NONLINEAR POTENTIAL DISTRIBUTION FOR HIGH CURRENT BEAM TRANSPORT WITHOUT EMITTANCE GROWTH

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Abstract

Beam dynamics in nonlinear uniform focusing channel is studied from the point of view of keeping emittance and brightness of high current beam. Conservation of beam emittance is treated as a problem of proper matching of the beam with the uniform focusing channel. To obtain matching conditions for the beam with arbitrary distribution function, it is necessary to accept that the potential of external focusing field contains higher order terms than quadratic. The solution for external potential is attained from stationary Vlasov equation for beam distribution function and Poisson equation for electrostatic beam potential. Possible variants to create required nonlinear potential distribution are discussed. Analytical approach is illustrated by the results of particle-in-cell simulation.

Introduction

Nonlinear beam dynamics play an important role in limitation of intensity and brightness of particle beams. Single particle aberrations result in filamentation in phase space. Injection of non uniform space-charge dominated beam into a uniform linear focusing channel results in beam emittance growth due to intrinsic mismatching of the beam with the channel [1]. For a beam matched with a uniform focusing channel the Hamiltonian

$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{k^2}{2}(x^2 + y^2) + q\,U_b(x,y) \qquad (1)$$

is a constant of motion and any function of Hamiltonian provides self-consistent solution for scalar potential of the beam U_b and beam phase space distribution. If external focusing field is linear, only KV distribution [2] gives elliptical phase space projections. Other function of Hamiltonian provide distributions which result in non linear space charge forces of the beam and do not have elliptical symmetry. For example distribution

$$f = f_o \quad H \leqslant H_o, \qquad f = 0 \quad H > H_o \qquad (2)$$

yield phase space projections which are more close to rectangle than ellipse [2]

$$\frac{p_x^2}{p^2} + \frac{I_o(\frac{x}{R})}{I_o(\frac{R}{R_o})} = 1 \qquad (3)$$

where $I_o(\xi)$ is a modified Bessel function. It is interesting to verify whether it is possible to match the beam with more realistic distribution function with focusing channel. Instead of finding self-consistent distributions in linear focusing channel one can try to adjust external potential in such a way that it will maintain given beam distribution. As a sequence, higher order terms in external potential function appear. So the problem of beam matching with arbitrary distribution function can be solved if focusing field is not linear anymore.

Matched Beam with Arbitrary Distribution Function

Consider the beam of particles with charge q and mass m which propagates in uniform focusing channel with longitudinal velocity $v=\beta c$. We assume that the beam is matched with the channel therefore Hamiltonian

$$H = \frac{p_x^2 + p_y^2}{2m} + q\,U(x,y) \qquad (4)$$

is a constant of motion. Total potential U is a sum of external potential of focusing structure U_{ext} and self potential of the beam U_b

$$U = U_{ext} + U_b\ . \qquad (5)$$

Distribution function of the beam

$$f(x,p_x,y,p_y) = \frac{dN(x,p_x,y,p_y)}{dx\,dp_x\,dy\,dp_y} \qquad (6)$$

in general case will result in nonlinear space charge density distribution which produces nonlinear space charge forces. The purpose of this study is to find external potential (in general case nonlinear as well) which maintains the initial beam distribution and hence conserves the beam emittance.

Vlasov equation for time-independent distribution function is given by

$$\frac{df}{dt} = \frac{\partial f}{\partial x}v_x + \frac{\partial f}{\partial y}v_y - q\left(\frac{\partial f}{\partial p_x}\frac{\partial U}{\partial x} + \frac{\partial f}{\partial p_y}\frac{\partial U}{\partial y}\right) = 0 \qquad (7)$$

where the partial derivative of distribution function over time t is omitted. We assume that the distribution function is given and therefore self potential of the beam is known from Poisson equation:

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$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_b}{\partial r}\right) = -\frac{\rho(r)}{\varepsilon_o} \qquad (8)$$

After calculation of the self potential of the beam and substitution of the distribution function in Vlasov equation one can find the total potential of the structure U(x,y). Subtraction of the self potential of the beam from total potential

$$U_{ext} = U - U_b \qquad (9)$$

gives the expression for external potential of the structure which is required to keep the initial distribution of the beam and therefore to conserve beam emittance.

Matched Gaussian and "Water Bag" Distributions

Let us consider Gaussian distribution function

$$f = f_0 \exp\left(-2\frac{x^2+y^2}{R^2} - 2\frac{p_x^2+p_y^2}{p_0^2}\right). \qquad (10)$$

This distribution gives elliptical projections at phase planes (x,p_x), (y,p_y) with normalized RMS beam emittance $V = \pi R p_0/mc$ and is characterized by nonlinear self field. Substituting the distribution function into Vlasov equation (7) provides equation for the total potential of the structure:

$$\frac{mc^2}{q}(x p_x + y p_y) = \frac{R^4}{V^2}\left[p_x\frac{\partial U}{\partial x} + p_y\frac{\partial U}{\partial y}\right]. \qquad (11)$$

It is easy to find that potential

$$U(x,y) = \frac{mc^2}{q}\frac{V^2}{R^4}\left(\frac{x^2+y^2}{2}\right) \qquad (12)$$

satisfies eq. (11). Self potential of the beam is obtained from Poisson equation which gives

$$E_b = -\frac{\partial U_b}{\partial r} = \frac{I}{2\pi\varepsilon_o\beta c}\frac{1}{r}\left[1-\exp\left(-2\frac{r^2}{R^2}\right)\right] \qquad (13)$$

where $r^2 = x^2 + y^2$ is a square of beam radius. Subtraction of the self field of the beam from external field provides expression for external field :

$$E_{ext} = -\frac{mc^2}{qR}\left[\frac{V^2 r}{R^3} + 2\frac{I}{I_c\beta}\frac{R}{r}\left(1-\exp\left(-2\frac{r^2}{R^2}\right)\right)\right] \qquad (14)$$

or for external potential

$$U_{ext}(r) = \frac{mc^2}{q}\left[\left(\frac{V^2}{2R^4} + \frac{2I}{I_c\beta R^2}\right)r^2 + \frac{2I}{I_c\beta}\left(-\frac{r^4}{2R^4} + \frac{2}{9}\frac{r^6}{R^6} + \ldots + \frac{(-1)^{k+1}2^k r^{2k}}{2k\,k!\,R^{2k}}\right)\right] \qquad (15)$$

where $I_c = A/Z\cdot 3.13\cdot 10^7$ Amp is a characteristic value of beam current. External potential of the structure consists of two parts: quadratic (which produces linear focusing) and higher order terms which describes nonlinear focusing. Linear part depends on the values of beam emittance and beam current while nonlinear part depends on beam current only. It means that external field has to compensate nonlinearities of self field of the beam and produce required linear focusing of the beam which should keep elliptical beam phase space distribution.

Analogous result can be obtained for the beam with "water bag" distribution with the same RMS beam parameters defined by :

$$f = f_0, \quad \frac{2}{3}\left(\frac{x^2+y^2}{R^2} + \frac{p_x^2+p_y^2}{p_0^2}\right) \leqslant 1$$

$$f = 0, \quad \frac{2}{3}\left(\frac{x^2+y^2}{R^2} + \frac{p_x^2+p_y^2}{p_0^2}\right) > 1 . \qquad (16)$$

Total potential for this case is given by the same expression (12) as for Gaussian beam which gives the following external potential of the structure to keep initial beam distribution

$$U_{ext}(r) = \frac{mc^2}{q}\left[\frac{V^2}{2R^4}r^2 + \frac{4Ir^2}{3I_c\beta R^2}\left(1-\frac{r^2}{6R^2}\right)\right]. \qquad (17)$$

Numerical Example

Beam dynamics simulation in nonlinear focusing channel was executed using general-purpose particle-in-cell code BEAMPATH [3]. Beam of particles was represented as a collection of 10000 trajectories. Equations of motion were integrated using combination of "leap-frog" method for acceleration in electric field and implicit method of second order for particle rotation in magnetic field. Initial particle distributions were produced by random number generator [4] which creates different distributions with elliptical symmetry in 4D phase space. The space charge problem for z-uniform field was solved in rectangular domain with the Dirichlet boundary conditions on the surface of infinite pipe. The rectangular region was divided into uniform meshes of dimensions 256 x 256. Charge of every particle was area-weighted among the four neighboring points. Poisson's equation was solved by Fast Fourier Transformation (FFT) of space charge distribution and FFT synthesis of beam potential.

At fig.1,2 the results of beam dynamics study of Gaussian beam in linear and nonlinear focusing channel are presented. The matched conditions for linear focusing channel were chosen as for equivalent KV beam with the same root-mean-square sizes.

From results of simulations it is seen that mismatched beam in linear focusing channel is characterized by 50% emittance growth accompanied by halo formation while matched beam in nonlinear channel does not have any emittance growth.

Discussion

Potential of external focusing field obtained from the above consideration produces linear focusing field near axis which drops non linearly with deviation from the axis of the structure. Axial-symmetric electrostatic and magnetostatic lenses have aberrations which increase the focusing with radius[5]. Higher order multipole lenses (sextupoles, octupoles, etc.) have azimuth variation of potential $U_{ext} = U_o r^n \cos n\theta$ which increases focusing in one direction and decreases it after azimuth angle shift $\Delta\theta=\pi/n$. One of the idealized way to produce required potential is to introduce inside the transport channel the opposite charged cloud of heavy particles. For example to focus the beam with Gaussian distribution (10) space charge distribution of focusing cloud should be as follow

$$\rho_{ext} = \frac{I_c}{2\pi c R^2}\left[\frac{V^2}{R^2} + \frac{4I}{\beta I_c}\exp\left(-2\frac{r^2}{R^2}\right)\right]. \quad (18)$$

The other ways to create required nonlinear external potential will be examined later.

Conclusions

Matched conditions for the beam with elliptical phase space projections but nonlinear space charge forces in uniform focusing channel impose the focusing field to include nonlinear terms of higher order than quadratic. The focusing field produces linear focusing near axis of the structure which has to drop non-linearly away from the axis. External potential of focusing field to keep the beam emittance with nonlinear space charge distribution is given. Different examples like Gaussian and "water bag" distribution in 4D phase space are considered. Results of particle-in-cell simulation confirms the conservation of beam emittance in nonlinear external field.

References

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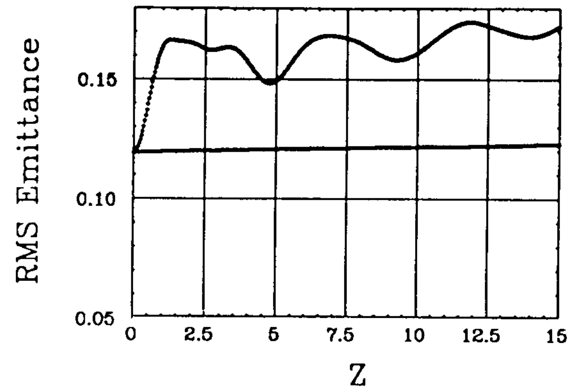

Fig. 1. Emittance growth of the Gaussian beam in linear focusing channel (upper curve) and emittance conservation in nonlinear focusing channel (lower curve).

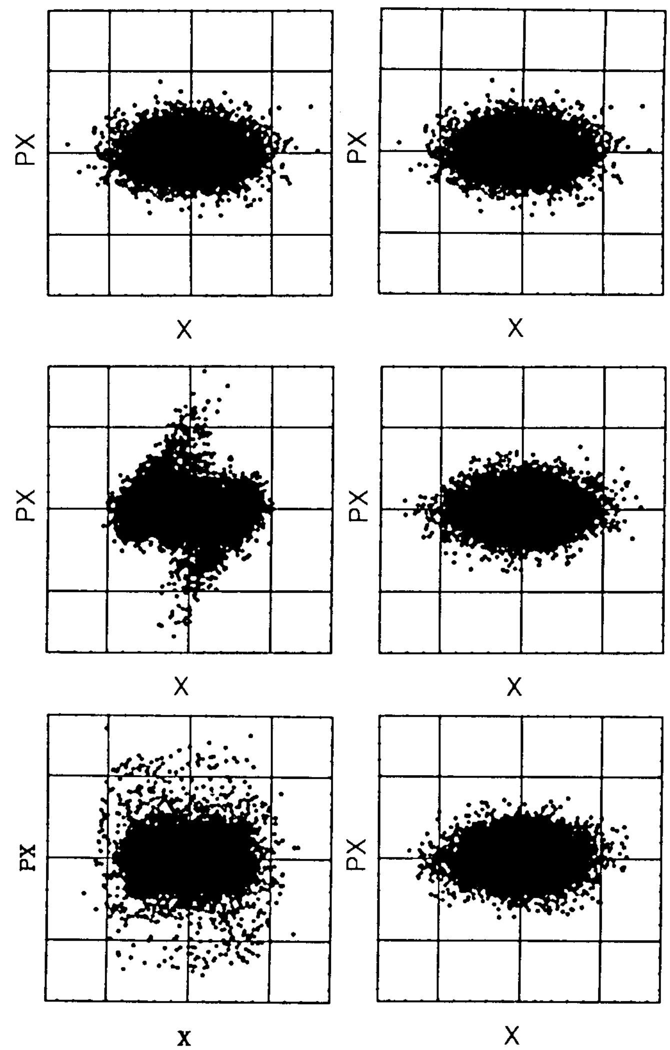

Fig. 2. Mismatching of Gaussian beam in linear focusing channel (left column) and matching of the same beam with nonlinear focusing channel (right column)