On the Dynamics of Space-Charge dominated Beams

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Abstract

A space-charge dominated beam is indubitably a *Complex System*, both *Chaos Dynamics* and *Plasma Physics* can be used to explain its behaviour. It is shown that the nonlinear resonances induce local instabilities then mixing property and stochastic motions, nonlinear space-charge waves which lead to metaequilibria and thermalization of the particle system. Results obtained using the particle-core model and a self-consistent PIC code (RENOIR) are presented and compared.

1. INTRODUCTION

For the new generations of high-current CW proton or deuteron linear accelerators, the most important aim is to reduce beam losses to an extremely low fraction of the total beam (< 10^{-7} / m) in order to limit the radioactivity in the machine area. To reach this objective, the basic phenomena which lead to emittance growth and halo formation must be understood, tools to study them must be built.

Recently, new insights have been obtained using the particle-core model (PCM) [1]. This method can be briefly summarized by : -i- at first, using the KV envelope equation, computation of the beam core evolution in the focusing channel to be studied, -ii- then, analysis of the behaviour of test particles injected into or around the beam core.

Ref.[1] and [2] concern the evolution of a zero-emittance beam in a continuous focusing channel. For this N=1.5 degrees of freedom system, a test particle is an "oscillator" coupled to a periodic nonlinear perturbing force induced by the mismatched core. In ref.[2], the different sorts of trajectories are analysed using Poincaré sections and the leading role of the $\upsilon = \omega_{particle}/\omega_{core} = 1/2$ resonance is pointed out. Chaotic and halo areas formed "under almost any perturbation" and always bounded by invariant tori (KAM curves) are also described.

Ref. [3] and [4] analyse the behaviour of a matched beam in a FODO channel. Like for the previous study, chaotic trajectories are induced by the beam-core oscillation. Nevertheless, the results obtained for a continuous focusing channel cannot be extrapolated because the phase-space topology changes sharply. For a FODO channel : -i- both number and order of the resonances which are present around the beam core are determined by the phase advances with and without space charge, -ii- the coupling force induced by space charge leads to a N=2.5 nonautonomous system in which Arnol'd diffusion takes place [4]. For N>2, the invariant tori no longer divide the space, they intersect to form the so-called Arnol'd web and the "system may move as far as desired across the surface of equal energy" [5]. Arnol'd diffusion has then to be studied with great care because it permits diffusion of particles far from the beam core.

The PCM is a simplified model in which the N-body (N-particle) system is studied as a restricted three-body problem (the halo particules are nonperturbing). It might be used to study 3D problems with low computation time in order to estimate diffusions concerning an extremely low fraction of the total beam. Before to undertake such studies, the PCM validity domain must be checked. In the following sections, results obtained with this model will be compared to those obtained using the RENOIR PIC code in the case of a finite-emittance beam in a continuous focusing channel.

2. PARTICLE-CORE MODEL RESULTS

Dimensionless equations of motion for both beam core and test particles are given in ref.[6]. They depend only on the space-charge tune depression $\eta = k/k_0$ where k and k_0 are respectively the phase advances per unit length with and without space charge. The core-radius oscillation being characterized by $k_c^2 = 2(k_0^2 + k^2)$, the tune (υ) of a test particle injected inside the core is $\upsilon_i = k/k_c = \eta/\sqrt{2(1+\eta^2)}$. For particles injected further and further, the influence of the space-charge force becomes smaller and smaller then υ tends toward $\upsilon_\infty = k_0/k_c = 1/\sqrt{2(1+\eta^2)}$. The tune of the test particles is then such that $\upsilon_i \leq \upsilon < \upsilon_\infty$ and the resonances which are present around the beam core are determined by η. Figure 1 shows that the $\upsilon = 1/2$ resonance (always present) can overlap with the $\upsilon = 1/4$ resonance when $\eta < 0.4$.

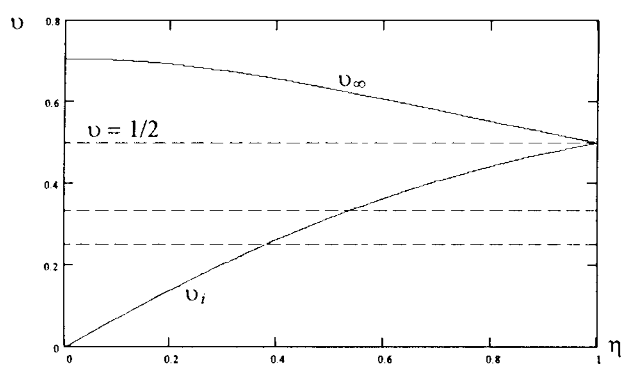

Fig.1 : tune spread around the beam core versus η.

The system can be seen as a "test-particle oscillator coupled by the space charge to the beam-core oscillator" [3]. This becomes obvious when the Poincaré sections drawn for different η values [6] are compared to those drawn for a system of two coupled oscillators with the Hamiltonian :

$$H = (p_1^2 + \omega^2 x_1^2)/2 + (p_2^2 + \omega^2 x_2^2)/2 + \alpha(x_1^4 + x_2^4)/4 + \varepsilon x_1^2 x_2^2/2$$

(ref.[5]-p.234). The different sorts of trajectories, the importance of the $\upsilon = 1/2$ resonance and the accumulation of particules in the vicinity of its hyperbolic fixed points, already described in ref.[2] for $\eta = 0$, can be also observed in these figures.

Figure 2 shows calculations of phase advances ($\sigma = 2\pi\upsilon$) for $\eta = 0.1$ and $\eta = 0.5$. To draw this figure, the evolution of particles injected with increasing amplitudes (x'=0) is computed over one beam-core oscillation, then σ is calculated. The results are given for both KV and Gaussian density distributions ($\rho(r)$).

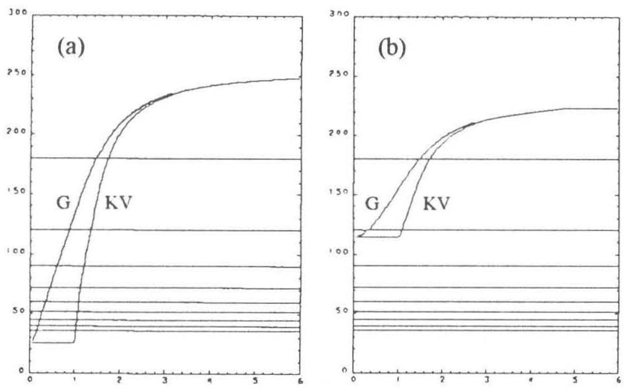

Fig.2 : Phase advances ($\sigma°$) vs amplitude for $\eta = 0.1$ (a) and $\eta = 0.5$ (b), KV and Gaussian distributions with $R_0 = R_{rms} = 1$.

Figure 3 shows a cloud of test particules in the transverse phase space after 33 beam-core oscillations. To draw it, 2000 test particles have been injected with a mismatch parameter M=1.5, the space-charge tune depression is $\eta = k(r \sim 0)/k_0 = .5$, the beam-core density distribution is Gaussian and the initial "painting" is limited to 3 R_{rms}. An analysis of the trajectories shows that the degree of stochasticity is weak for $\eta = .5$, nevertheless, the "whorls" and "tendrils" formed around the elliptic and hyperbolic fixed points lead to the complex pattern shown in figure 3.

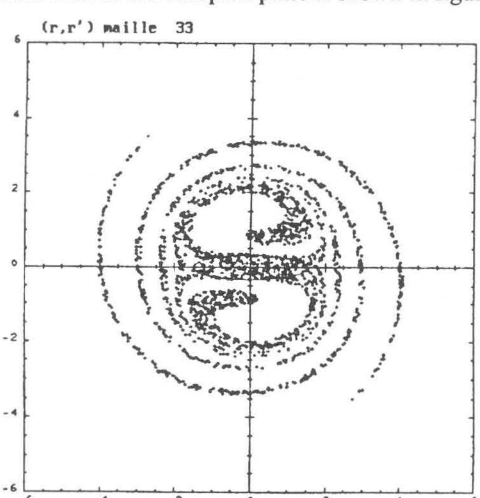

Fig.3: Phase-space portrait after 33 core oscillations ($\eta = .5$)

3. RENOIR PIC CODE RESULTS

For a complete description of a N-charged-particle system, the coordinate $\vec{r}_k(t)$ and velocity $\vec{v}_k(t)$ of each particle must be given as a function of time. The system is then completly described by the distribution function :

$$f(\vec{r},\vec{v},t) = \Sigma_{k=1,N}\ \delta(\vec{r} - \vec{r}_k(t))\ \delta(\vec{v} - \vec{v}_k(t)) \qquad (1)$$

which is solution of the Klimontovich-Duprée and Maxwell equations for the microscopic fields. Clearly, it is difficult and costly to solve directly this 6ND general system. Fortunatly, f can be expanded as a series of the plasma parameter $g = 1/n\lambda_D^3$ where n is the particle density and λ_D is the Debye shielding length; this development gives a chain of equations of order $g^0, g^1, \ldots$. In intense charged particle beams, g ($\sim 10^{-8}$) is always a "small parameter", therefore order 0 is already an accurate description of the system . This is just the 6D Vlasov- Maxwell system which is reduced in RENOIR to :

$$\text{Vlasov :}\quad \frac{\partial \tilde{f}}{\partial t} + \vec{v}_\perp . \vec{\nabla}\tilde{f} + \frac{q}{\gamma m}\left\{\vec{E}^{s+x} + \vec{v}\times\vec{B}^{s+x}\right\}_\perp . \vec{\nabla}_v \tilde{f} = 0 \qquad (2)$$

$$\text{Poisson :}\quad -\Delta\phi^s = \rho/\varepsilon_0 \ \text{ and } \ \vec{E}^s = -\vec{\nabla}\phi^s \qquad (3)$$

$$\text{Ampère :}\quad -\Delta\vec{A} = \mu_0\vec{j} \ \text{ and } \ \vec{B}^s = \vec{\nabla}\times\vec{A}^s \qquad (4)$$

plus boundary conditions

where $\tilde{f} = f(\vec{r},\vec{v},t) + o(g)$, $\phi \equiv \phi(\vec{r},t)$ and $\vec{A} \equiv \vec{A}(\vec{r},t)$ are respectively the scalar and vector potential, $\vec{E} \equiv \vec{E}(\vec{r},t)$ and $\vec{B} \equiv \vec{B}(\vec{r},t)$ are the electric and magnetic fields, $\rho = q\int\tilde{f}\,d\vec{v}$ and $\vec{j} = q\int\tilde{f}\,\vec{v}\,d\vec{v}$ are charge and current densities; the suffixes s and x refer to self and external fields.

Equations (2-4) point out that the N-particle system is a Vlasov's plasma in which metaequilibria can exist [7]. These "quasi-steady" equilibria develop both organized and chaotic structures in phase space, they do not fit exactly with the ideal Maxwell-Boltzman equilibrium. Without collisions, they can persist a long time, then be modified in a reversible way, then stabilized by some perturbations due to the "breathing-core"... The existence of these metaequilibria has been observed in the time evolution of the rms emittance.

Instead of solving directly the Vlasov equation (2) depending of Euler variables (r,v), the equations :

$$\frac{\partial \vec{r}_k}{\partial t} = \vec{v}_k = \frac{\vec{u}_k}{\gamma} \quad \text{and} \quad \frac{\partial \vec{u}_k}{\partial t} = \frac{q}{\gamma m}\left(\vec{E}_k^{s+x} + \frac{\vec{u}_k}{\gamma_k}\times\vec{B}_k^{s+x}\right)$$

can be used where $\vec{r}_k(t)$ and $\vec{v}_k(t)$ are Lagrangian variables which describe the characteristics of the Vlasov equation. Using (1), the charge and current densities can be written :

$$\rho(\vec{r},t) = \Sigma_k\, q\,\delta(\vec{r} - \vec{r}_k(t)) \ \text{ and } \ \vec{j}(\vec{r},t) = \Sigma_k\, q\,\vec{v}_k(t)\ \delta(\vec{r} - \vec{r}_k(t))$$

This reformulation performed without new approximation gives a still too large system to solve. The following crude approximations are done :

-i- the space coordinate ($\vec{r}$) is projected on a grid,

-ii- ρ and $\vec{j}$ are estimated at each grid node ($r = R_i$) using the projection $S(\vec{R}_i - \vec{r}_k)$: $\rho(\vec{R}_i,t) = \Sigma_k\, q\, S(\vec{R}_i - \vec{r}_k)$ and $\vec{j}(\vec{R}_i,t) = \Sigma_k\, q\,\vec{v}_k\, S(\vec{R}_i - \vec{r}_k)$

-iii- the fields $\vec{E}_k$ and $\vec{B}_k$ are calculated at each particle location ($\vec{r}_k$) using a restricion $S(\vec{R}_i - \vec{r}_k)$ from the grid nodes : $\vec{E}_k(\vec{r}_k(t)) = \Sigma_i\, \vec{E}(\vec{R}_i,t)\, S(\vec{R}_i - \vec{r}_k)$

These are ingredients of the PIC model. Attention must be payed on the fact that the system is purely Hamiltonian, numerical schemes must then be diffusionless on the whole. In RENOIR, the radial density evolution is simulated with relative density values as low as 10^{-12} using only 30000 macro-particles.

The dynamics in phase-space has been compared to those obtain with the PCM. This study shows that the particle motions are well explained by the analysis done in ref.[2] and section 2. For the first beam-core oscillations, "islands", KAM trajectories and accumulation of particles around hyperbolic fixed points can be observed but it is the system mixing property which is especially prominent (see figure 4). The particle motions are complicated by mutual core-halo perturbations but the global behaviour shown using the PCM can be observed. The self-similar structure of the distribution can be seen in figure 4 (η = .5, M=1.5 and initial Gaussian distribution extended up to 6 R_{rms}).

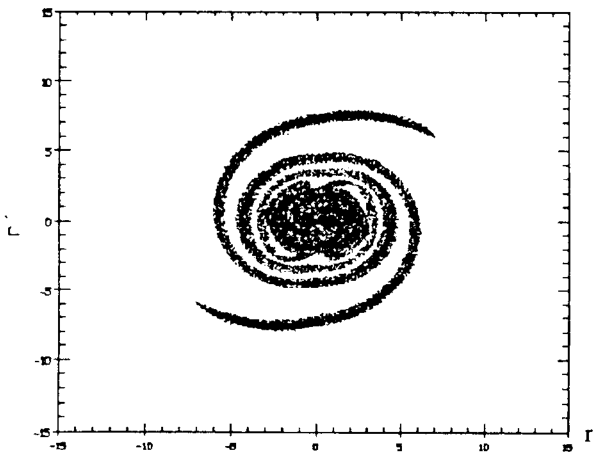

Fig.4 : phase-space portrait after 33 R_{rms} oscillations.

The density in the core vicinity tends rapidely (some core oscillations) toward an equilibrium. At the opposite, halo particles oscillate continuously and the presence of solitary waves (solitons) [7] which satisfy the Bohm-Gross dispersion relation [8] can be observed. Figure 5 shows these solitons (pointed by arrows) on the density profile, they are the result of particle motions on separatrices [5] which lead to the formation of aggregates of particles.

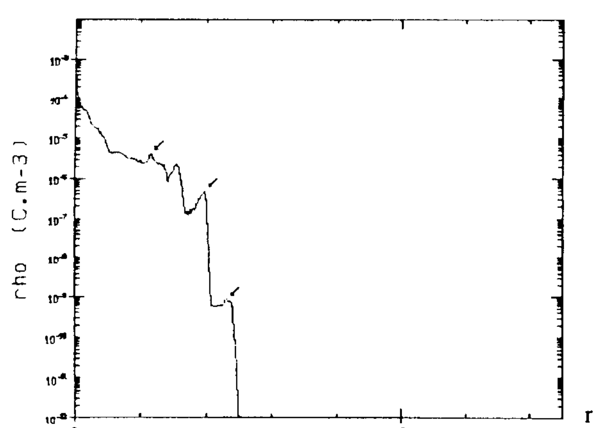

Fig.5 : Charge density versus r ($\rightarrow$solitons).

Application of the Vlasov equation to nonneutral plasmas and "plasma effects in intense beams" have been and still are discussed [9]. Reflection and transmission of space-charge waves have been observed experimentally for an electron beam [10]. The present study has described the mechanisms leading to the formation of solitary waves. These solitons which locally accelerate or deccelerate the particles indicate a possible source of thermalization.

4. CONCLUSION

Space-charge dominated beams are obviously *complex systems*. The Poincaré sections show a complex mixture of chaotic and regular trajectories, a self-similar (fractal) structure of elliptic and hyperbolic points induced by nonlinear resonances. This property is inherent in all nonlinear systems (K-systems) for which local instability leads to mixing property, to the formation of "whorls" and "tendrils". This complexity is the result of particle motions in the neighbourhood of the separatrices associated with each resonance. Very close to a separatrix, the period of oscillations tends to infinity and the particles form solitons around the hyperbolic fixed points; even small changes in initial position give a completely different behaviour. This is the *causa finalis* of local instability which induces mixing property and stochasticity [5].

A Poincaré surface of section obtained thanks to the PCM gives the description of the particle trajectories at a given time. When the beam progresses, the core distribution is modified and η increases. Figure 2 shows that the position of the resonances changes, that some of them disappear. In spite of this evolution of the phase-space topology, a first comparison with the self-consistant PIC code RENOIR has shown that the beam behaviour is quite well described by the PCM. Further studies will enable to determine the PCM validity limits as a function of the beam characteristics.

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