SPACE CHARGE EFFECT OF BUNCHED BEAM WITH NONUNIFORM DISTRIBUTION IN BOTH LONGITUDINAL AND TRANSVERSE DIRECTIONS *

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Abstract

The nonlinear space charge effect of bunched beam with nonuniform density distribution in both longitudinal and transverse directions is discussed in this paper. Some useful formulae are derived for calculating the potential induced by a cylinder model of space charge in the waveguide of a linac with longitudinal density distributions of waterbag(WB) or parabolic(PA) type combining with transverse density distributions of Kapchinskij-Vladimirskij(K-V), waterbag, parabolic and Gaussian(GA) types, respectively.

Introduction

The nonlinear space charge effect of bunched beam is one of the basic factors determining the beam dynamics in many intense beam facilities, such as Free Electron Laser(FEL), Inertial Confined Fusion(ICF), Heavy Ion Fusion(HIF) and some microwave devices. Although many study results published[1-6], some dealt with uniform density distribution in the bunched beam, and the others concerning the calculation of space charge field were based on the assumption of a bunch in free space or a continuous beam in a tube.

In Ref.7, the general calculation formulae for the nonuniform density space charge effect in the waveguide of electron linac have been developed. Furthermore, in the Ref.8 the space charge effect of the disk and cylinder models with different transverse density distributions, including waterbag(WB), parabolic(PA) and Gaussian(GA) distributions, was discussed. However, longitudinal uniform distribution was still adopted in Ref.8. So it is necessary to develop the theory on space charge effect of a bunched beam with nonuniform distribution in both longitudinal and transverse directions.

General formulae

The potential induced by a cylinder space charge bunch with uniform distribution ρ can be written as follows[1]:

$$\varphi_0=\rho f_0(r,z;b,\frac{L}{2}) \tag{1}$$

where b and L/2 are the boundaries of the bunch in r and z axes respectively, f_0 is the potential induced by a unit space charge density, and the subscript " 0 " stands for uniform density distribution.

According to the general formulae in Ref.7, for the space charge bunch model with nonuniform charge density distribution $\rho(r,z)=\rho(r)\rho(z)$, we have the induced potential as follows:

$$\varphi=\int_0^{\frac{L}{2}}\int_0^{b}\rho(\xi,\zeta)\frac{\partial^2 f_0(r,z;\xi,\zeta)}{\partial\xi\partial\zeta}d\xi d\zeta \tag{2}$$

Substituting the potential induced by a uniform density cylinder space charge, [1] we can rewrite the general formulae for calculating the potential induced by a space charge bunch model with nonuniform density distribution as follows:

$$\varphi_{1,2}=\frac{2}{\varepsilon_0 a}\int_0^{\frac{L}{2}}\int_0^{b}\rho(\xi,\zeta)\ S\ e^{-k_i|z|}d\xi d\zeta,\ (|z|>\frac{L}{2}) \tag{3}$$

$$\varphi_3=\frac{2}{\varepsilon_0 a}\int_0^{|z|}\int_0^{b}\rho(\xi,\zeta)\ S\ e^{-k_i|z|}d\xi d\zeta+$$
$$+\frac{2}{\varepsilon_0 a}\int_{|z|}^{\frac{L}{2}}\int_0^{b}\rho(\xi,\zeta)\ S\ e^{-k_i\zeta}d\xi d\zeta,\ (|z|<\frac{L}{2}) \tag{4}$$

where

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$$S=\sum_{l=1}^{\infty}\frac{\xi J_0(k_l\xi)J_0(k_lr)}{(k_la)J_1^2(k_la)}ch(k_lz). \qquad (5)$$

Employing the eqs.(3) and (4), we have derived the potentials induced by a cylinder model of space charge with nonuniform distributions in both longitudinal and transverse directions in a waveguide of a linac as follows in which the subscript "1,2" stands for the potential in the region of (|z|>L/2), while the subscript "3" for the potential in the region of (|z|<L/2).

Longitudinal WB distribution

Transverse (K-V) distribution

$$\varphi_{1,2}=\frac{2ab\rho_{kv,wb}}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_1(k_lb)J_0(k_lr)}{(k_la)^3J_1^2(k_la)}P_{wb}, \qquad (6)$$

$$\varphi_3=\frac{2ab\rho_{kv,wb}}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_1(k_lb)J_0(k_lr)}{(k_la)^3J_1^2(k_la)}Q_{wb}, \qquad (7)$$

where $\rho_{kv,wb}=\frac{3q}{2\pi b^2L}$, total charge q and

$$P_{wb}=\frac{2}{k_l(L/2)^2}\left(\frac{L}{2}ch\frac{k_lL}{2}-\frac{1}{k_l}sh\frac{k_lL}{2}\right)e^{-k_l|z|}, \qquad (8)$$

$$Q_{wb}=\left(1-\frac{z^2}{(L/2)^2}-\frac{2}{k_l^2(L/2)^2}\right)+$$

$$+\left(\frac{2}{k_l(L/2)}+\frac{2}{k_l^2(L/2)^2}\right)e^{-k_lL/2}ch(K_lz). \qquad (9)$$

Transverse WB distribution

$$\varphi_{1,2}=\frac{4a^2\rho_{wb,wb}}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_2(k_lb)J_0(k_lr)}{(k_la)^4J_1^2(k_la)}P_{wb}, \qquad (10)$$

$$\varphi_3=\frac{4a^2\rho_{wb,wb}}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_2(k_lb)J_0(k_lr)}{(k_la)^4J_1^2(k_la)}Q_{wb}, \qquad (11)$$

where $\rho_{wb,wb}=\frac{3q}{\pi b^2L}$.

Transverse PA distribution

$$\varphi_{1,2}=\frac{16a^3\rho_{pa,wb}}{\varepsilon_0 b}\sum_{l=1}^{\infty}\frac{J_3(k_lb)J_0(k_lr)}{(k_la)^5J_1^2(k_la)}P_{wb}, \qquad (12)$$

$$\varphi_3=\frac{16a^3\rho_{pa,wb}}{\varepsilon_0 b}\sum_{l=1}^{\infty}\frac{J_3(k_lb)J_0(k_lr)}{(k_la)^5J_1^2(k_la)}Q_{wb}. \qquad (13)$$

where $\rho_{pa,wb}=\frac{9q}{2\pi b^2L}$

Transverse GA distribution

$$\varphi_{1,2}=\frac{2\rho_{ga,wb}\alpha^2}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_0(k_lr)}{(k_la)^2J_1^2(k_la)}e^{-k_l^2\alpha^2/2}P_{wb}, \qquad (14)$$

$$\varphi_3=\frac{2\rho_{ga,wb}\alpha^2}{\varepsilon_0}\sum_{l=1}^{\infty}\frac{J_0(k_lr)}{(k_la)^2J_1^2(k_la)}e^{-k_l^2\alpha^2/2}Q_{wb}, \qquad (15)$$

where $\rho_{ga,wb}=\frac{3q}{4\pi\alpha^2L}$, $\alpha^2=\langle x^2\rangle$

Longitudinal PA distribution

The analogous formulae can be obtained for potential induced by a bunched beam with longitudinal PA distribution, while transverse density distributions of K-V, WB, PA and GA types, respectively, if P_{wb}, Q_{wb}, and $\rho_{kv,wb}$, $\rho_{wb,wb}$, $\rho_{pa,wb}$, $\rho_{ga,wb}$ in Eq.(6), Eq.(7), and Eq.(10) to Eq.(15) are correspondingly replaced by

$$P_{pa}=\left[\frac{8}{(k_lL/2)^2}sh\frac{k_lL}{2}+\frac{24}{(k_lL/2)^4}sh\frac{k_lL}{2}-\right.$$

$$\left.-\frac{24}{(k_lL/2)^3}ch\frac{k_lL}{2}\right]e^{-k_l|z|},\qquad(16)$$

$$Q_{pa}=1-\frac{2}{(L/2)^2}\left(z^2+\frac{2}{k_l^2}\right)+$$

$$+\frac{1}{(L/2)^4}\left(z^2+\frac{12z^2}{k_l^2}+\frac{24}{k_l^4}\right)-$$

$$-\frac{32}{(k_lL)^2}e^{-k_lL/2}\left(1+\frac{6}{k_lL}+\frac{12}{(k_lL)^2}\right)chk_lz,\qquad(17)$$

and

$$\rho_{kv,pa}=\frac{15q}{8\pi b^2L},\ \rho_{wb,pa}=\frac{15q}{4\pi b^2L}$$

$$\rho_{pa,pa}=\frac{45q}{8\pi b^2L},\ \rho_{ga,pa}=\frac{15q}{16\pi\alpha^2L}$$

respectively.

As L/2 approaches to infinity , the above potential formulae eq.(6) to eq.(17) degenerated into the potentials induced by the cylindrical space charge with longitudinal uniform distribution while uniform(K-V), waterbag, parabolic and Gaussian distributions in transverse direction, respectively, in Ref.8. Furthermore, as L/2 and b approach to zero, the above equations agree to the potential of a point charge in Ref.1.

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