SURVEY OF HALO FORMATION STUDIES IN HIGH INTENSITY PROTON LINACS*

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Abstract

There has been recent interest in using high intensity ion linacs for a variety of purposes, including production of tritium, transmutation of radioactive wastes and as drivers for fission and fusion energy devices. Since beam spill will cause unacceptably large activation, it has become necessary to understand, and if possible, to avoid the generation of beam halos which were first seen at LAMPF about 20 years ago and which arise in most numerical orbit simulations. We review the history of halos, which were first considered a byproduct of the more serious emittance growth, studied by Jameson and others at LAMPF. More recently, simulations have been run by Reiser, Wangler and O'Connell, among others, illustrating the development of halos, particularly when there are core oscillation arising from discontinuities in the parameters of the structure and/or the beam transport system. Finally, we present a likely model for the growth of halos which arise from a resonant interaction between the driving oscillation of the core and the individual ions which traverse the core. An essential part of the model is the amplitude dependence of the individual ion oscillations in the core.

Introduction

High current, high duty factor ion linacs have become increasingly attractive in recent years. Among possible applications are ion beam drivers, production of tritium, transmutation of radioactive wastes and production of radioactive isotopes for medical use.

Obviously, it is desirable to accelerate the maximum possible current in such linacs. Much work has been done to explore the optimum transverse phase space distribution in such beams. In particular, the K-V distribution[1] is simplest to analyze, since its projection into real space has a uniform density and therefore linear space charge forces. The stability of the K-V distribution has been analyzed and approximately confirmed by numerical simulations. Nevertheless it appears that, particularly at high currents, the K-V and other equililbrium distributions evolve to ones with rounded edges and tails. In many cases involving high peak current, the distribution spins off a cluster of particles in the form of a halo surrounding a dense core. This halo is seen in simulations as well as in actual linacs, such as LAMPF.[2] And efforts to remove the halo by collimation have been largely unsuccessful since the halos almost always regenerate.

It is clear that the halos can produce unacceptably high levels of radioactivity in high current, high duty factor linacs. For this reason considerable effort has recently been devoted to exploring their detailed structure and understanding the mechanism or mechanisms by which the halos are produced.[3-7] What has been learned is that halos are most likely to be produced at transition locations, such as where there are discontinuities in frequency, structure geometry, transverse focusing pattern, accelerating gradient and phase, etc.

In the present paper, we survey recent efforts to explore and understand the properties of halos. Certain truths are obvious, and are best seen from a Lorentz frame traveling with the charge bunch. In this frame, the motion of individual particles is non-relativistic and therefore affected only by the electric field. For a core which is matched in phase space, and which therefore is time independent in the moving frame, it is not possible to exchange energy between the core and individual particles traversing the core. However, if the core is time dependent, such as would be the case if there were a mismatch, or a misalignment, such an energy transfer could occur. As a result, all recent efforts to study the development of a halo assume some sort of interaction between a core oscillation and the motion of individual particles.

Observations

The primary experimental information about halos comes from observations on LAMPF. The design current of 17 m.a. with 6% duty factor made it clear that activitation would be excessive even for a beam spill of 1 part in 10^4. Therefore considerable attention was devoted to removing the tail of the transverse distribution by collimation and to monitoring the transverse emittance of the beam. The emittance was seen to grow, in spite of efforts to align the drift tubes, focusing magnets, and beam cen-

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ter. A great deal of effort went into understanding the effects of non-linear resonances[8] and equipartitioning.[9] At this point halos were first identified. Although efforts to remove the halos were unsuccessful (they quickly regenerated) the matter was considered only an annoying curiosity, since relatively few particles populated the halo, causing no significant activation.

When recent interest arose in increasing the current by a factor of 100 or more, efforts to understand the development of halos were renewed. These efforts included extensive numerical simulations as well as theoretical efforts to identify the underlying mechanism(s).

Numerical Simulations

The most extensive simulations are those performed by Jameson[2]. He first considers a zero emittance laminar beam and follows the progress of particles which initially travel in a mismatched channel parallel to the z axis in the r, r' and x, x' or y, y' phase planes, as well as in x, y space.

An example is shown in Fig. 1 where the development of a halo is visible. When the halo is scraped off, continuing the simulation shows the halo regenerating. Jameson also considers a beam with finite emittance, where the singularities at $r = 0$ are no longer present, and again shows the development of a halo and regeneration after scraping. He discusses the phase space evolution qualitatively and includes some of the elements of non-linear resonance[8] and equipartition[9], which are important in understanding the emittance growth.

Simulations have also been performed by Cucchetti, Reiser and Wangler[4] who show the increase of emittance resulting from a mismatch, and suggest this as a likely source of halos observed in real beams. O'Connell, et al.[6] have also performed simulations starting with a zero emittance Gaussian beam. An example of their results is shown in Fig. 2, where the development of a halo is clearly visible. They, as well as Jameson[2], characterize the various orbits as betatron-like, plasma-like and hybrid. As we shall see later, these patterns are directly related to the existence of approximate integrals of the motion in the presence of a dominant non-linear resonance and the resulting phase plane trajectories. Kehne, Reiser and Rudd[5] also explore halo formation experimentally.

Recent computations by Lagniel[7] for a zero emittance K-V beam using test particles show a phase plane with considerable structure as well as the onset of chaos at high space charge. He also points out the similarity between the halo/beam problem and the astronomical three-body problem. And Chen and Davison[10] have performed simulations of the envelope equation for a K-V beam with periodic focusing which also show the onset of chaotic behavior as well as resonant structure in the Poincaré phase plane plots, an example of which is shown in Fig. 3.

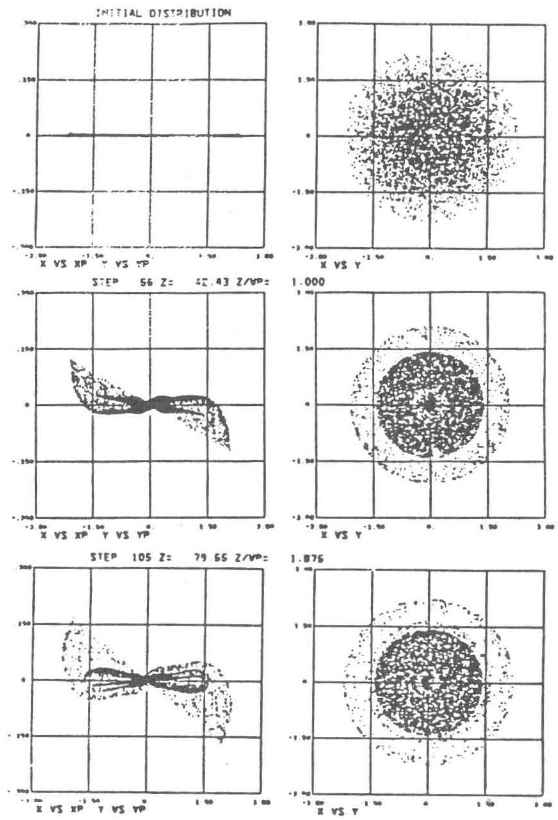

Figure 1: Evolution of a laminar beam in a mismatched channel at $z = 0, 1.0, 1.875$ plamsa periods.

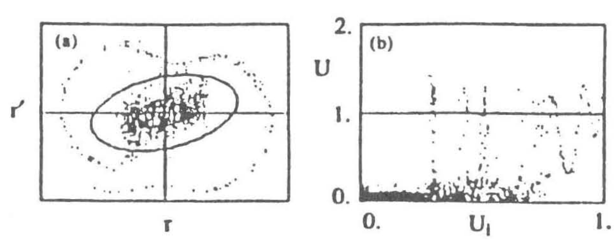

Figure 2: Results at 20 plasma periods from multiparticle simulation of an initial Gaussian-density laminar beam in a uniform linear focusing channel. The initial rms size in x is larger than the matched value by a factor $M = 1.5$. We show a) $r - r'$ phase space, b) particle energy U versus the initial particle energy U_i.

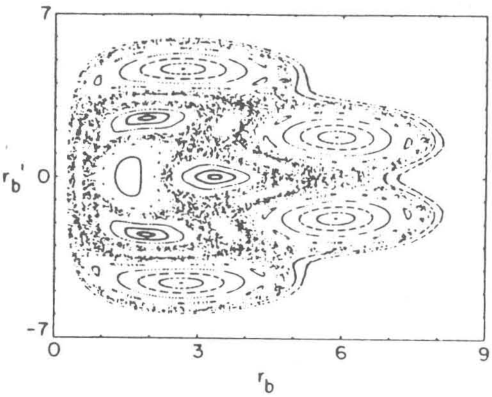

Figure 3: Poincaré surface-of-section plot of the beam envelope oscillations for the choice of system parameters corresponding to $K = 5$, $\eta = \frac{1}{6}$, and $\kappa_z(0) = 24.2(\sigma_0 = 115°)$.

Theoretical Considerations

The simulations make it clear that halos are related to the resonant interaction of individual particle motion with beam oscillations caused by a mismatch. All theoretical efforts start with this premise.

A matched beam which enters a region of mismatch will generally start a complicated pattern of collective oscillations. The individual particles which interact with these oscillations will cause a coupling of all these modes, and the system will eventually settle down to a new equilibrium distribution. But in a linac, there is unlikely to be enough time to reach the new equilibrium and the beam will undergo very complicated turbulent motion which apparently ejects particles from the core into some sort of halo.

Bohn and Delayen[11] analyze this complicated process, which has some similarity to the collisionless relaxation of stellar systems, by invoking the Fokker-Planck equation to account for the rapid evolution of the coarse-grained distribution function. This is apparently equivalent to neglecting non-linear coupling between fluctuations in the particle distribution and in the electromagnetic field. They then make simplifying assumptions to estimate the relaxation-rate and diffusion coefficients. Finally they construct a model of turbulent heating which involves a "diffusive temperature" and plan in the future to explore numerical solutions to the coupled Fokker-Planck and Poisson equations.

The model which I propose is based on the supposition that one mismatch mode of oscillation is more important than the others, and that the interaction between this mode and individual particles takes place before the mode is damped or otherwise modified by interaction with other oscillation modes. A logical candidate for this mode is the breathing mode in which the size of the distribution oscillates much like a beam envelope. We also assume a K-V distribution in the 2-D phase space. Its attraction is directly related to the linear fields within the beam core, making analytical results feasible. Even though it is well known that actual distributions have tails and that the non-monotonic (in the total energy) K-V distribution has instabilities which other self consistent distributions do not[12], the breathing mode will be very similar to that in other self consistent distributions, e.g. the defocussing term will oscillate periodically in strength. We also assume a smoothed external force, although we acknowledge that periodic variation of the force gradient can also interact with individual particle motion. Finally, we recognize that individual particles which transverse the core have frequencies which are amplitude dependent. This feature plays an essential role in the resonant interaction between individual particles and the breathing core.

Breathing Mode Interaction with Individual Particle Motion[13]

A General Form

We consider an azimuthally symmetric K-V core of radius a for which the equation of motion of an ion is

$$x'' + k^2 x = x \left\{ \begin{array}{ccc} \kappa/a^2 & , & r \leq a \\ \kappa/r^2 & , & r \geq a \end{array} \right\}, \tag{1}$$

where the prime stands for d/dz, and k is the wave number of the transverse motion in the absence of space charge. The perveance of the beam,

$$\kappa = \frac{eI}{2\pi\epsilon_0 m v^3}, \tag{2}$$

is a dimensionless parameter proportional to the current I, where e, m, v are the charge, mass, ion velocity and ϵ_0 is the permittivity of free space. The equation for y is identical to Eq. (1).

We now assume a core oscillation of wave number p of the form

$$a \longrightarrow a(1 - \epsilon \cos pz) \tag{3}$$

and expand a^{-2} in Eq. (1) to first order in ϵ, the relative oscillation amplitude. After some algebra, Eq. (1) can be written as

$$x'' + q^2 x = -\frac{\kappa}{a^2} x \left(1 - \frac{a^2}{r^2}\right) \Theta(r-a) + \frac{2\epsilon\kappa}{a^2} x \cos pz\, \Theta(a-r), \tag{4}$$

where $\Theta(u) = 1, 0$ for $u > 0, u < 0$ and where $q = \sqrt{k^2 - \kappa/a^2}$ is the oscillation wave number within the core.

With the radial forces of Eq. (4), we see that the angular momentum is constant. Writing $Lqa^2 \equiv xy' - x'y = r^2\theta'$, the equation for radial motion becomes

$$\begin{aligned} r'' + q^2\left(r - \frac{L^2 a^4}{r^3}\right) &= -\frac{\kappa}{a^2} r \left(1 - \frac{a^2}{r^2}\right) \Theta(r-a) \\ &+ 2\frac{\epsilon\kappa}{a^2} r \cos pz\, \Theta(a-r). \end{aligned} \tag{5}$$

The first term on the right makes the oscillation wave number depend on amplitude and the second term allows for energy transfer between the core and the oscillating ion. It can be shown[13] that the distribution in L for a K-V beam is uniform in the range $-\frac{1}{2} \leq L \leq \frac{1}{2}$.

B Phase-Amplitude Method

Although the analysis can be carried out with $L \neq 0$, we expect the largest effect for $L = 0$ and therefore quote the results in this case. We set

$$r/a = A\sin(qz+\alpha)\ ,\ r'/a = qA\cos(qz+\alpha) \qquad (6)$$

and obtain expressions for A' and α' in terms of the right hand side of Eq. (5.6). We then average over all oscillations except for the wave number $2q-p$ and are able to obtain an integral of the motion C which allows us to write

$$h(w)\epsilon\cos\Psi = \Delta - b(w) - C/w. \qquad (7)$$

Here $w = r^2/a^2$ and $\Psi = (2q-p)z + 2\alpha$. Also $\Delta = [1+\sqrt{(1+k^2/q^2)/2}]^{-1}$ is proportional to the resonance separation $p-2q$, given as a relatively insensitive fucntion of the tune depression q/k. We have $h(w) = 1$ and $b(w) = 0$ for $w \leq 1$, and for $w \geq 1$ have

$$h(w) = 1 - \frac{2}{\pi}\left[\tan^{-1}\sqrt{w-1} + \frac{(w-2)\sqrt{w-1}}{w^2}\right], \qquad (8)$$

$$b(w) = \frac{2}{\pi w}\int_1^w \frac{du}{u}[(u-2)\tan^{-1}\sqrt{u-1}+\sqrt{u-1}]. \qquad (9)$$

The quantity $b(w)$ is related to the amplitude dependence of the single particle wave number brought about by the non-linearity of the particle motion outside the core.

C Halo Formation

Figure 4 shows a plot of $\epsilon\cos\Psi$ vs. w for $\Delta = 0.35$, for different values of C. Particles with low initial amplitude remain bounded by the lines $\epsilon\cos\Psi = \pm\epsilon$, whereas particles which start at P or beyond will undergo excursion as far as R. In terms of the phase plot, the origin and S are stable fixed points and Q is an unstable fixed point. This is seen more clearly in the polar plot of w vs. Ψ of Fig. 5 for $\epsilon = 0.3$ and the r vs. r' stroboscopic plot of Fig. 6, where the thick solid and dashed curves form a separatrix.

The phenomenon of halo formation is now expected to take place more or less as follows:

1. Particles within the inner separatrix (bounded by the thick solid curve) will wander to the separatrix due to any one of a variety of current dependent non-linear instabilities which cause tails to grow in the phase space distribution.

2. Simulations indicate that, at high current, chaos develops in the vicinity of the inner separatrix thus permitting particles to cross the separatrix and flow primarily along the outer separatrix (thick dashed curve).

3. In this way, particles which escape the inner separatrix form a low density region between r_Q and r_R, the radii corresponding to Q and R, terminated by a somewhat higher density ring or halo at r_R.

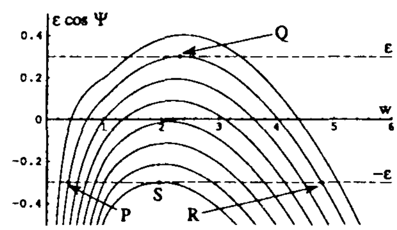

Figure 4: Plot of $\epsilon\cos\Psi$ vs. w with $\Delta = 0.35$, $L = 0$.

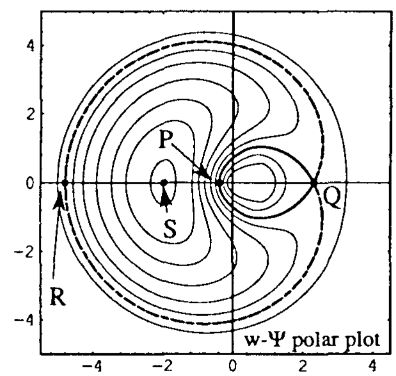

Figure 5: Polar plot of w vs. Ψ for the trajectories corresponding to the parametric resonance with $\Delta = 0.35$, $L = 0$, $\epsilon = 0.3$.

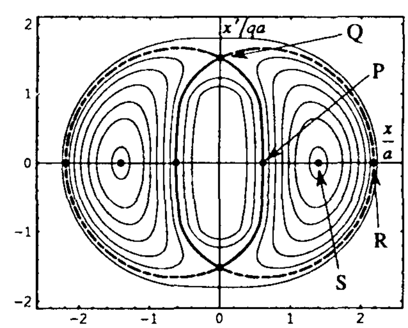

Figure 6: Stroboscopic plot of x, x' trajectories for $\Delta = 0.35$, $L = 0$, $\epsilon = 0.3$.

D Simulations

Simulations have been performed by Wangler and Ryne[14] by integrating the envelope equation and the equation for the individual particle motion, with various initial conditions. A sample of their results is shown in the stroboscopic plot of Fig. 7 which clearly agrees with the features of the analytic result in Fig. 6. In addition there is evidence of some chaotic motion near the unstable fixed point making it easier to cross the inner separatrix.

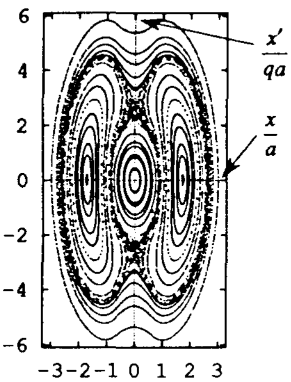

Figure 7: Stroboscopic plot of x, x' trajectories from simulations with $q/k = 0.4$ and mismatch of 1.4.

E Additional Comments

It seems clear that the proposed model contains the essential features of the phenonemon responsible for halo formation. At the present time we can account for the halo parameters quantitatively, but have not addressed the diffusion of particles across the core boundary. Moreover, we have not yet been able to predict the circumstances which lead to chaotic motion.

Summary

We have reviewed the observations of halos and some of the simulation results which help us understand the features of particle and core motion which are related to the development of a halo. We have also reviewed the efforts to construct a global theory involving the evolution of many core modes. Obviously more work would be useful along those lines. And finally we reviewed a specific model involving a resonant interaction between a particle undergoing non-linear motion through the core and the core breathing oscillation, which duplicates the halo parameters seen in recent simulations. What is needed in the future is an understanding of how the regions outside the core start to get populated and an analysis of the parameters at which chaotic motion is expected to start.

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