavity, a 9 cells 1.3 GHz Superconducting Cavity, drives the beam up to 14 MeV. Three Solenoidal Lenses provide the beam focusing during the drifting to the Capture Cavity.

We deal at present only with the Capture Cavity element. This case is well suited for a test of our model, as the beam consists of a train of not fully relativistic bunches. Moreover, for the single bunch there are already complete information obtained with the code PARMELA [4]. It is therefore possible to make a comparison with the results given by HOMDYN. In the next sections we present some results of the computation for the single bunch case and for a short train of 200 bunches. We deal with the interaction with the fundamental pass-band modes (9 - modes) only. A detailed study taking into account more bunches and a complete set of higher order modes under beam tubes cut-off frequency is under way.

### Single-Bunch Computations

In Tab. 1 input data and the comparison of PARMELA and HOMDYN output data are reported for a reference case. The agreement seems to be satisfactory even with the dramatic approximation we make on the beam description but in 3 minutes of CPU time on a VAX 7000 system. We consider in fact only 20 slices on a cylindrical bunch to compute energy spread and rms normalized emittance  $\epsilon_d$  degradation due to phase correlation.

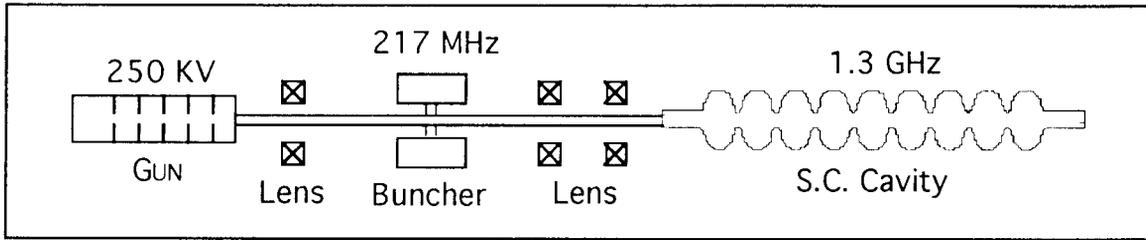


Fig. 1 Sketch of the TTF Injector

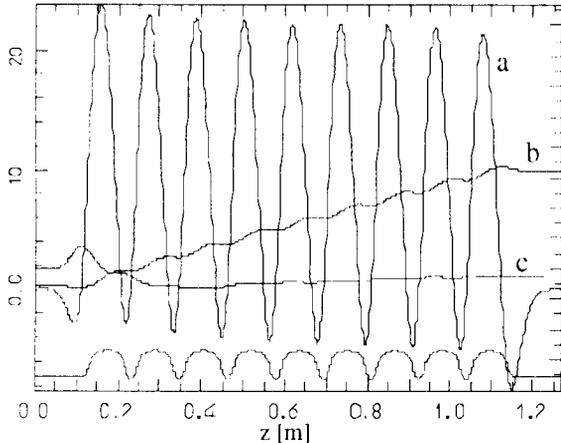


Fig. 2 a-Field seen by the bunch [MV/m], b-Bunch energy [MeV], c- $\Delta\gamma/\gamma$  [%].

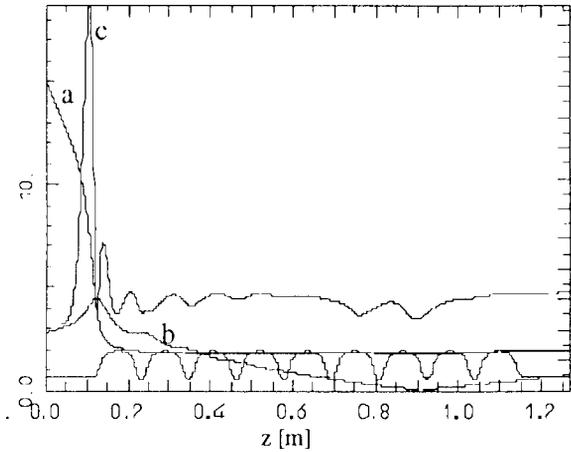


Fig. 3 a-Bunch length [mm], b-Bunch radius [mm], c-rms norm.emittance [mm mrad]

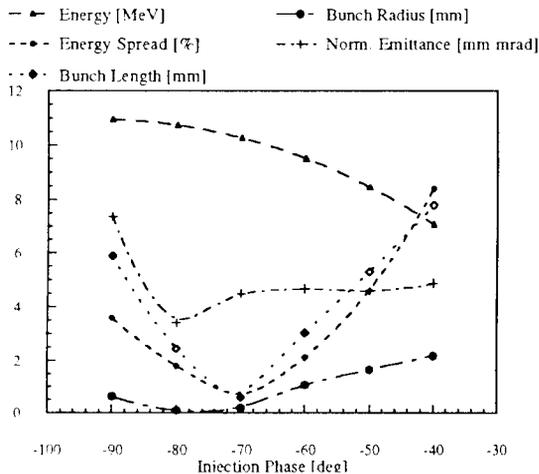


Fig. 4 Output parameters versus the injection phase at a constant cavity voltage (12 MV)

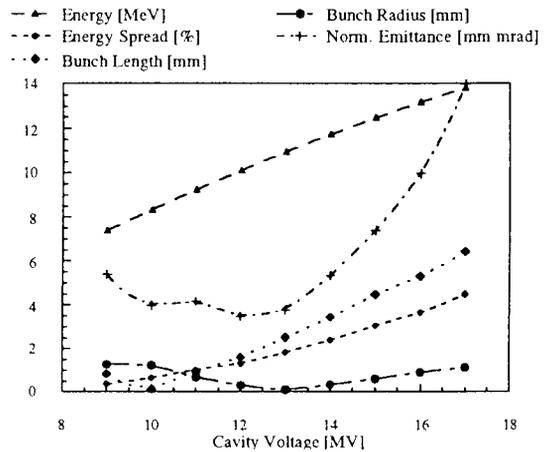


Fig. 5 Output parameters versus the cavity voltage at a constant phase (-80°)

Assuming linear transverse component of the RF fields we define  $\epsilon_d$  as:

$$\epsilon_d = \frac{\langle \beta \gamma \rangle}{N} \sqrt{\sum_i R_i^2 \sum_i (\alpha_i R_i)^2 - \left( \sum_i \alpha_i R_i^2 \right)^2}$$

$$\epsilon_n = \sqrt{\epsilon_{n,0}^2 + \epsilon_d^2}$$

where  $N$  is the number of slices,  $R_i$  are the slice radius,  $\alpha_i = R'_i/R_i$  and  $\epsilon_{n,0}$  the thermal emittance

In Fig. 2 the field seen by the bunch, the energy and the energy spread evolution are shown. In Fig. 3 the bunch length and radius evolution and the normalized emittance are shown. In both figures the cavity shape is also reported.

The decelerating field seen by the bunch at the entrance of the first cell, that at the injection phase considered (-65°) is more intense for the bunch head than for the tail, acts as

bunching and defocusing field. In the subsequent cells a strong RF focusing effect occurs that leads to a waist inside the cavity. Moreover the field unflatness seen by the bunch is due to phase slippage of the non relativistic beam

In Fig 4 the output parameters are plotted versus the injection phase at a constant cavity voltage (12 MV) and in Fig. 5 the same parameters are plotted versus the cavity voltage at a constant phase (-80°)

TABLE 1  
Single Bunch Data

	Input	Parmela Out	Homdyn Out
E <sub>acc</sub> [MV/m]	10		
Energy [MeV]	0.234	9.9	9.9
ΔE/E [%]	16	2.5	3
Length [mm]	15	2	1.9
Radius [mm]	2.8	0.6	0.7
rms ε <sub>n</sub> [mm mrad]	3.2	4.05	4.75
Charge [pC]	37		
N. of Macroparticle		500	20

Multi-Bunch Computations

When treating the multi-bunch case we assume that the RF power is pulsed and beam is injected with the right delay with respect to the RF pulse to achieve balance between the rising generator voltage V<sub>g</sub> and the beam loading voltage V<sub>b</sub> resulting in a constant accelerating field V<sub>acc</sub>=V<sub>g</sub>cosφ<sub>g</sub>-V<sub>b</sub> as proposed for the main TTF Linac [3](see Fig. 6).

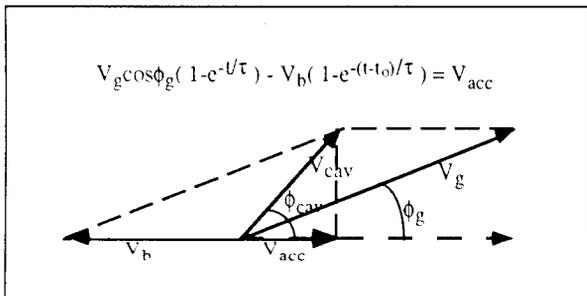


Fig. 6 Vectors diagram for the cavity voltage computation

For the ultra relativistic case is easy to show that the right delay for an off-crest beam is given by

$$t_0 = \tau \ln \frac{V_g \cos \phi_g}{V_b}$$

where  $\tau = 2Q/\omega$  is the cavity filling time. This relation holds only approximately for non-relativistic beams. At the injection time  $t_0 < \tau$  a steady state field corresponding to a standing wave pattern in the cavity has not yet been reached and field propagation effects could perturb the energy gained by different bunches in particular in the low beam energy interaction area. We take in to account the RF generator contribution in the field equation as a driving sinusoidal

current source located on the cavity axis under the first cell iris which drives all the n resonant modes with an incident amplitude  $|\alpha_1|^g$ , phase  $\phi_g$ , detuning shift  $\Omega_{1,n} = (\omega_1 - \omega_n)$  and coupling proportional to the ratio of any field form factor  $\hat{e}_n(z_g)$  to the fundamental (n=1) form factor  $\hat{e}_1(z_g)$  on the coupler position. The superposition of the resonant modes accounts also for field propagation effects [1].

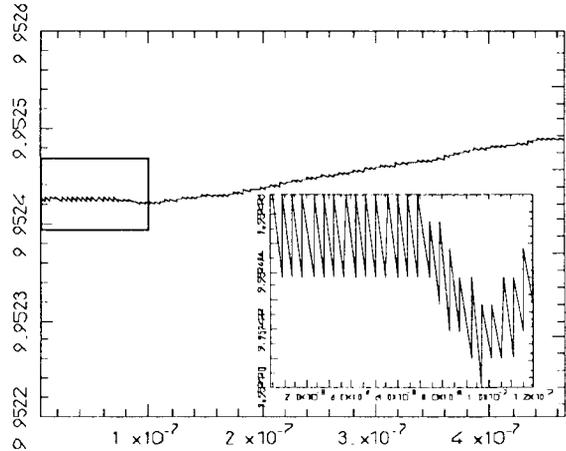


Fig.7 Cavity voltage V<sub>cav</sub> [MV] versus time [sec].

In Fig.7 the cavity voltage V<sub>c</sub> is shown versus time and the beam loading is evident in the enlarged picture. The cavity re-filling from bunch to bunch is not a problem because the bunch charge is very low, but one can observe a slight oscillation of the cavity voltage after a delay time of the order of 150 nsec, corresponding to the back and forth propagation time of wave front in the cavity, see ref. [5]. This introduces an energy spread in the train. Another source of train energy spread is a not perfect compensation of the beam loading resulting from an error in the injection time t<sub>0</sub>, that introduces a slope on the cavity voltage. But first simulations show that multi-bunch effects are not a problem. More detailed study are under way.

References

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