DENSITY UNIFORMING OF LARGE MOMENTUM SPREAD BEAM

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Abstract

A method of beam intensity transformation for making uniform irradiated area is discussed. Combination of linear (quadrupoles) and nonlinear (octupoles, dodecapoles, etc.) lenses produces required variation of transverse beam momentum which results in beam density equalization after beam drift. Influence of transverse and longitudinal momentum spread on uniformity of final beam distribution is studied. Conditions, when arbitrary value of beam emittance does not affect the final uniformity of the beam at the target are found.

Introduction

Intensity distribution of accelerated beam is characterized by high concentration of particles near axis of structure and beam halo at the periphery . Many practical applications of accelerated beams require highly uniform irradiated zone where every local element of the target accepts equal number of particles per unit of time. A useful method of beam intensity redistribution in a transport channel containing linear and nonlinear focusing lenses to provide flattened distribution at the target was considered in ref. [1-5]. Nonlinear transverse velocity modulation due to nonlinear elements (octupoles, dodecapoles, etc.) force the peripheral particles to move faster to the axis than inner beam particles. During the drift after modulation the beam halo is eliminated and the boundaries of the beam become more pronounced. Superposition of transverse momentum modulation in two orthogonal planes provides rectangular beam spot at the target with high uniformity. Previous analytical studies [4,5] covered kinematics relationship between final and initial beam distribution via nonlinear channel parameters and neglecting beam emittance. This paper analyses non-zero value of particle momentum spread (both longitudinal and transverse) on flattening of beam distribution.

Redistribution of Beam Intensity Neglecting Momentum Spread

Relationship between initial and final beam distribution in a nonlinear optics channel ignoring beam emittance was obtained in ref. [4]. Suppose the beam of particles with charge q and mass m has transverse momentum modulation at starting point z=0

$$p_x = p_{xo} + a_2 x_o + a_3 x_o^2 + a_4 x_o^3 + ... + a_n x_o^{n-1} \qquad (1)$$

where p_{xo} and x_o are initial transverse momentum and position of particle. After a drift to a distance z the x-coordinate of the particle becomes

$$x = x_o + \frac{z}{p_z}(p_{xo} + a_2 x_o + a_3 x_o^2 + a_4 x_o^3 + ... + a_n x_o^{n-1}) \ . \qquad (2)$$

The number of particles dN inside the element (x,x+dx) is invariable hence the particle density $\rho(x) = dN/dx$ at any z is connected with the initial density $\rho(x_o)$ by condition $\rho(x) = \rho(x_o)\, dx_o/dx$ or

$$\rho(x) = \rho(x_o) \frac{1}{1 + \alpha_2 + 2\alpha_3 x_o + 3\alpha_4 x_o^2 .. + (n-1)\alpha_n x_o^{n-2}} \qquad (3)$$

where notation is used: $\alpha_n = a_n z/p_z$. Coefficients α_2, α_3, α_4,... for flattening beam density distribution can be obtained from Taylor expansion of initial beam profile. For example if the initial distribution of particles is Gaussian

$$\rho_o \exp(-\frac{x_o^2}{2\sigma^2}) = \rho_o (1 - \frac{x_o^2}{2\sigma^2} + \frac{x_o^4}{8\sigma^4} + .. + \frac{(-1)^k x_o^{2k}}{2^k k!\, \sigma^{2k}}) \qquad (4)$$

the coefficients to provide constant distribution at the target $\rho(x) = P$ are [5]:

$$\alpha_2 = \frac{\rho_o}{P} - 1; \quad \alpha_4 = -\frac{1+\alpha_2}{6\sigma^2}; \quad \alpha_6 = \frac{1+\alpha_2}{40\sigma^4};$$

$$\alpha_{2k+2} = \frac{(-1)^k (1+\alpha_2)}{2^k k!(2k+1)\sigma^{2k}}; \qquad k = 1,2,3... \qquad (5)$$

Coefficients α_n can be expressed through parameters of multipole magnetic lenses. If extension of the beam is provided by combination of two magnetic quadrupoles at distance L between them and nonlinear transverse momentum modulation is provided by higher order magnetic multipoles then coefficients are

$$\alpha_2 = S_2^2 L z; \qquad \alpha_n = S_n z, \quad n=3,4,5.. \qquad (6)$$

where n is an order of multipole (n=2 for quadrupole, n=3 for sextupole, etc.) and S_n is the strength of an n-th order multipole

$$S_n = \frac{B_o d_n}{R^{n-1} (Br)} \qquad (7)$$

of length d_n, pole tip field B_o and radius R for a beam with particle rigidity $Br = p_z/q$.

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Effect of Longitudinal Momentum Spread

The final distribution of particles in a transport channel (see eq.3) $\rho(x) = \rho(x_0)/g(x_0)$ depends on the initial distribution $\rho(x_0)$ and the values of modulation coefficients $\alpha_2, \alpha_3, ...$ through the function

$$g(x_0) = 1 + \alpha_2 + 2\alpha_3 x_0 + 3\alpha_4 x_0^2 + ... + (n-1)\alpha_n x_0^{n-2} \quad . \qquad (8)$$

Distortion of final beam distribution under deviation of parameters from their fixed values is a matter of interest. Relative error of the flattened distribution follows from eqs. (3),(8):

$$\frac{d\rho(x)}{\rho(x)} = \frac{d\rho(x_0)}{\rho(x_0)} - \frac{dg(x_0)}{g(x_0)} \quad . \qquad (9)$$

The first term in eq.(9) is due to distortion of initial beam distribution . For Gaussian beam (4) it gives:

$$\frac{d\rho(x_0)}{\rho(x_0)} = \frac{x_0^2}{\sigma^2}\left(\frac{d\sigma}{\sigma}\right) \qquad (10)$$

which means strong dependence of final beam distribution on small deviation in initial distribution.

The second term can be expressed as follows:

$$\frac{dg}{g} = \frac{d\alpha_2 + 2x_0\, d\alpha_3 + ... + (n-1)x_0^{n-2}\, d\alpha_n}{1 + \alpha_2 + 2\alpha_3 x_0 + 3\alpha_4 x_0^2 + ... + (n-1)\alpha_n x_0^{n-2}} \quad . \qquad (11)$$

Modulation coefficients $\alpha_2, \alpha_3, ...$ depend on beam line parameters L,z, d_n and particle rigidity Br . Let us take into account only spread of longitudinal momentum of particles $\delta = - dp_z/p_z$ neglecting all other terms resulting in deviation of modulation coefficients from their adjusted values. Equation (11) can be rewritten as

$$\frac{dg}{g} = \left(\frac{\alpha_2 + g - 1}{g}\right)\delta \quad . \qquad (12)$$

For initial Gaussian distribution and strongly expanded beam ($\alpha_2 >> 0$) the relative change of the beam distribution is

$$\frac{dg}{g} = \left(1 + \frac{1}{\exp(-x_0^2/2\sigma^2)}\right)\delta \qquad (13)$$

and for weakly enlarged beam ($\alpha_2 \sim 0$)

$$\frac{dg}{g} = \left(1 - \frac{1}{\exp(-x_0^2/2\sigma^2)}\right)\delta \quad . \qquad (14)$$

In both cases the distortion of beam distribution takes place but smaller than distortion due to deviation in initial beam distribution because functions (13), (14) are more flat near axis of the structure than function (10) .

Effect of Transverse Beam Emittance

Consider the influence of transverse beam momentum spread on final beam distribution uniformity. Suppose the initial beam phase space distribution function can be presented as a product of two independent functions, i.e. the distributions at x_0 and p_{x0} are independent on each other:

$$f(x_0, p_{x0}) = \frac{dN(x_0, p_{x0})}{dx_0\, dp_{x0}} = \rho(x_0)\, w(p_{x0}) \quad . \qquad (15)$$

Formulas (1), (2) describe the canonical transformation from old variables (x_0, p_{x0}) to new variables (x, p_x). According to Liouville's theorem the number of particles inside the phase space element is invariable

$$dN = f(x_0, p_{x0})\, dx_0\, dp_{x0} = f(x, p_x)\, dx\, dp_x \qquad (16)$$

and Jacobian of transformation $\partial(x, p_x)/\partial(x_0, p_{x0}) = 1$. The distribution function of the beam at the target is connected with the initial distribution function by

$$f(x_0, p_{x0}, 0) = f\,[x - \frac{p_x}{m}t,\ p_x - a_2(x - \frac{p_x}{m}t) - a_3(x - \frac{p_x}{m}t)^2 - ..] \quad . \qquad (17)$$

To obtain beam distribution in real space one has to integrate distribution function over momentum:

$$\rho(x) = \int f\, dp_x = \int \frac{\rho(x_0)}{\left(\frac{dp_{x0}}{dp_x}\right)} w(p_{x0})\, dp_{x0} = \qquad (18)$$

$$\int \left[\frac{\rho(x_0)}{1 + \alpha_2 + 2\alpha_3 x_0 + 3\alpha_4 x_0^2 + ... + (n-1)\alpha_n x_0^{n-2}}\right] w(p_{x0})\, dp_{x0}$$

The expression in square brackets is the final beam distribution in real space for zero-emittance case (see eq. 3). Appropriate choice of coefficients $\alpha_2, \alpha_3, \alpha_4$ results this expression to be constant so it can be removed out of integral (18). The rest function which is still under integral is not a function of x therefore the final beam distribution for non-zero emittance is again constant:

$$\rho(x) = P\int w(p_{x0})\, dp_{x0} = \text{const} \quad . \qquad (19)$$

The result of this consideration is that it is possible to obtain uniformity of the beam with any value of beam emittance. This statement has to be commented. Consider again the zero-emittance phase space layer of initial beam distribution (15) for particles with the value of initial momentum p_{fix}

$$df(x_0, p_{x0}, 0) = w_0\, \rho(x_0)\, \delta(p_{x0} - p_{fix}) \quad . \qquad (20)$$

From eq. (1), (2), (3) it follows that this distribution can be transformed into uniform distribution but particles devoted to this phase space layer will be shifted in x coordinate from the initial position after beam drift at $\Delta x(p_{fix}) = t\, p_{fix} / m$. Every phase space layer will be transformed in the same manner but bias towards x will be different for every layer. The resultant transformation is a superposition of all flattened distributions from all layers. If we are not interested in margins of the final distribution, the result of that superposition will be uniform distribution for all beam with arbitrary value of beam emittance as it described by eq. (19). But the margins of the beam will be overlapped partially so the final uniform distribution $\rho(x)$ of the whole beam will be dropped at the boundaries of the beam. To avoid it one has to limit drift distance of the beam in such a way that the bias of phase space layers should be smaller than transverse size of the beam: $z\, p_{x\ max} / p_z < R$ or

$$z < \frac{R^2 \beta_z}{\varepsilon} \qquad (21)$$

where $\varepsilon = \pi\, p_{x\ max} R / mc$ is a normalized beam emittance. At fig. 1,2 the results of beam phase space transformation for zero and non-zero beam emittance are presented. Calculation were executed using particle-in-cell code BEAMPATH [6] for nonlinear beam optics study. Beam of particles was represented as a collection of 10000 particles. Initial particle distribution in x and p_x was Gaussian. The nonlinear elements up to the 52th order were included in simulation to provide required transverse momentum modulation. As it shown a flattening of beam density distribution can be achieved in both cases.

Conclusions

The nonlinear optics method for equalization of beam intensity distribution was discussed. The kinematics relationship between initial and final beam distribution for non-zero value of beam emittance was given. It was shown that final beam uniformity is critically dependent on initial beam distribution and less dependent on longitudinal momentum spread of the beam. Under certain conditions the transverse beam emittance does not affect the flattening of beam distribution but the drift distance of the beam after nonlinear transverse momentum modulation is limited.

References

[1] Ph.F.Meads, Jr., IEEE Trans. Nucl.Science, NS-30, No.4, (1983), 2838.
[2] B.Sherrill, J.Bailey, E.Kashy, C.Leakeas, Nucl. Instr. Meth. B40/41, (1989), 1004.
[3] A.Jason, B.Blind, E.Svaton, Proc. 1988 Linear Acceler. Conference, CEBAF Report 89-001, 192.
[4] Y.Batygin, Nucl. Instr. Meth. , B79 (1993), 770.
[5] Y.Batygin, Proc. of the 1993 Particle Accelerator Conference, Washington, D.C.,(1993) ,50.
[6] Y.Batygin, Proc. EPAC92, Berlin, (1992),822.

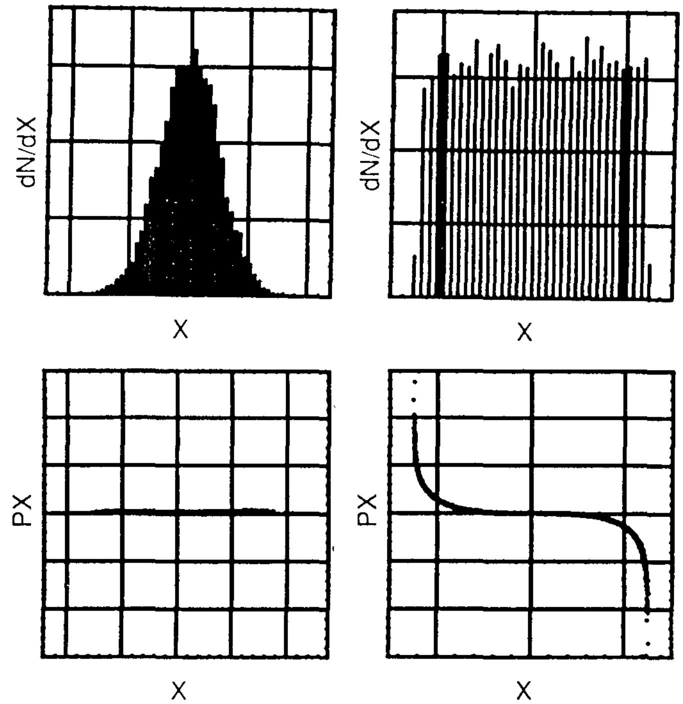

Fig.1. Density uniforming of zero-emittance beam. Upper part illustrates initial (left) and final (right) beam density distribution. At the lower part initial (left) and final (right) phase space projections are presented.

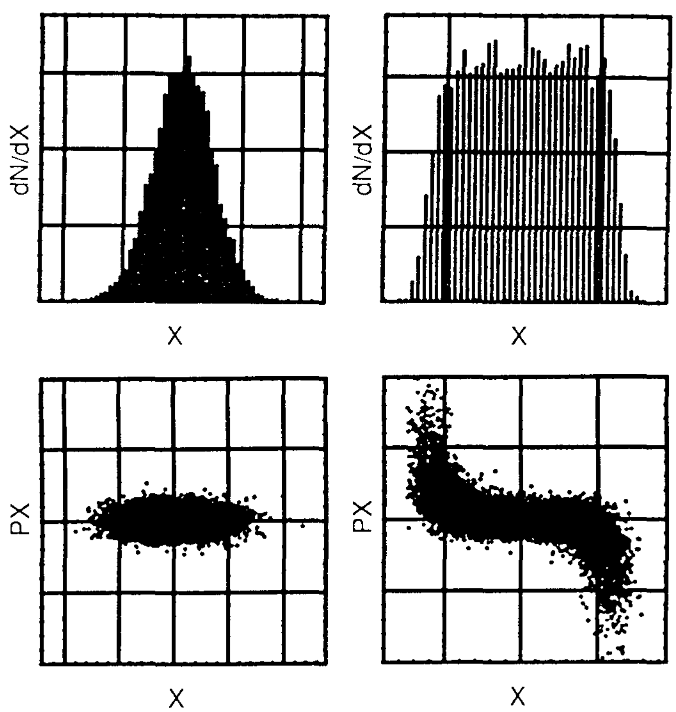

Fig. 2. Density unifroming of non-zero emittance beam. The meaning of the plots are the same as at fig. 1.