DESIGN OF EQUIPARTITIONED HIGH-CURRENT RF LINACS

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Abstract

Beams in rf linacs are generally not in thermal equilibrium due to the asymmetry in the transverse-to-longitudinal focusing strengths in the conventional linac design. In the high-current linacs for advanced accelerator applications space-charge coupling forces tend to drive the beam towards an equipartitioned state which may cause significant emittance growth and halo formation. In this paper, design scenarios for equipartitioned high-current rf linacs that differ significantly from conventional designs are examined. Analytic scaling laws, design relations, and results for a proton linac in the 2 to 1000 MeV range will be presented.

Introduction

In conventional rf linacs the beams are usually not in thermal equilibrium as the longitudinal temperature $T_{\|}$ in the bunch is typically much smaller than the transverse temperature $T_{\perp}$. This anisotropy exists in both electron and ion linacs, but we will focus our analysis on an ion linac, e.g., a high-current proton, H^-, or deuteron machine. The temperature anisotropy, measured by the ratio $T_{\perp}/T_{\|}$, is then further enhanced by the acceleration process and the conventional focusing strategy in the drift-tube linac (DTL) and coupled-cavity linac (CCL) sections.

Thermodynamically, a beam wants to be equilibrated ($T_{\perp} = T_{\|}$). In high-current linacs, the space-charge forces couple the longitudinal and transverse motion and produce an equipartitioning effect. This leads to emittance growth and halo formation which is unacceptable when high beam quality or "hands-on maintenance" is required such as in the machines being considered for Accelerator Driven Transmutation of Radionuclear Waste (ATW) [1], where particle losses to the walls must remain less than 10^{-9} amperes per meter.

After discussing the prevailing rf linac design philosophy, we will present the main features of an alternative design strategy in which the beam is in three-dimensional thermal equilibrium throughout the entire system.

Equipartitioning and Emittance Growth in the Conventional Linac Design

The applied transverse and longitudinal focusing forces keeping the bunches confined during the acceleration process in an rf linac can be represented by the wave numbers $k_{x0} = 2\pi/\lambda_{x0}$ and $k_{z0} = 2\pi/\lambda_{z0}$, respectively. They define the wavelengths λ_{x0} and λ_{z0} of the transverse and longitudinal particle oscillations in the absence of space charge. The transverse focusing wave number, k_{x0}, is determined by the phase advance, σ_{x0}, of the transverse ("betatron") oscillations without space charge in one period of length $S = n\beta\lambda$ by

$$k_{x0} = \frac{\sigma_{x0}}{n\beta_0\lambda}. \qquad (1)$$

Here, $\beta_0 = v_0/c$ is the ratio of the beam centroid velocity v_0 to the speed of light c, $\lambda = c/f$ is the wavelength, and f the frequency of the accelerating rf field while n is an integer (usually $n \geq 2$) that depends on the machine design. The longitudinal focusing wave number is given by

$$k_{z0} = \left(-\frac{2\pi q E_m \sin\phi_0}{\lambda m c^2 \beta_0^3 \gamma_0^3}\right)^{1/2}. \qquad (2)$$

With the applied accelerating rf electric field defined by $E_{az} = E_m \cos\phi$, E_m is the peak field; ϕ_0 is the phase angle of the beam-centroid ("synchronous") particle ($\phi_0 < 0$ for longitudinal focusing), $\gamma_0 = (1-\beta_0^2)^{1/2}$ is the relativistic energy factor, q the charge, and m the mass of the particle.

For a given average current, emittance, and rf wavelength, λ, the two wave numbers determine the size of the bunch and the beam physics. Specifically, if $\tilde{x}$ and $\tilde{z}$ denote the transverse and longitudinal rms widths of the bunch, $a = \sqrt{5}\tilde{x}$ and $z_m = \sqrt{5}\tilde{z}$ the semi-axes of the equivalent uniform-density ellipsoidal bunch, the aspect ratio $z_m/a = \tilde{z}/\tilde{x}$ is directly related to the ratio k_{x0}/k_{z0}.

The traditional rf linac design is based on a constant phase advance σ_{x0} and a constant product of field gradient E_m and $|\sin\phi_0|$, that is we get the scaling

$$k_{x0} \propto \frac{1}{\beta_0}, \qquad k_{z0} \propto \frac{1}{(\beta_0\gamma_0)^{3/2}}, \qquad (3)$$

or

$$\frac{k_{x0}}{k_{z0}} \propto \beta_0^{1/2}\gamma_0^{3/2}. \tag{4}$$

Thus, the ratio of transverse-to-longitudinal focusing strength increases with energy and, as a result, the aspect ratio z_m/a also increases: the radius a usually becomes smaller while the bunch length $2z_m$ becomes larger as the beam is accelerated. Thermodynamically, such a beam is not equilibrated and the difference between transverse and longitudinal temperature increases during the acceleration process. The resulting equipartitioning process due to the space-charge coupling forces leads to significant emittance growth, and halo formation with particle losses to the drift tube walls. This effect was first demonstrated and studied systematically in computer simulation work by R. Jameson [2] who tried to correlate his findings with the instability modes due to coupling forces in an anisotropic K-V beam investigated theoretically by I. Hoffmann [3]. Further work following these early papers, such as the computer studies by Wangler et al [4] and by Jameson [5], have added a wealth of information on the equipartitioning effect. But so far in the framework of the traditional linac design (σ_{x0} = const, $E_m|\sin\phi_0|$ = const), no satisfactory solution with tolerable minimal emittance growth has been found.

Scaling Laws and Design Relations for an Equipartitioned RF Linac

Theoretically, for a matched beam in a smooth-focusing system, the transverse and longitudinal temperatures, $T_\perp$, $T_\|$, can be related to the beam widths, a, z_m, and normalized emittances ϵ_{nx}, ϵ_{nz}, by [6]

$$\frac{T_\perp}{T_\|} = \frac{\epsilon_{nx}^2}{\epsilon_{nz}^2}\frac{\gamma_0^2 z_m^2}{a^2}. \tag{5}$$

A two-temperature Boltzmann distribution with coupled space-charge forces is not a stationary solution of the Vlasov equation [6]. The beam is equipartitioned ($T_\perp = T_\|$) when

$$\frac{\epsilon_{nx}}{\epsilon_{nz}}\frac{\gamma_0 z_m}{a} = 1. \tag{6}$$

For a space-charge dominated beam with small aspect ratio of $0.8 \leq \gamma_0 z_m/a \leq 4$, one can find analytic relations for a and z_m which yield for the ratio z_m/a the relation [6]

$$\frac{z_m}{a} = \frac{2}{3\gamma_0}\left(\frac{k_{x0}^2}{k_{z0}^2} + \frac{1}{2}\right). \tag{7}$$

By using the equipartitioning condition (6) we obtain

$$\frac{k_{x0}}{k_{z0}} = \left(\frac{3}{2}\frac{\epsilon_{nz}}{\epsilon_{nx}} - \frac{1}{2}\right)^{1/2}. \tag{8}$$

If the beam is in thermal equilibrium, the normalized emittances do not change and the ratio of the focusing wave numbers must remain constant during the acceleration process. Thus, in contrast to the scaling (3), (4) for the conventional design, we must vary the transverse focusing like the longitudinal focusing, as

$$k_{x0} \propto \frac{1}{(\beta_0\gamma_0)^{3/2}} \tag{9}$$

so that k_{x0}/k_{z0} remains constant while the energy changes, in accordance with Eq. (8).

As an example, consider a high-current rf linac accelerating protons from a nonrelativistic energy of 2 MeV ($\gamma_0 \approx 1$, $\beta_0 \approx 0.065$) to a relativistic energy of 938 MeV ($\gamma_0 \approx 2$, $\beta_0 \approx 0.866$). Assume $\epsilon_{nz}/\epsilon_{nx} = 2$ so that $k_{x0}/k_{z0} = \sqrt{2.5} \approx 1.58$ while at the same time both k_{x0} and k_{z0} decrease along the linac as $(\beta_0\gamma_0)^{-3/2}$. From (7) the bunch width ratio for this design is $z_m/a = 2$ at injection and decreases to $z_m/a = 1$ at full energy. The analysis shows that the beam radius scales as [6]

$$a \propto \frac{N^{1/3}}{k_{z0}^{2/3}}\frac{1}{\beta_0^{2/3}\gamma_0^{2/3}} \propto N^{1/3}\beta_0^{1/3}\gamma_0^{1/3} \tag{10}$$

and the longitudinal semi-axis as

$$z_m \propto \frac{N^{1/3}}{k_{z0}^{2/3}}\frac{1}{\beta_0^{2/3}\gamma_0^{5/3}} \propto \frac{N^{1/3}\beta_0^{1/3}}{\gamma_0^{2/3}}, \tag{11}$$

where N is the number of particles in the bunch. Thus, for a given particle number N or average current $\bar{I} = qNc/\lambda$, one finds that the bunch radius a increases from 2 MeV to 938 MeV by a factor of about 3 while z_m increases by about 1.5. This increase can be reduced by making a transition at an appropriate energy to a linac that operates at a higher frequency (shorter wavelength). For a given total power loss in the walls, one gets the scaling $E_m \propto (1/\lambda)^{3/4}$. Using the more conservative scaling $E_m \propto (1/\lambda)^{1/2}$, combined with the factor λ in the denominator of Eq. (2), one finds that k_{z0} scales with the rf wavelength as

$$k_{z0} \propto \frac{1}{\lambda^{3/4}} \tag{12}$$

so that the radius varies as

$$a \propto \lambda^{1/2}. \tag{13}$$

Thus, if one steps up the frequency by a factor of 4, as in LAMPF (200 MHz to 800 MHz), the semi-axes of the bunch will increase by only 50%. For example, using numbers similar to the study by Wangler et al [4], i.e. a 200 MHz drift tube linac with average current of $\bar{I}$=100 mA, emittances $\epsilon_{nx} = 6.85 \times 10^{-7}$ m-rad, $\epsilon_{nz} = 1.37 \times 10^{-6}$ m-rad, field gradient E_m=1.6 MV/m, and phase $\phi_0 = -40°$, one finds that the bunch radius and half-length at 2 MeV are $a \approx 2.5$ mm and $z_m \approx 5.0$ mm. For our equipartitioned linac design with a frequency change to 800 MHz one would then have $a = z_m \approx 3.8$ mm at 938 MeV. These numbers fit very well with typical drift-tube bore radii of 2 to 4 cm and they indicate that an equilibrated linac is a realistic alternative to the conventional design. In superconducting linacs, where wall losses are negligible, the gradient can be generally higher, say 4-6 MV/m so that the final beam size can be further reduced.

Summary and Conclusions

Our results and the conclusions can be summarized as follows:

1. The prevailing rf ion linac design (with σ_{x0}=const, $E_m|\sin\phi_0|$=const) is intrinsically anisotropic in temperature and unstable for high-current operation. Theoretically, such a two-temperature beam does not satisfy the stationary Vlasov equation and is therefore mismatched by definition even if transverse and longitudinal rms matching is done at injection. Equipartitioning, which is a manifestation of the instability, results in unavoidable emittance growth and halo formation.

2. As a logical alternative, a design strategy is proposed in which the beam is in three-dimensional equilibrium from injection to full energy. Such a beam is in a state of minimum total energy and stable against perturbations that would cause undesirable emittance growth and halo formation.

3. The parameters for the equilibrated beam look very attractive and simple analytical formulas can be derived to calculate the bunch radius a and semi-length z_m as a function of average current, focusing wave numbers, and kinetic energy.

4. Careful transverse and longitudinal matching of rms width, divergence, and beam profile in the transitions between accelerator sections will be necessary to keep the beam in thermal equilibrium and avoid creation of free energy that could cause emittance growth and halo formation [7].

5. The proposed design strategy for an equilibrated beam is relevant not only to ATW linacs but also to other high-current linac applications such as spallation neutron sources, high-energy colliders, heavy-ion fusion drivers, and to high-intensity electron rf linacs as well.

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References

[1] R.A. Jameson, G.P. Lawrence, and C.D. Bowman, *Nucl. Inst. and Meth. in Phys. Res. B* **68**, 474-480 (1992).

[2] R.A. Jameson, *IEEE Trans. Nucl. Sci.* **28**, 2408-2412 (1981).

[3] I. Hofmann, *IEEE Trans. Nucl. Sci.* **28**, 2399-2401 (1981).

[4] T.P. Wangler, T.S. Bhatia, G.H Neuschaefer, and M. Pabst, *Conference Record of the 1989 IEEE Particle Accelerator Conference*, 89CH2669-0, (1989), pp. 1748-1751.

[5] R.A. Jameson, *AIP Conference Proceedings* **279**, 969-998 (1993).

[6] M. Reiser, M., *Theory and Design of Charged Particle Beams*, New York: John Wiley & Sons, Inc., 1994, Appendix 4.

[7] M. Reiser, J. Appl. Phys. **70**, 1919-1923 (1991).